ALGEBRA 2

UNIT 3

SYSTEMS OF EQUATIONS

Unit Essential Questions:

- How does representing functions graphically help you solve a system of equations?
- How does writing equivalent equations help you solve a system of equations?

SECTION 3.1: SOLVING SYSTEMS USING GRAPHS

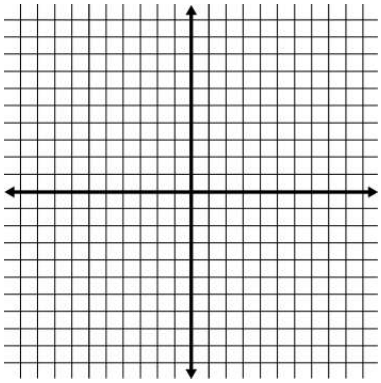
MACC.912.A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

RATING	LEARNING SCALE
4	I am able to • solve systems of equations by graphing in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • solve systems of equations by graphing
2	I am able to • solve systems of equations by graphing with help
1	I am able to • understand the definition of a system of equations

WARM UP

Graph each equation. Use one coordinate plane for all 3 graphs.

$y = 2x - 2$ $\quad$ $y = -x$ $\quad$ $y = -\frac{1}{2}x + 4$



KEY CONCEPTS AND VOCABULARY

_____- a set of two or more equations that use the same variables.

_____- when the graph of each equation of a system is a line.

_____ – a set of values for the variables that makes all the equations true.

EXAMPLES

EXAMPLE 1: CHECKING SOLUTIONS TO A SYSTEM

Check whether the ordered pair is a solution of a system.

a) $\left(1, \frac{1}{2}\right)$

$3x + 4y = 5$

$-4x + 6y = -1$

b) $\left(-\frac{1}{3}, \frac{3}{8}\right)$

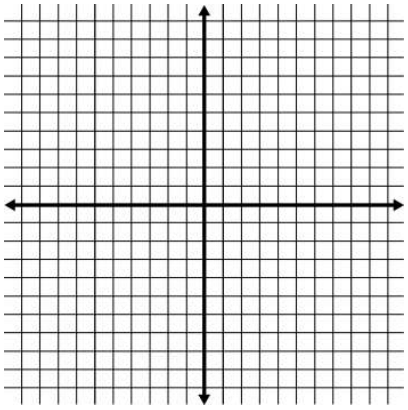
$3x + 8y = 2$

$9x - 16y = -9$

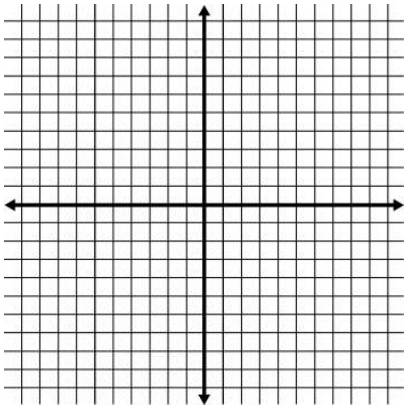
EXAMPLE 2: SOLVING SYSTEMS BY GRAPHING

Solve the system by graphing.

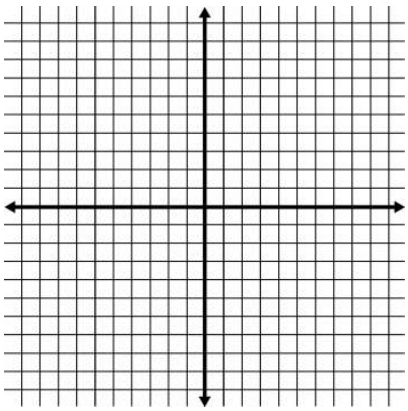
a) $\begin{cases} y = x \\ y = 2x + 2 \end{cases}$



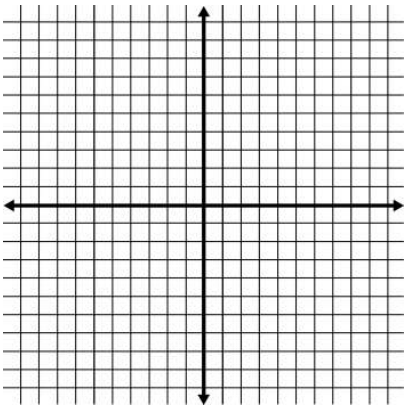
b) $\begin{cases} -3x + 2y = 8 \\ x + 4y = -12 \end{cases}$



c) $\begin{cases} x + y = 4 \\ y = -x + 1 \end{cases}$



d) $\begin{cases} y = 3x + 7 \\ -2y + 6x = -14 \end{cases}$

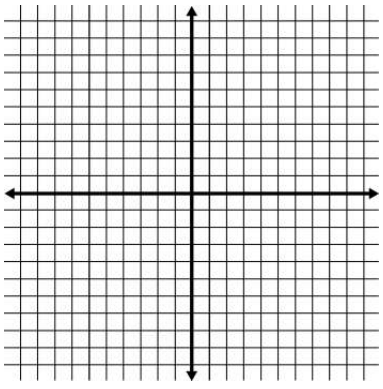


EXAMPLE 3: WRITING AND SOLVING SYSTEMS FOR REAL WORLD SITUATIONS

Mulan and Lilo are competing to see who can sell the most candy bars for a fundraiser. Mulan sold 4 candy bars on the first day and 2 each day after that. Lilo sold 7 on the first day and 1 each day after that.

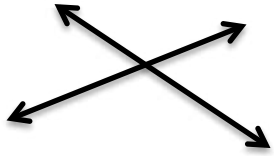
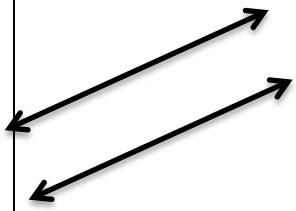
a) Write an equation for the number of candy bars each person sold.

b) Graph each equation



c) Solve the system. Interpret your solution

KEY CONCEPTS AND VOCABULARY

Graph	Number of Solutions	Name of System	Slopes are… y-intercepts are…
			
			
			

EXAMPLES

EXAMPLE 4: CLASSIFYING SYSTEMS OF EQUATIONS

Classify the system without graphing.

a) $\begin{cases} y = 3x + 2 \\ -6x + 2y = 4 \end{cases}$

b) $\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 3.2: SOLVING SYSTEMS ALGEBRAICALLY

MACC.912.A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

MACC.912.A-REI.C.5: Prove that, given a system with two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

RATING	LEARNING SCALE
4	I am able to • solve systems of equations algebraically in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • solve systems of equations using substitution and elimination
2	I am able to • solve systems of equations using substitution and elimination with help
1	I am able to • understand the definition of systems of equations

WARM UP

Evaluate for $a = 2$ and $b = -6$

1) $2(7a - b)$ 2) $a + b$ 3) b^2

KEY CONCEPTS AND VOCABULARY

_____ – means to plug in or replace a variable with an expression.

Steps For Solving Systems Using Substitution:

- Solve for one variable in one equation
- Substitute that variable into the other equation.
- Solve for a variable
- Substitute the answer to solve for the other variable

EXAMPLES

EXAMPLE 1: SOLVING SYSTEMS BY SUBSTITUTION

Solve the system by substitution.

a) $\begin{cases} y = x \\ y = -x + 2 \end{cases}$

b) $\begin{cases} x + 3y = 5 \\ -2x + 4y = 0 \end{cases}$

Solve the system by substitution.

c) $\begin{cases} 3(x+y)-5=x-6 \\ -4x+y=3-3x \end{cases}$

EXAMPLE 2: WRITING AND SOLVING SYSTEMS FOR REAL WORLD SITUATIONS

A high school band program sold a total of 350 tickets for a jazz concert. Student tickets were $5 each and general admission tickets were $8 each. If the band boosters collected $2050, how many of each ticket did they sell?

EXAMPLE 3: WRITE AND SOLVE A SYSTEM OF EQUATIONS

The sum of two angles is 180°. The measure of one angle is 34° greater than the measure of the other angle.

a) Define the variables and write equations to model the situation.

b) Find the measure of each angle.

KEY CONCEPTS AND VOCABULARY

_____- using the Addition Property of Equality, or use additive inverses to cancel a variable.

Steps For Solving Systems Using Elimination:

- Put each equation in standard form
- Modify (if needed) one or both equations so that a variable will be eliminated
- Add equations
- Solve for a variable
- Substitute the variable back into an equation to solve for the other variable

EXAMPLES

EXAMPLE 4: SOLVING SYSTEMS USING ELIMINATION

Use the elimination method to solve the system.

a) $\begin{cases} 3x + y = -9 \\ -3x - 2y = 12 \end{cases}$

b) $\begin{cases} 2m + 4n = -4 \\ 3m + 5n = -3 \end{cases}$

c) $\begin{cases} -3x + 5y = 6 \\ 6x - 10y = 0 \end{cases}$

EXAMPLE 5: WRITING AND SOLVING SYSTEMS FOR REAL WORLD SITUATIONS

You have a coin jar with quarters and dimes. You choose 10 coins that are worth $2.05. How many of each type do you have?

EXAMPLE 6: SOLVING A SYSTEM IN A CHALLENGING PROBLEM

Find a and b so that the system below has the unique solution (-2, 3).

$$\begin{cases} ax + by = 17 \\ 2ax - by = -11 \end{cases}$$

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 3.3: SYSTEMS OF INEQUALITIES

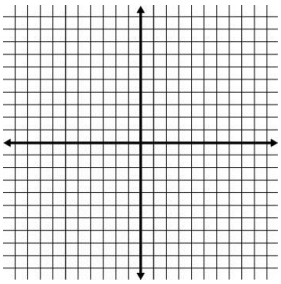
MACC.912.A-REI.D.12: Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

RATING	LEARNING SCALE
4	I am able to • graph linear inequalities in two variables in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • graph linear inequalities in two variables
2	I am able to • graph linear inequalities in two variables with help
1	I am able to • understand how to graph a boundary line of a linear inequality

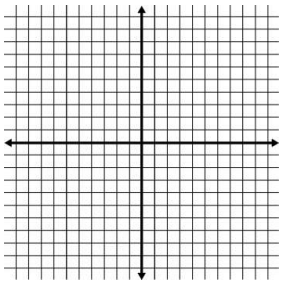
WARM UP

Graph each inequality.

1) $y \geq 2x + 5$



2) $9x + 3y \geq -6$



KEY CONCEPTS AND VOCABULARY

You can solve a system of linear inequalities by _____.

Steps To Solving Systems Of Inequalities By Graphing:

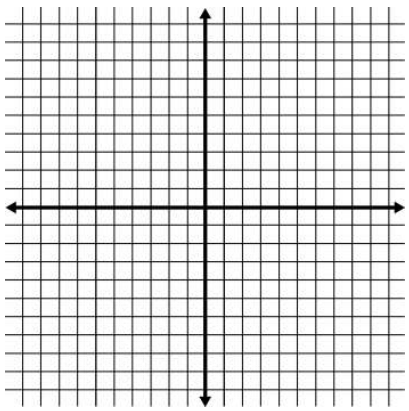
- First graph boundary lines and decide whether the boundary lines are included.
- Then decide which side of the boundary lines to shade.
  - If $<$ or $>$, boundary not included; use dashed line.
  - If $\leq$ or $\geq$, boundary included; use solid line.
- The solutions are any point where the shaded regions overlap.

EXAMPLES

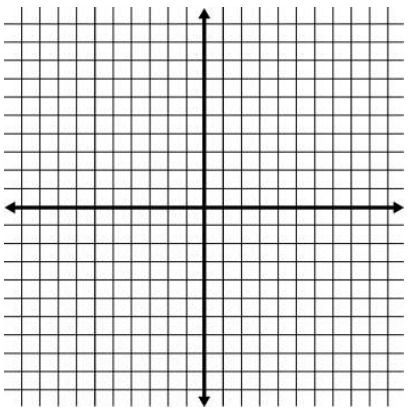
EXAMPLE 1: SOLVING SYSTEMS OF INEQUALITIES BY GRAPHING

Solve the system of inequalities by graphing.

a) $\begin{cases} y \geq 3 \\ y > 2x + 1 \end{cases}$

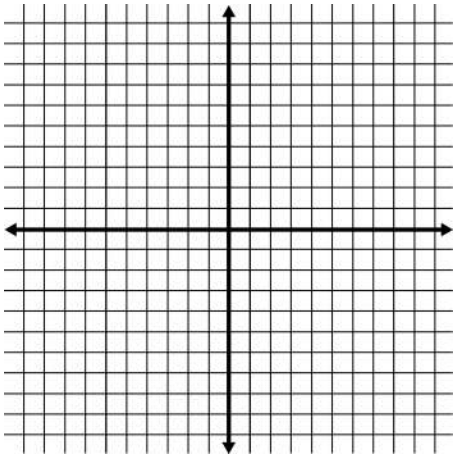


b) $\begin{cases} -x + y < -1 \\ x + y < 3 \end{cases}$



Solve the system of inequalities by graphing.

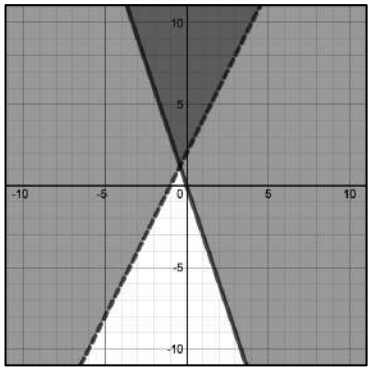
c) $\begin{cases} y \geq 3x - 4 \\ y \leq -\frac{1}{2}x + 3 \\ x > -2 \end{cases}$



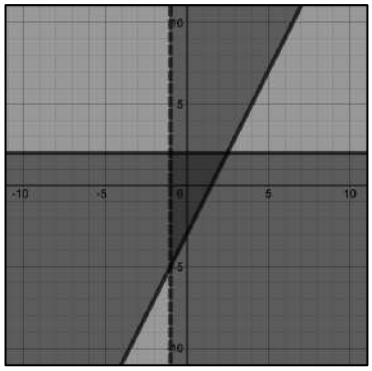
EXAMPLE 2: WRITING A SYSTEM OF EQUATIONS

Write a system of inequalities for graphs.

a)

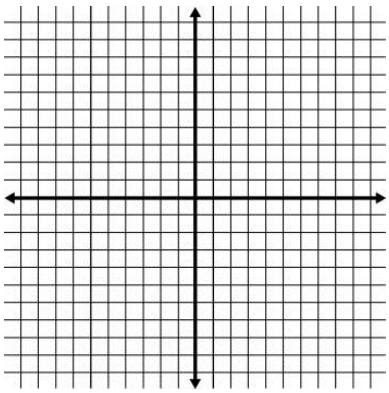


b)



EXAMPLE 3: SOLVING SYSTEMS OF INEQUALITIES FOR REAL WORLD SITUATIONS

You are working on time management skills for juggling after school activities and homework obligations. You cannot spend more than 5 hours doing homework and playing baseball. While planning, you decided that you should spend no more than 2 hours playing baseball and you need to spend at least 1 hour on homework. Create a graph to represent how you can spend your time.



RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

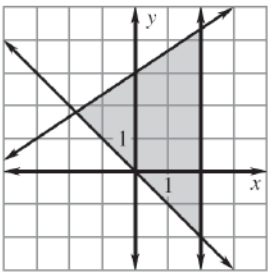
SECTION 3.4: LINEAR PROGRAMMING

MACC.912.A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

RATING	LEARNING SCALE
4	I am able to • solve problems using linear programming in real-world scenarios or more challenging problems that I have never previously attempted
TARGET 3	I am able to • solve problems using linear programming
2	I am able to • solve problems using linear programming with help
1	I am able to • understand that linear programming involves multiple linear inequalities

WARM UP

Write a system of inequalities for the shaded region shown in the graph.



KEY CONCEPTS AND VOCABULARY

_____– means finding the maximum or minimum value of some quantity

_____ **–** is the process of optimizing a linear **objective function** subject to a system of linear inequalities called **constraints**

_____ - The graph of the system of constraints

Find Values That Minimize Or Maximize An Object Function:

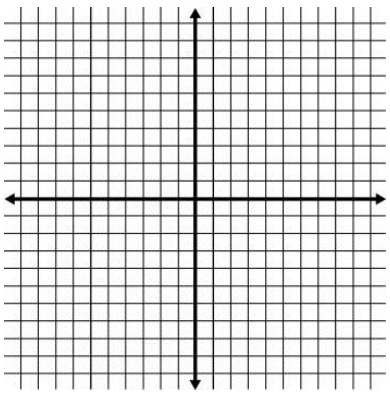
- Graph each inequality
- Find the vertices
- Substitute vertex value into objective function

EXAMPLES

EXAMPLE 1: USING LINEAR PROGRAMMING TO FIND THE MAXIMUM AND MINIMUM VALUE

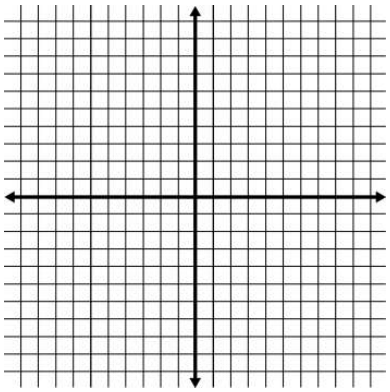
a) Find the minimum value and the maximum value of $C = -x + 3y$ subject to the following constraints:

$x \geq 0$
$y \geq 0$
$x + y \leq 8$



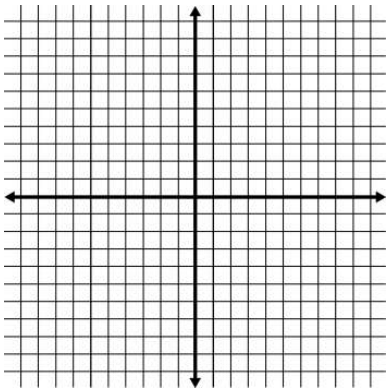
b) Find the minimum value and the maximum value of $C = x + 5y$ subject to the following constraints:

$x \geq 0$
$5 \geq x + y$
$y \leq 2x + 2$



c) Find the minimum value and the maximum value of $C = 2x - y$ subject to the following constraints:

$x \geq 0$
$y \geq x + 2$
$y \leq -x + 6$



EXAMPLE 2: USING LINEAR PROGRAMMING TO MAXIMIZE PROFIT

Suppose a lumber mill can turn out 600 units of product each week. To meet the needs of its regular customers, the mill must produce 150 units of lumber and 225 units of plywood. If the profit for each unit of lumber is $30 and the profit for each unit of plywood is $45, how many units of each type of wood product should the mill produce to maximize profit?

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 3.5: SYSTEMS WITH THREE VARIABLES

EXTENDS MACC.912.A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

RATING	LEARNING SCALE
4	I am able to • solve systems in three variables in real-world scenarios or more challenging problems that I have never previously attempted
TARGET 3	I am able to • solve systems in three variables
2	I am able to • solve systems in three variables with help
1	I am able to • understand that a system with three variables has three equations

WARM UP

Find the x-intercepts and y-intercepts of the graph of each equation.

1) $y = 2x + 6$ 2) $2x + 9y = 36$ 3) $y = x - 1$

KEY CONCEPTS AND VOCABULARY

Steps for Solving Systems with Three Variables:

- Write each equation in **standard form**.
- Choose a **PAIR** of equations and eliminate one of the variables.
- Choose a different pair of equations and eliminate the **SAME** variable.
- Use substitution or elimination method to solve for one variable.
- Substitute the value you found in step 4 into one of the equations from step 4 and solve for the other variable.
- Then substitute the two answers you found into the one of the original equations and solve for the remaining variable.
- Put answer into **ordered triple**. (x, y, z).

EXAMPLES

EXAMPLE 1: SOLVING A SYSTEM WITH THREE VARIABLES

Solve the system.

a) $\begin{cases} 2x - y + z = 4 \\ x + 3y - z = 11 \\ 4x + y - z = 14 \end{cases}$

Solve the system.

b) $\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$

EXAMPLE 2: SOLVING A SYSTEM WITH THREE VARIABLES IN A REAL WORLD SITUATION

In 1998, Cynthia Cooper of the WNBA Houston Comets basketball team scored 680 points for the season. She hit 413 of her 1-point, 2-point and 3-point attempts. She made 40% of her 160 3-point field goal attempts. How many 1-, 2-, and 3-point baskets did she complete? Set up a system of equations and then solve.

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1