ALGEBRA 2

UNIT 1

EXPRESSIONS, EQUATIONS, AND INEQUALITIES

ANSWER KEY

<u>Unit Essential Questions:</u>

- How do variables help you model real-world situations?
- How can you use properties of real numbers to simplify algebraic expressions?
- How do you solve an equation or inequality?

SECTION 1.1: PATTERNS AND EXPRESSIONS

<u>MACC.912.A-SSE.A.2</u>: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

RATING	LEARNING SCALE
4	I am able to • use variables to represent variable quantities in real world situations and in patterns or more challenging problems that I have never previously attempted
TARGET 3	I am able to • identify and describe patterns using diagrams, tables, words, numbers, and algebraic expressions
2	I am able to • identify and describe patterns using diagrams, tables, words, numbers, and algebraic expressions with help
1	I am able to • understand that I can use diagrams, tables, words, numbers, and algebraic expressions to identify patterns

WARM UP

1) $12+2(-6)$ 2) $6\frac{2}{5}+4\frac{3}{10}$ 3) $61-(-11)$ 4) $\frac{5}{3}-\frac{13}{4}$

0 107/10 72 -19/12

KEY CONCEPTS AND VOCABULARY

Variable - a symbol, usually a letter, that represents one or more numbers

Numerical Expression - a mathematical phrase that contains numbers and operation symbols

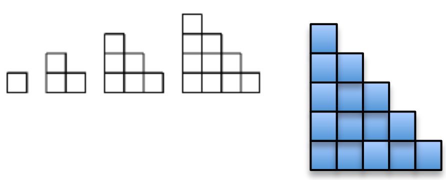
Algebraic Expression - a mathematical phrase that contains one or more variables

EXAMPLES

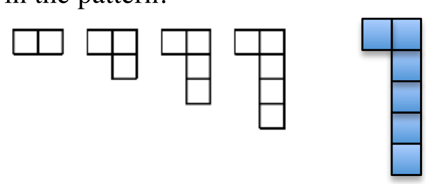
<u>EXAMPLE 1: RECOGNIZING PATTERNS GIVEN FIGURES</u>

Describe each pattern using words. Draw the next figure in the pattern.

a)



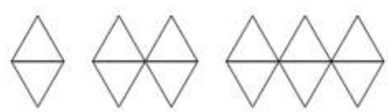
b)



a) The bottom row increases by one b) One square is added to the right bottom

<u>EXAMPLE 2: RECOGNIZING PATTERNS BY CREATING A TABLE</u>

These figures are made with toothpicks.



a) How many toothpicks are in the 20th figure? Use a table of values with a process column to justify your answer.

The 20th figure has 100 toothpicks.

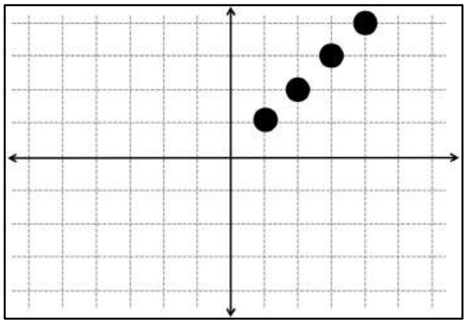
Figure	Process	Output
1	1 x 5	5
2	2 x 5	10
3	3 x 5	15
…	…	…
20	20 x 5	100

b) What expression describes the number of toothpicks in the *n*th figure? $5n$

EXAMPLE 3: RECOGNIZING PATTERNS GIVEN A GRAPH

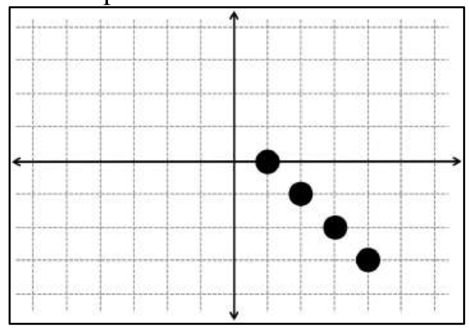
Identify a pattern by making a table of the inputs and outputs. Include a process column.

a)



Input	Process	Output
1	$1+0$	1
2	$2+0$	2
3	$3+0$	3
4	$4+0$	4

b)



Input	Process	Output
1	$-(1)+1$	0
2	$-(2)+1$	-1
3	$-(3)+1$	-2
4	$-(4)-5$	-3

EXAMPLE 4: RECOGNIZING A PATTERN IN A SEQUENCE OF INTEGERS

Identify a pattern and find the next three numbers in the pattern.

a) 2, 4, 8, 16, …

multiply by 2;
32, 64, 128

b) 4, 8, 12, 16, …

add 4;
20, 24, 28

c) 5, 25, 125, 625, …

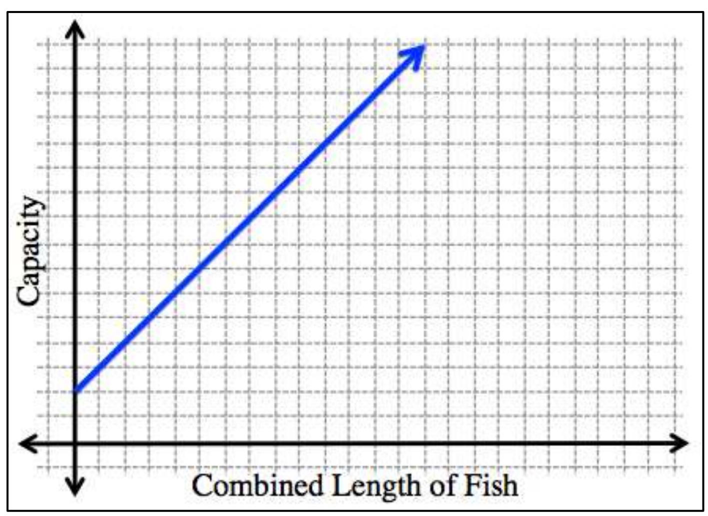
multiply by 5;
3,125, 15,625, 78,125

EXAMPLE 5: RECOGNIZING PATTERNS IN A REAL-WORLD SITUATION

You need to set up an aquarium and you need to decide what size tank to buy. The graph shows tank sizes using a rule that relates the capacity of the tank to the combined lengths of fish it can hold.

If you want six 2-in platys, three 1-in guppies, and two 3-in loaches, what is the smallest capacity tank you can buy?

23-gallon tank



RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 1.2: PROPERTIES OF REAL NUMBERS

MACC.912.N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and the product of a nonzero rational number and an irrational number is irrational.

RATING	LEARNING SCALE
4	I am able to • use properties of real numbers to perform algebraic operations
TARGET 3	I am able to • identify properties of real numbers
2	I am able to • identify properties of real numbers with help
1	I am able to • understand that real numbers have several special subsets related in particular ways

WARM UP

Write each number as a percent.

1) 0.5   2) 0.25   3) $\frac{1}{3}$ 4) $1\frac{2}{5}$ 5) 1.72   6) 1.23

50%   25%   33.3%   140%   172%   123%

KEY CONCEPTS AND VOCABULARY

SUBSETS OF REAL NUMBERS		
NAME	DESCRIPTION	EXAMPLES
Natural Numbers	Counting Numbers	1, 2, 3, 4, ….
Whole Numbers	Counting Numbers with Zero	0, 1, 2, 3, 4, …
Integers	Positive and Negative Whole Numbers	…, -3, 2, -1, 0, 1, 2, 3, …
Rational Numbers	Numbers that can be written as a fractions (Terminating or repeating decimals)	1/2, 5, –0.25, $0.\overline{3}$
Irrational Numbers	Numbers that cannot be written as a fraction (non-repeating, never-ending decimals)	$\sqrt{2}, \pi, \sqrt{3}, 4\pi, \frac{\sqrt{10}}{2}$

EXAMPLES

EXAMPLE 1: IDENTIFYING SUBSETS OF REAL NUMBERS

Your math class is selling pies to raise money to go to a math competition. Which subset of real numbers best describes the number of pies p that your class sells?

Whole

EXAMPLE 2: CLASSIFYING NUMBERS INTO SUBSETS OF REAL NUMBERS

For each number, place a check in the column that the number belongs to. Remember the numbers may belong to more than one set.

#	Number	Real	Whole	Natural	Integer	Rational	Irrational
a)	-9	✔			✔	✔	
b)	4	✔	✔	✔	✔	✔	
c)	$\sqrt{81}$	✔	✔	✔	✔	✔	
d)	$\frac{2}{5}$	✔				✔	
e)	$\frac{\sqrt{10}}{2}$	✔					✔
f)	0	✔	✔		✔	✔	
g)	$\frac{-\sqrt{4}}{2}$	✔			✔	✔	
h)	$3\pi+1$	✔					✔

EXAMPLE 3: OPERATIONS OF REAL NUMBERS

Show each statement is false by providing a counterexample.

a) The difference of two natural numbers is a natural number.

Example: $5 - 10 = -5$

b) The product of two irrational numbers is irrational.

Example: $\sqrt{2}\cdot\sqrt{8}=\sqrt{16}=4$

c) The product of a rational number and an irrational number is rational.

Example: $2\cdot 5\pi = 10\pi$

d) The sum of a rational number and an irrational number is rational.

Example: $3+(\pi+2)=\pi+5$

KEY CONCEPTS AND VOCABULARY

COMMUTATIVE PROPERTY The order in which you add or multiply does not matter. For any numbers a and b, $a + b = b + a$ and $ab = ba$
ASSOCIATIVE PROPERTY The way three or more numbers are grouped when adding or multiplying does not matter. For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

ADDITIVE IDENTITY For any number *a*, the sum of *a* and 0 is *a* $a + 0 = 0 + a = a$ or $7 + 0 = 0 + 7 = 7$	**MULTIPLICATIVE IDENTITY** For any number a, the product of a and 1 is a. $a \cdot 1 = 1 \cdot a = a$ or $14 \cdot 1 = 1 \cdot 14 = 14$
ADDITIVE INVERSE A number and its opposite are additive inverses of each other, $a + (-a) = 0$ or $5 + (-5) = 0$	**MULTIPLICATIVE INVERSE (RECIPROCALS)** For every number, $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product is one. $\frac{a}{b} \cdot \frac{b}{a} = 1$ or $\frac{2}{3} \cdot \frac{3}{2} = 1$

MULTIPLICATIVE PROPERTY OF ZERO Anything times zero is zero. $a \cdot 0 = 0 \cdot a = 0$ or $2 \cdot 0 = 0 \cdot 2 = 0$

EXAMPLES

EXAMPLE 4: USING PROPERTIES TO FIND UNKNOWN QUANTITIES

Name the property then find the value of the unknown.

a) $n \text{ x } 12 = 0$

Multiplicative Property of Zero;
$n = 0$

b) $7 + (3 + z) = (7 + 3) + 4$

Associative Property;
$z = 4$

c) $0 + n = 8$

Additive Identity;
$n = 8$

EXAMPLE 5: PROVIDING A COUNTEREXAMPLE

Is the statement true or false? If it is false, give a counterexample.

a) For all real numbers, $a + b = ab$

False,
if a = 1 and b = 3; 1 + 3 = 4 but 1 x 3 = 3

b) For all real numbers, $a(1) = a$

True, Multiplicative Identity

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 1.3: ALGEBRAIC EXPRESSIONS

MACC.912.A-SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

RATING	LEARNING SCALE
4	I am able to • evaluate and simplify algebraic expressions and apply them to real world problems
TARGET 3	I am able to • evaluate algebraic expressions • simplify algebraic expressions
2	I am able to • evaluate algebraic expressions with help • simplify algebraic expressions with help
1	I am able to • understand that you can represent real world quantities and mathematical phrases using algebraic expressions

WARM UP

Use order of operations to simplify.

1) $3 \div 4 + 6 \div 4$

9/4

2) $5[(2 + 5) \div 3]$

35/3

3) $40 + 24 \div 8 - 2^2 - 1$

38

KEY CONCEPTS AND VOCABULARY

Term - an expression that is a number, a variable, or the product of a number and one or more variables

Coefficient - the numerical factor of a term

Constant Term - a term with no variable

Like Terms - the same variables raised to the same power

EXAMPLES

EXAMPLE 1: WRITING AN ALGEBRAIC EXPRESSION

Write an algebraic expression that models each word phrase.

a) six less than a number w

$w - 6$

b) the product of 11 and the difference of 4 and a number r

$11(4 - r)$

EXAMPLE 2: EVALUATING AN ALGEBRAIC EXPRESSION

Evaluate each expression for the given values of the variables.

a) $5(a+7)-3b;\ a=1$ and $b=9$

13

b) $x^2+\frac{y}{3}; x=\frac{1}{2}$ and $y=2$

$\frac{11}{12}$

EXAMPLE 3: WRITING AND SIMPLIFYING EXPRESSIONS

Use the expression *twice the sum of 4x and y increased by six times the difference of 2x and 3y*.

a) Write an algebraic expression for the verbal expression.

$2(4x + y) + 6(2x - 3y)$

b) Simplify the expression.

$20x - 16y$

EXAMPLE 4: WRITING EXPRESSIONS IN SIMPLEST FORM

Simplify the following.

a) $2(a - 7) + 3a + a$

$6a - 14$

b) $9y - 5 + 8 + 2y - 11y$

3

c) $3h^2 - 5h + 4 - 8h^2 - 12$

$-5h^2 - 5h - 8$

d) $2x^2 - \frac{3}{4}x + \frac{x}{4}$

$2x^2 - \frac{1}{2}x$

e) $9x^2y + 2xy^2 - 4xy^2 + x^2y$

$10x^2y - 2xy^2$

f) $-(d - 3d + 8) + 2(d + 6)$

$4d + 4$

EXAMPLE 5: WRITING AN ALGEBRAIC EXPRESSION IN REAL WORLD SITUATIONS

Write an algebraic expression to model the situation.

You fill your car with gasoline at a service station for $2.75 per gallon. You pay with a $50 bill. How much change will you receive if you buy *g* gallons of gasoline? How much change will you receive if you buy 14 gallons?

$\$50 - 2.75g$
$11.50

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 1.4: SOLVING EQUATIONS

<u>MACC.912.A-CED.A.1.</u>: Create equations and inequalities in one variable and use them to solve problems
<u>MACC.912. A-CED.A.4</u>: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

RATING	LEARNING SCALE
4	I am able to • solve real world problems by writing equations
TARGET 3	I am able to • solve equations • solve problems by writing equations
2	I am able to • solve equations with help • solve problems by writing equations with help
1	I am able to • understand that you can use properties of equality and inverse operations to solve equations

WARM UP

Simplify.

1) $4x+3x-4$

$7x-4$

2) $-\frac{p}{3}+\frac{q}{3}-\frac{2p}{3}-q$

$-p-\frac{2q}{3}$

3) $-2(4+b)+4(b-5)$

$2b-28$

KEY CONCEPTS AND VOCABULARY

PROPERTY	DEFINITION
Reflexive	a = a
Symmetric	If a = b, then b = a
Transitive	If a = b and b = c, then a = c
Substitution	If a = b, then you can replace a with b and vise versa
Addition/ Subtraction	If a = b, then a + c = b + c and a – c = b – c
Multiplication/ Division	If a = b and c = 0, then ac = bc and a/c = b/c

EXAMPLES

<u>EXAMPLE 1: SOLVING MULTI-STEP EQUATIONS</u>

Solve each equation. Check your answers.

a) $8z + 12 = 5z - 21$

$z = -11$

b) $7b - 6(11 - 2b) = 10$

$b = 4$

c) $10k - 7 = 2(13 - 5k)$

$k = 33/20$

KEY CONCEPTS AND VOCABULARY

Identity - an equation that is true for every value of the variable.

Literal Equation - an equation that uses at least 2 letters as variables. You can solve for any variable "in terms of" the other variables.

EXAMPLES

EXAMPLE 2: DETERMINING IF AN EQUATION IS ALWAYS, SOMETIMES, OR NEVER TRUE

Determine whether the equation is *sometimes, always,* or *never* true.

a) $3x - 5 = -2$

Sometimes

b) $2x - 3 = 5 + 2x$

Never

c) $6x - 3(2 + 2x) = -6$

Always

EXAMPLE 3: SOLVING A LITERAL EQUATION

Solve each formula for the indicated variable.

a) $\frac{3}{4}(y-2) = x$; for y

$y = \frac{4}{3}x + 2$

b) $\frac{y-7}{x} = x^2$; for y

$y = x^3 + 7$

c) $ax + bx - 3 = -4$; for x

$x = \frac{-1}{a+b}$

d) $V = \frac{1}{3}\pi r^2 h$; for h

$h = \frac{3V}{\pi r^2}$

KEY CONCEPTS AND VOCABULARY

STEPS FOR SOLVING PROBLEMS

- Read the entire problem and identify important information
- Set up an equation
- Solve
- Check the answer

EXAMPLES

EXAMPLE 4: WRITING AND SOLVING EQUATIONS IN REAL WORLD SITUATIONS

a) The first part of a play is 35 minutes longer than the second part. If the entire play is 155 minutes long, how long is the first part of the play? Write an equation to solve the problem.

Let x = first part of the play

$x + (x - 35) = 155$

95 minutes

b) A desktop computer now sells for 15% less than it did last year. The current price is \$425. What was the price of the computer last year?

\$500

c) Two buses leave Houston at the same time and travel in opposite directions. One bus averages 55 mi/h and the other bus averages 45 mi/h. When will they be 400 mi apart?

4 hours

d) The length of a rectangle is 3cm greater than its width. The perimeter is 24cm. What are the dimensions of the rectangle?

7.5cm by 4.5cm

e) Find four consecutive odd integers with a sum of 232.

55, 57, 59, 61

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 1.5: SOLVING INEQUALITIES

MACC.912.A-CED.A.1.: Create equations and inequalities in one variable and use them to solve problems

RATING	LEARNING SCALE
4	I am able to • solve real world problems by writing inequalities
TARGET 3	I am able to • solve and graph inequalities • write and solve compound inequalities
2	I am able to • solve and graph inequalities with help • write and solve compound inequalities with help
1	I am able to • understand that you can use properties of inequality and inverse operations to solve inequalities

WARM UP

You want to download some new songs on your MP3 player. Each song will use about 4 MB of space. You have 6.5 GB of 25 GB available on our MP3 player. At most, how many songs can you download? (1 GB = 1024 MB)

1664 songs

KEY CONCEPTS AND VOCABULARY

Inequalities can be expressed using interval notation.

[value 1, value 2] Brackets indicate the value is included

(value 1, value 2) Parenthesis indicate the value is NOT included

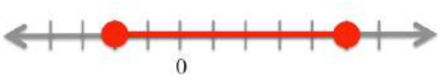
EXAMPLES

EXAMPLE 1: EXPRESSING AN INEQUALITY IN INTERVAL NOTATION

Graph and express in interval notation

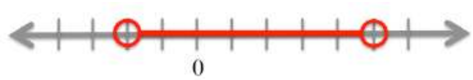
a) $-2 \leq x \leq 5$

[-2, 5]



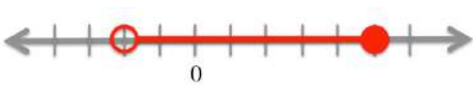
b) $-2 < x < 5$

(-2, 5)



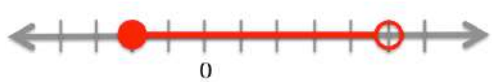
c) $-2 < x \leq 5$

(-2, 5]



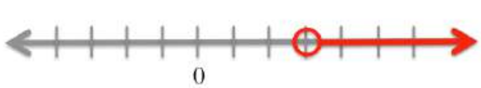
d) $-2 \leq x < 5$

[-2, 5)



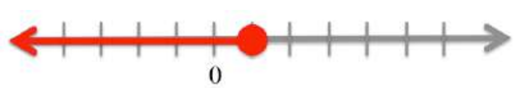
e) $x > 3$

(3,∞)



f) $x \leq 1$

(-∞,1]



KEY CONCEPTS AND VOCABULARY

WRITING AND GRAPHING INEQUALITIES			
SYMBOLS	WORDS	INTERVAL NOTATION	GRAPH
$x > 4$	x is greater than 4	$(4, \infty)$	 0
$x \geq 4$	x is greater than or equal to 4	$[4, \infty)$	 0
$x < 4$	x is less than 4	$(-\infty, 4)$	 0
$x \leq 4$	x is less than or equal to 4	$(-\infty, 4]$	 0

EXAMPLES

EXAMPLE 2: WRITING AN INEQUALITY

Write an inequality that represents the sentence.

a) The product of 12 and a number is less than 6.

$12x < 6$

b) The sum of a number and 2 is no less than the product of 9 and the same number.

$x + 2 \geq 9x$

EXAMPLE 3: SOLVING AN INEQUALITY

Solve each inequality. Write the solution in interval notation. Graph the solution.

a) $3x - 8 > 1$

$x > 3$
$(3, \infty)$

b) $3v \leq 5v + 18$

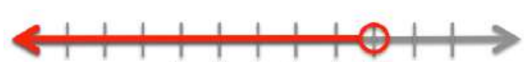
$v \geq -9$
$[-9, \infty)$

c) $7 - x \geq 24$

$x \leq -17$
$(-\infty, -17]$

d) $2(y - 3) + 7 < 21$

$y < 10$
$(-\infty, 10)$



EXAMPLE 4: DETERMINING IF AN INEQUALITY IS ALWAYS, SOMETIMES, OR NEVER TRUE

Is the inequality *always*, *sometimes*, or *never* true?

a) $-2(3x + 1) > -6x + 7$

Never

b) $5(2x - 3) - 7x \leq 3x + 8$

Always

c) $6(2x - 1) \geq 3x + 12$

Sometimes

KEY CONCEPTS AND VOCABULARY

Compound Inequalities - two inequalities joined with the word *and* or the word *or*

AND **means that a solution makes BOTH inequalities true.**
OR **means that a solution makes EITHER inequality true.**

EXAMPLES

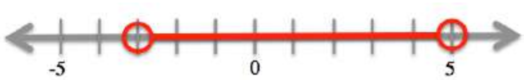
EXAMPLE 5: SOLVING COMPOUND INEQUALITIES

Solve each compound inequality. Write the solution in interval notation. Graph the solution.

a) $4r > -12$ and $2r < 10$

$r > -3$ and $r < 5$

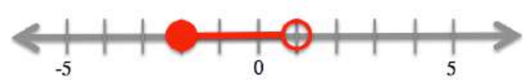
$(-3,5)$



b) $5z \geq -10$ and $3z < 3$

$z \geq -2$ and $z < 1$

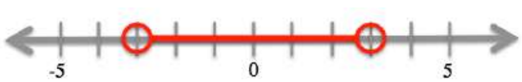
$[-2,1)$



c) $-2 < x + 1 < 4$

$-3 < x < 3$

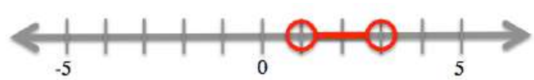
$(-3,3)$



d) $3 < 5x - 2 < 13$

$1 < x < 3$

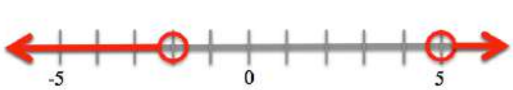
$(1,3)$



e) $3x < -6$ or $7x > 35$

$x < -2$ or $x > 5$

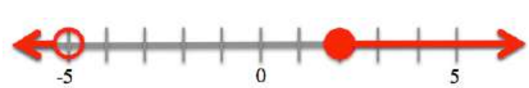
$(-\infty,-2) \cup (5,\infty)$



f) $5p \geq 10$ or $-2p > 10$

$p \geq 2$ or $p < -5$

$(-\infty,-5) \cup [2,\infty)$



RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 1.6: ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

MACC.912.A-CED.A.1.: Create equations and inequalities in one variable and use them to solve problems

RATING	LEARNING SCALE
4	I am able to • solve real world problems by writing equations and inequalities involving absolute value
TARGET 3	I am able to • write and solve equations involving absolute value • write and solve inequalities involving absolute value
2	I am able to • write and solve equations involving absolute value with help • write and solve inequalities involving absolute value with help
1	I am able to • understand that an absolute value quantity is nonnegative

WARM UP

Solve each equation.

1) $6x-6(10-x)=15$

$x = 25/4$

2) $12x-4=2(11+x)$

$x = 13/5$

KEY CONCEPTS AND VOCABULARY

Absolute Value - the distance from zero on the number line. Written |x|

Extraneous Solution - a solution derived from an original equation that is NOT a solution to the original equation.

STEPS TO SOLVE AN ABSOLUTE VALUE EQUATION

- Isolate the absolute value expression
- Write as two equations (set expression in the absolute value to the positive and negative - absolute value sign goes away)
- Solve for each equation
- Check for extraneous solutions

EXAMPLES

EXAMPLE 1: SOLVING ABSOLUTE VALUE EQUATIONS

Solve. Check your answers.

a) $3|x+2|-1=8$

$x=1, x=-5$

b) $|3x+2|=4x+5$

$x=-1$

KEY CONCEPTS AND VOCABULARY

STEPS TO SOLVE AN ABSOLUTE VALUE INEQUALITY

- Isolate the absolute value expression
- Write as a compound inequality
  - $|A| < b$ or $|A| \le b$: write the compound inequality as *AND*
  - $|A| > b$ or $|A| \ge b$: write the compound inequality as *OR*
- Solve the inequalities

EXAMPLES

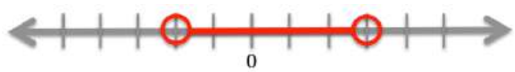
EXAMPLE 2: SOLVING ABSOLUTE VALUE INEQUALITIES

Solve the inequality. Graph the solutions.

a) $|2x-1| < 5$

$x < 3$ and $x > -2$

$(-2,3)$



b) $2|2y-5|+6 \ge 16$

$y \ge 5$ or $y \le 0$

$(-\infty,0] \cup [5,\infty)$

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1