

Probability, Permutation, Combination
REVIEW
Algebra 2

Name Key
Block _____ Date _____

1. Given the three letters W, H, and O:

- a. List all the permutations of these three letters using two at a time.

WH HW OW

$\boxed{6}$

- b. List all the combinations of these three letters using two at a time.

WO HO OH

WH
WO OH

$\boxed{3}$

2. In a class of 25 students how many ways can three students be picked, 1 to be a team leader, 1 to be a recorder, and 1 to be a reporter?

$$25P_3 = \boxed{13,800}$$

3. There is a variety pack of a dozen different donuts sitting on the kitchen table. Stanley picks 4 donuts to take to work. How many ways can he do this?

$$12C_4 = \boxed{495}$$

4. Dr. Zweistein has 14 students in a physics class. Over the course of the year, Dr. Zweistein would like to arrange the lab groups so that every student has the opportunity to work with every other student in groups of two. How many different two-person lab groups are there?

$$14C_2 = \boxed{91}$$

5. The Rails Club, a group of 25 train fanatics, is to choose four of their members to be on the Board of Directors.

- a. How many different possible boards could the Rails choose?

$$25C_4 = \boxed{12,650}$$

- b. How many different possible boards could the Rails choose if there are to be a Chair, Vice Chair, Treasurer, and Secretary?

$$25P_4 = \boxed{303,600}$$

6. At a burger specialty restaurant, the toppings options are catsup, mayonnaise, mustard, tomatoes, onions, lettuce, mushrooms, Swiss cheese, cheddar cheese, steak sauce, and guacamole. If you can choose between a 4-oz burger or a 6-oz burger and want a mix of 4 toppings, how many different burgers can you order?

$$2(11C_4) = 2(330) = \boxed{660 \text{ ways}}$$

order doesn't matter: 5 winning, 40 losing

7. For a particular lottery, the winning numbers are selected by a machine that randomly chooses 5 table-tennis balls from among 45, numbered 1 to 45. The lottery pays off if you match 5, 4, or 3 of the numbers.

a. What is the probability that you will match all 5 of the winning numbers?

$$\frac{5C_5}{45C_5} = \frac{1}{45C_5} = \frac{1}{1,221,759}$$

b. How many different winning number combinations are there? winning vs losing

$$5C_5 + 5C_4 \cdot 40C_1 + 5C_3 \cdot 40C_2 = 8001$$

c. What is the probability that you will match exactly 4 of the 5 winning numbers?

$$\frac{5C_4 \cdot 40C_1}{45C_5} = \frac{200}{1,221,759}$$

8. Two fair coins are flipped.

a. List all the possible outcomes of this experiment.

{ HT HH }
{ TH TT }

b. What is the probability that one coin lands heads and the other one lands tails?

$\frac{1}{2}$ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~ ~~1/2~~

c. What is the probability that both coins land tails?

$\frac{1}{4}$

d. What is the probability that both coins land heads?

$\frac{1}{4}$

9. In any given situation, when all the possible choices are considered, what do all the probabilities have to add up to? Why?

1 → makes up all possible outcomes

10. A class has 24 students, 14 girls and 10 boys. Three students are randomly selected to represent the class at a teachers' meeting.

a. What is the probability that there are 2 girls and 1 boy selected?

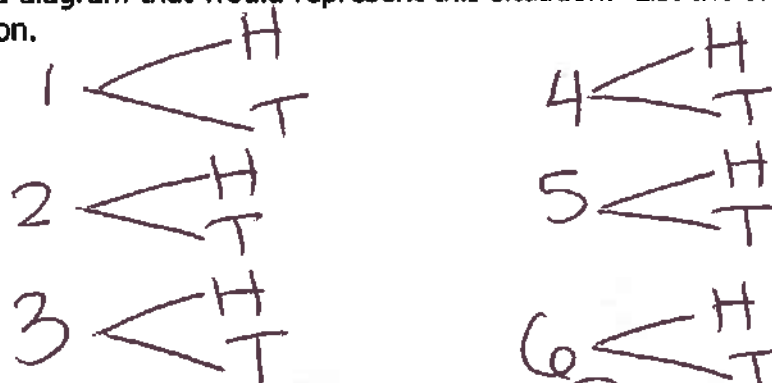
$$\frac{14C_2 \cdot 10C_1}{24C_3} = \frac{45}{1012} \quad \text{or} \quad \left(\frac{14}{24} \cdot \frac{13}{23} \cdot \frac{10}{22} \right) \times 3$$

b. What is the probability that all three students selected are boys?

$$\frac{10C_3}{24C_3} = \frac{15}{253} \quad \text{or} \quad \frac{10}{24} \cdot \frac{9}{23} \cdot \frac{8}{22}$$

11. Suppose you roll a die then flip a coin.

- a. Draw a diagram that would represent this situation. List the sample space for this situation.



- b. What is the probability of rolling a 5 on the die and getting heads on the coin?

$$\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

- c. What is the probability of rolling a 5 on the die or getting heads on the coin?

$$\frac{1}{6} + \frac{1}{2} - \frac{1}{12} = \frac{8}{12} - \frac{1}{12} = \frac{7}{12}$$

- d. What is the probability of getting tails on the coin and either a 3 or a 4 on the die?

$$\frac{1}{2} \cdot \left(\frac{1}{6} + \frac{1}{6} \right) = \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{6}$$

- e. What is the probability of getting a 5 on the die and a 6 on the die?

0

12. Find the number of 5-card hands that are possible if you want...

- a. 5 face cards

$$12C_5 = 792$$

- b. 1 ace and 4 cards that are not aces

$$4C_1 \cdot 48C_4 = 778,320$$

- c. at most 1 queen

$$4C_1 \cdot 48C_4 + 48C_5 = 2,490,624$$

- d. 4 kings and 1 other card

$$48$$

- e. 5 hearts or 5 diamonds

$$13C_5 + 13C_5 \text{ or } 2(13C_5) = 2,574$$

- f. at least 1 spade

$$13C_1 \cdot 39C_4 + 13C_2 \cdot 39C_3 + 13C_3 \cdot 39C_2 + 13C_4 \cdot 39C_1 + 13C_5$$

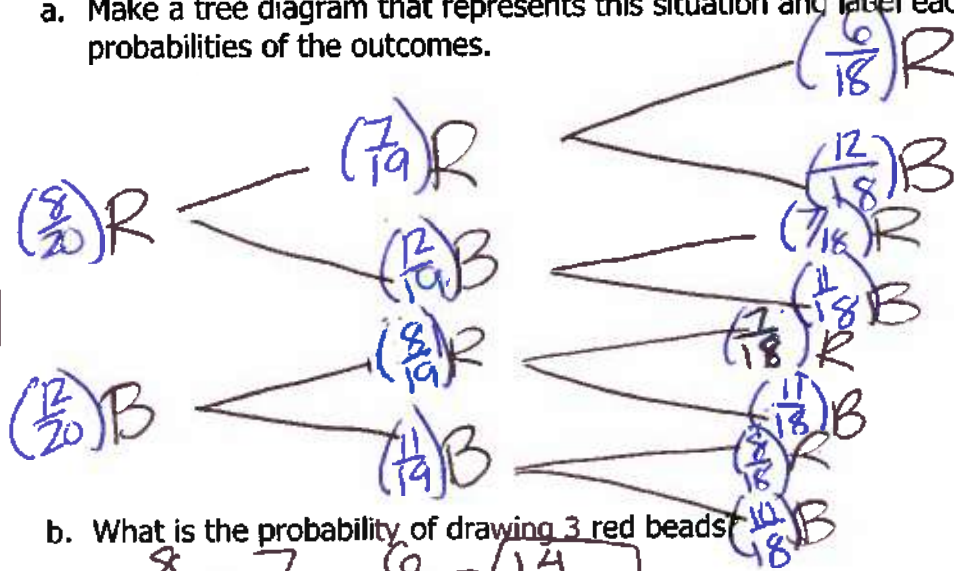
OR

$$52C_5 - 39C_5 \leftarrow \text{not spades} = 2,023,203$$

Assume order matters

13. Assume you have 8 red beads and 12 blue beads in a bag and you will be drawing 3 beads from the bag one at a time without replacement.

- a. Make a tree diagram that represents this situation and label each branch with the probabilities of the outcomes.



- b. What is the probability of drawing 3 red beads?

$$\frac{8}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} = \frac{14}{285}$$

- c. What is the probability of drawing 2 red and 1 blue?

$$\frac{8}{20} \cdot \frac{7}{19} \cdot \frac{12}{18} = \frac{28}{285}$$

- d. What is the probability of exactly 2 being the same color?

$$\frac{(8^2 \cdot 12 + 12^2 \cdot 8)}{(20^3)} = \frac{72}{95}$$

- e. What is the probability of drawing two red given the fact that your first bead drawn was blue?

$$P(2R | 1B) = \frac{8}{19} \cdot \frac{7}{18} = \frac{28}{171}$$

14. A bag contains 20 marbles, 8 white, 10 blue, and 2 red.

- a. If one marble is pulled from the bag, what is the probability that it is white?

$$\frac{8}{20} = \frac{2}{5}$$

- b. If one marble is pulled from the bag, what is the probability that it is red?

$$\frac{2}{20} = \frac{1}{10}$$

- c. If two marbles are pulled from the bag (with replacement), what is the probability that both marbles are blue?

$$\frac{10}{20} \cdot \frac{10}{20} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

- d. If two marbles are pulled from the bag (without replacement), what is the probability that both marbles are blue?

$$\frac{10}{20} \cdot \frac{9}{19} = \frac{9}{38}$$

"& then" is a key