# Algebra I Unit 4: Linear Equations, Inequalities, and Their Solutions

Time Frame: Approximately six weeks



## **Unit Description**

This unit focuses on the various forms for writing the equation of a line (point-slope, slope-intercept, two-point, and standard form) and how to interpret slope in each of these settings, as well as interpreting the *y*-intercept as the fixed cost, initial value, or sequence starting-point value. The algorithmic methods for finding slope and the equation of a line are emphasized. This leads to a study of linear data analysis with scatter plots, linear regression, interpolation and extrapolation of values. Linear equalities and inequalities are addressed through coordinate geometry. Linear and absolute value inequalities in one-variable are considered and their solutions graphed as intervals (open and closed) on the line. Linear inequalities in two-variables with applications are also introduced

### **Student Understandings**

Given information, students can write equations for and graph linear relationships. In addition, they can discuss the nature of slope as a rate of change and the *y*-intercept as a fixed cost, initial value, or beginning point in a sequence of values that differ by the value of the slope. Students learn the basic approaches to writing the equation of a line (two-points, point-slope, slope-intercept, and standard form). They graph linear inequalities in one variable (2x+3>-x+5) and |x|>3 on the number line and two variables on a coordinate system. Students construct and analyze scatterplots and determine the relationship among the data elements represented on the scatterplot.

#### **Guiding Questions**

- 1. Can students write the equation of a linear function given appropriate information to determine slope and intercept?
- 2. Can students use the basic methods for writing the equation of a line (two-point, slope-intercept, point-slope, and standard form)?
- 3. Can students discuss the meanings of slope and intercepts in the context of an application problem?
- 4. Can students relate linear inequalities in one and two variables to real-world settings?
- 5. Can students perform the symbolic manipulations needed to solve linear and absolute value inequalities and graph their solutions on the number line and the coordinate system?
- 6. Can students represent data of two quantitative variables on a scatterplot and describe how they are related?
- 7. Can students distinguish between correlation and causation? (2013 2014)

Unit 3 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

| Grade-Level Expectations                      |                                                                                                                                                                                                                            |  |  |
|-----------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| GLE # GLE Text and Benchmarks                 |                                                                                                                                                                                                                            |  |  |
| Number and Number Relations                   |                                                                                                                                                                                                                            |  |  |
| 4.                                            | Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)                                                          |  |  |
| 5.                                            | Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)                                                                                              |  |  |
| Algebra                                       |                                                                                                                                                                                                                            |  |  |
| 11.                                           | Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)                                                                                                                                     |  |  |
| 13.                                           | Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)                                                                                    |  |  |
| 14.                                           | Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)                                                                                                         |  |  |
| 15.                                           | Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)                                                                                            |  |  |
| Measurement                                   |                                                                                                                                                                                                                            |  |  |
| 21.                                           | Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)                                                                                                                    |  |  |
| Geometry                                      |                                                                                                                                                                                                                            |  |  |
| 24.                                           | Graph a line when the slope and a point or when two points are known (G-3-H)                                                                                                                                               |  |  |
| 25.                                           | Explain slope as a representation of "rate of change" (G-3-H) (A-1-H)                                                                                                                                                      |  |  |
| Data Analysis, Probability, and Discrete Math |                                                                                                                                                                                                                            |  |  |
| 29.                                           | Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)                                                                                                  |  |  |
| Patterns, Relations, and Functions            |                                                                                                                                                                                                                            |  |  |
| 38.                                           | Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)                                                                                                             |  |  |
| 39.                                           | Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)                                                                                                               |  |  |
|                                               | CCSS for Mathematical Content                                                                                                                                                                                              |  |  |
| CCSS#                                         | CCSS Text                                                                                                                                                                                                                  |  |  |
| Quantities                                    |                                                                                                                                                                                                                            |  |  |
| N-Q.2                                         | Define appropriate quantities for the purpose of descriptive modeling.                                                                                                                                                     |  |  |
| Seeing Structure in Expressions               |                                                                                                                                                                                                                            |  |  |
| A-SSE.2                                       | Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ . |  |  |
| Creating Equations                            |                                                                                                                                                                                                                            |  |  |
| A-CED.1                                       | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and                                                                       |  |  |

|                                                                                          | simple rational and exponential functions.                                                                |  |  |  |
|------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|--|--|--|
| A-CED.2                                                                                  | Create equations in two or more variables to represent relationships between                              |  |  |  |
|                                                                                          | quantities; graph equations on coordinate axes with labels and scales.                                    |  |  |  |
| A-CED.3                                                                                  | Represent constraints by equations or inequalities, and by systems of equations                           |  |  |  |
|                                                                                          | and/or inequalities, and interpret solutions as viable or non-viable options in a                         |  |  |  |
|                                                                                          | modeling context. For example, represent inequalities describing nutritional and                          |  |  |  |
|                                                                                          | cost constraints on combinations of different foods.                                                      |  |  |  |
| A-CED.4                                                                                  | Rearrange formulas to highlight a quantity interest, using the same reasoning as                          |  |  |  |
|                                                                                          | in solving equations For example, rearrange Ohm's law $V = IR$ to highlight                               |  |  |  |
|                                                                                          | resistance R.                                                                                             |  |  |  |
| Reasoning with Equations and Inequalities                                                |                                                                                                           |  |  |  |
| A-REI.10                                                                                 | Understand that the graph of an equation in two variables is the set of all its                           |  |  |  |
|                                                                                          | solutions plotted in the coordinate plane, often forming a curve (which could be                          |  |  |  |
|                                                                                          | a line).                                                                                                  |  |  |  |
| Linear, Quadratic, and Exponential Models                                                |                                                                                                           |  |  |  |
| F-LE.1                                                                                   | Distinguish between situations that can be modeled with linear functions and                              |  |  |  |
|                                                                                          | with exponential functions.                                                                               |  |  |  |
| F-LE. 2                                                                                  | Construct linear and exponential functions, including arithmetic and geometric sequences, given           |  |  |  |
|                                                                                          | a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |  |  |  |
| Interpreting Categorical and Quantitative Data                                           |                                                                                                           |  |  |  |
| S-ID.6                                                                                   | Represent data on two quantitative variables on a scatter plot, and describe how                          |  |  |  |
|                                                                                          | the variables are related.                                                                                |  |  |  |
| S-ID.7                                                                                   | Interpret the slope (rate of change) and the intercept (constant term) of a linear                        |  |  |  |
|                                                                                          | model in the context of the data                                                                          |  |  |  |
| S-ID.9                                                                                   | Distinguish between correlation and causation                                                             |  |  |  |
|                                                                                          | ELA CCSS                                                                                                  |  |  |  |
| CCSS#                                                                                    | CCSS Text                                                                                                 |  |  |  |
| Reading Stand                                                                            | lards for Literacy in Science and Technical Subjects 6–12                                                 |  |  |  |
| RST.9-10.7                                                                               | Translate quantitative or technical information expressed in words in a text into                         |  |  |  |
|                                                                                          | visual form (e.g., a table or chart) and translate information expressed visually                         |  |  |  |
|                                                                                          | or mathematically (e.g., in an equation) into words.                                                      |  |  |  |
| Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects |                                                                                                           |  |  |  |
| 6-12                                                                                     |                                                                                                           |  |  |  |
| WHST.9-10.6                                                                              | Use technology, including the Internet, to produce, publish, and update                                   |  |  |  |
|                                                                                          | individual or shared writing products, taking advantage of technology's capacity                          |  |  |  |
|                                                                                          | to link to other information and to display information flexibly and dynamically                          |  |  |  |
|                                                                                          |                                                                                                           |  |  |  |

#### **Sample Activities**

Activity 1: Generating Equations (GLEs: <u>13</u>, 24, 25; CCSS: <u>A. CED.1</u>, <u>A.CED.2</u>, A.CED.3, A-REI.10, F-LE.2; ELA: RST. 9-10.7)

Materials List: paper, pencil, graph paper, geo-board (optional), colored rubber bands (needed if using geo-boards), Vocabulary Self- Awareness Chart BLM, Generating Equations BLM

Begin the unit by having the students complete a *vocabulary self-awareness chart* (view literacy strategy descriptions). The *vocabulary self-awareness chart* allows you and your students to assess prior knowledge by having students complete the chart before beginning to work with the linear equations. A chart with suggested list of terms to start with is included as a BLM. Over the course of the unit students should be encouraged to revisit the chart to update the chart as their knowledge concerning linear equations grows. At the end of the unit students will have a resource for studying for tests and other assessments. After students have been introduced to the rather extensive language of linear equations, students will then be introduced to and instructed in writing and analyzing linear equations in point-slope, slope-intercept and standard forms.

Remind the students that the slope of a line is the ratio of the change in the vertical distance between two points on a line to the change in horizontal distance between the two points. Use a geo-board or graph paper to model the concept. Ask the students to think of the pegs on the geo-board as points in a coordinate plane and explain that the lower left peg represents the point (1, 1). Ask the students to locate the pegs representing the pair (1,1) and the pair (3,5) and place a rubber band around the pegs to model the line segment joining (1,1) and (3,5). Ask them to use a different colored rubber band to show the horizontal from the *x*-value of the first endpoint to the *x*-value of the second endpoint. Students should use a third different colored rubber band to show the distance from the *y*-value of the first endpoint to *y*-value of the second endpoint. Ask the students to find the value of the change in *y*-values (3) and the change in *x*-values (2) and show

that the defined slope ratio is  $\frac{3}{2}$ . Ask students to use this procedure to find the slope of the segment from the point (5, 2) and (1,4). Lead the students to discover that, because the line moves downward from left to right, the change in y would produce a negative value and the slope ratio is negative. Show the class that if the computations above are generalized, the

formula  $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$  where  $x_2$  is not equal to  $x_1$  could determine the slope of the line passing through the two points.

When student understanding of slope is evident, ask them to find the slope between a specific point  $(x_1, y_1)$  and a general point (x, y). Guide them to the conclusion that this slope would be  $m = \frac{(y-y_1)}{(x-x_1)}$ . Work with the students to algebraically transform this equation into its equivalent form  $(y-y_1)=m(x-x_1)$ . Explain that this is the point-slope form for the equation of a line and that it may be used to write the equation of a line when a point on the line and the slope of a line are known. Guide the students through the determination of the equation of the line with a slope of 2 and passing through the point with coordinates (3, 4).

Have students use *split-page notetaking* (view literacy strategy descriptions) as they work through the process of finding the equation of the line when given two points on the line. They should perform the calculations on the left side of the page and write a verbal explanation of each step on the right side of the page. An example of what split-page note-taking might look like in this situation is shown below. The *split-page notetaking* guide may be used by students to quiz each other regarding the processes of changing from one form of an equation to another. It may also be used in preparation for tests and other assessments.

| Problem:                                                                |                                                |
|-------------------------------------------------------------------------|------------------------------------------------|
| Find the equation of the line that passes                               |                                                |
| through the points $(4, 7)$ and $(-2, -11)$ .                           |                                                |
| Write your answer in slope-intercept form                               |                                                |
| and in standard form.                                                   |                                                |
| 7-(-11) 18                                                              | Find the slope of the line.                    |
| $m = \frac{7 - (-11)}{4 - (-2)} = \frac{18}{6} = 3$                     | $m = \frac{(y_2 - y_1)}{(y_2 - y_1)}$          |
| 1 (2)                                                                   | Formula: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ |
| y-7=3(x-4)                                                              | Find the equation of the line using the slope  |
| y - 7 = 3x - 12                                                         | and one of the original points.                |
| $   \begin{array}{c cccc}     +7 & +7 \\     y = 3x - 5   \end{array} $ | Point-slope formula: $(y-y_1)=m(x-x_1)$        |
|                                                                         | Slope-intercept form: $y = mx + b$             |
|                                                                         | Simplify equation to slope-intercept form.     |
| y = 3x - 5                                                              | Rewrite the equations in standard form         |
| $\int_{-3x}^{-3x} + y = -5$                                             | Standard form: $Ax + By = C$ , where           |
| 3x - y = 5                                                              | A > 0, A, B, C cannot be fractions or          |
|                                                                         | decimals, A and B cannot both equal 0          |
|                                                                         | simultaneously                                 |

Remind students again about how to use their split-page notes to review by covering content in one column and using the other column to recall the covered information. Students can also use their notes to quiz each other in preparation for tests and other class activities.

Ask the students to use a coordinate grid and graph several non-vertical lines. Guide the students to the discovery that all non-vertical lines will intersect the *y*-axis at some point and inform them that this point is called the *y*-intercept. Pick out several points along the *y*-axis and write their coordinates. Through questioning, allow the students to infer that all points on the *y*-axis have *x*-coordinates of 0. Then, establish that a general point of the *y*-intercept of a line could be expressed as (0, b). Ask the students to write and simplify the equation of the line with slope *m* and passing through the point (0, b). Using the point-slope form for the equation of a line,  $(y-y_1)=m(x-x_1)$ , have students insert the point (0, b) ((y-b)=m(x-0)) and solve for *y*, producing the slope-intercept form for the equation, y=mx+b. Place the students in small groups and have them work cooperatively to write equations of lines when given the slope and the *y*-intercept.

Introduce the standard form of a linear equation, Ax + By = C, using the definition from the *split-page notetaking* above. Provide students with the Generating Equations BLM for completion, checking, and discussion. Have students practice converting linear equations into point-slope, slope-intercept, and standard forms. If additional practice is required, use an algebra textbook as a reference to provide students with more practice in finding the equation of a line given a point and the slope and also given two points. Continue to have students write their answer in each of the three forms.

## Activity 2: Points, Slopes, and Lines (GLE: 24; CCSS: A-REI. 10)

Materials List: paper, pencil, graph paper

Provide students with opportunities to plot graphs using either a known slope and a point or two points. When given a slope and a point, help students start at the given point and use the slope to move to a second point. Have students label the second point. Then have them connect these two points to produce a graph of the line with the given slope which passes through the given point. When given two points, ask students to plot them and then connect them with a line. Next, have students determine the slope of the line by counting vertical and horizontal movement from one of the plotted points to the other plotted point. Repeat this activity with various slopes and points. Then give students an equation in slope-intercept form and provide discussion for graphing a line when the equation is in and slope-intercept form. Use an algebra textbook as a reference to provide more opportunities for students to practice graphing linear equations.

Using an equation in slope-intercept form, like y = 2x + 3, ask students to solve the equation. At first students may not understand how to solve the equation, or may solve the equation for x or y. Ask students what it means to solve an equation (to find values for each variable in the equation that make the equation true). Then ask students how many solutions they believe there are to the equation. Have students find values that make the equation true. If students are struggling, give students a simpler equation like x + y = 9 and have them create a table of values that would make the equation true. Guide students to understanding that the values which make the equation true form ordered pairs because they are values for x and y. Have students plot the ordered pairs they created and ask them to make observations about the points. Students should see that they can connect the points with a line. Lead students in a discussion about the points on the graph representing the solutions to the equation. Be sure to discuss those points that can be found between the ordered pairs they plotted and have students find fraction or decimal values for x and y that make the equation true. Repeat this process with more difficult equations. As students graph equations in the future, ask students to find solutions to the equations.

Have students complete a *RAFT writing* (view literacy strategy descriptions) assignment using the following information:

Role – Horizontal line Audience – Vertical line Format – letter Topic – Our looks are similar but our slopes are incredibly different

Have students share their writing with the class, and lead a class discussion on the accuracy of their information. A *RAFT* writing sample is given in Unit 1 Activity 6.

#### 2013-2014

Activity 3: Processes (CCSS: A-SSE.2, A-CED.4)

Materials List: paper, pencil, Processes BLM

Have students follow the steps in a flow chart for putting a linear equation expressed in standard form into slope-intercept form. A flow chart is type of *graphic organizer* (view literacy strategy descriptions). Students have seen and used the flow chart several times previously and know that the flow chart enables them to learn the steps involved in a process. Students may use the flow chart to practice converting equations to the various forms. A sample flow chart that could be used is included as the Processes BLM. Next, have students work in pairs to create a flow chart of steps an "absent classmate" could use to convert a linear equation written in slope-intercept form to one in standard form. Review the following procedures: questions go in the diamonds; processes go in the rectangles; *yes* or *no* answers go on the connectors. Have a class discussion of the finished flow charts, and then have students construct another flow chart individually to convert a linear equation from point-slope form to standard form. Remind students that transforming the forms of equations requires them to use properties of equalities and other algebraic properties addressed in Unit 2. Have them exchange charts with another student and follow them to perform the conversion. After use in class, students may use the graphic organizers to prepare for assessments.

### Activity 4: You Sank My Battleship! (GLEs: 24, 38; CCSS: A-REI. 10)

Materials List: paper, pencil, Battleship BLM, manila file folder per group

In this activity, students will play a modified version of the game Battleship to practice graphing linear equations. Place students in groups of four and have them form teams of two. Provide each team with the Battleship BLM and a manila file folder to shield the other teams view. Have each team draw four battleships on their Battleship BLM. The four ships should consist of 5, 4, 3, and 2 points as indicated at the bottom of the Battleship BLM. The ships can be placed horizontally, vertically, or diagonally. Team A creates a linear equation and gives the equation to Team B. Team B will graph the equation and determine if the line goes through any of the battleships. Team B should then provide the Team A with information as to how many hits were made (i.e. if the line passed through any of the points which make up the ships) or if the line missed all of the ships. When all of the points on a ship are passed through, the ship sinks. The first team to sink all of the other team's battleships wins.

After teams have had an opportunity to play the game, discuss with students if there is a way to determine whether the graph of the equation would "hit" their ship without graphing the line.

Guide students to understand that if the line would pass through any point to "hit" their ship, the ordered pair would be a solution to the equation. Students could replace the variables *x* and *y* with the values in the ordered pair and see if the equation is true. Ask students which method they feel is the best to use and have them defend their choice.

# Activity 5: Applications (GLEs: 4, 5, 11, 13, 21, <u>24</u>, 25, 29, 38, 39; CCSS: A-REI.10, F-LE.1)

Materials List: paper, pencil, tape measures, graph paper, a piece of uncooked spaghetti, Applications BLM, Transparency Graphs BLM, graphing calculator (optional), Data Collection Sheet BLM from Activity 7

This activity includes an investigation that will involve applying the concepts learned in Activities 1 and 2. Students will investigate the linear relationship between a person's foot length and length of the arm from the elbow to fingertip. They will also collect and organize data, determine line of best fit, investigate slope and *y*-intercept, and use an equation to make predictions.

Initially this is done as an in-class activity. Place students in groups of four and have them use the Applications BLM to record their data collection. Have the students measure their foot length and arm length to the nearest millimeter (a class discussion of measurement techniques and of rounding measurements is appropriate). The foot length should be measured from the heel to the end of the big toe. The arm length should be from the elbow to the tip of the index finger. Have the class agree on a measuring technique so that all measures are somewhat standardized. Have students take measurements and compile their data into the tables where foot length is the independent variable and arm length is the dependent variable. Use the Transparency Graph BLM and have each student graph his/her personal data on the coordinate plane on the overhead. After all points are plotted, discuss what occurs. Ask questions like, "Looking at the graph, do you see any interesting characteristics? Does there appear to be a relationship? What happens to the *y*-values as the *x*-values increase?"

Talk about the line of best fit. The piece of spaghetti will be used as a tool to estimate the line of best fit. Allow the students to make suggestions as to where it will be placed on the graph. Once the line is placed, review the ideas of slope of a line, *y*-intercept, point-slope form of a line, dependent and independent variables, etc. Determine two points that are contained in the line of best fit, find the slope of the line, and use the point-slope formula to write the equation. Have students state the real-life meaning of the slope of the line. Explain that this equation could be used as a means of estimating the length of a person's arm when the length of his or her foot is known. Have the students take foot and arm measures of an individual not yet measured (often the teacher is a good candidate for these measures). Place the newly found foot length into the equation to estimate foot length and to compare the actual value with the measured value.

Conduct another linear experiment such as timing students in the class as they do the wave where the number of students would be the independent variable and time in seconds would be the dependent variable. Assign a student to be the timer. Have 5 students do the wave and have the student time them. Continue to increase the number of students doing the wave by five until the entire class has participated. Students may use the Data Collection Sheet BLM from Activity 7. After the entire class has conducted the experiment and collected the data, put students in small groups and have each group create the scatter plot, derive the linear equation for the data, state the real-life meaning of the slope, and calculate how long it would take 100 students to do the wave. Compare each group's lines of best fit. Have students identify the characteristics of the different lines that are the same or different. Also have them compare and contrast the linear functions they obtained algebraically in terms of their rates of change and y-intercepts. Many graphing calculators are programmed to use statistical processes to calculate lines of best fit. Students might find it interesting to input class data into the calculator and compare the calculator's estimate with theirs.

In their math *learning logs* (view literacy strategy descriptions), have students respond to the following prompt:

Describe some other examples that could be modeled with a scatter plot and a line of best fit. Give reasons for your choice and explain why you believe they could be linear models.

After students have completed their entries, have them share their explanations with the class. Guide a class discussion of each entry and have the class decide if the examples are truly indicative of linear examples.

# Activity 6: Linear Experiments (GLEs: 13, <u>15</u>, <u>25</u>, 29, 39; CCSS: <u>F-LE.2</u>, S-ID.6, S-ID.7; ELA: RST.9-10.7, WHST.9-10.6)

Materials List: Data Collection Sheet BLM, Experiment Descriptions BLM with descriptions cut apart, needed materials for the experiments, graph paper, pencils

Place students in groups and have them complete a variety of experiments. Copy the Experiment Descriptions BLM, cut the descriptions so they are on separate strips of paper, and give each group a different linear experiment. Provide each student with a copy of the Data Collection Sheet BLM. For each experiment, have the groups collect, record, and graph the data using the Data Collection Sheet BLM. Have the group discuss the meaning of the *y*-intercept and slope, identify independent and dependent variables, explain why the relationship is linear, write the equation, and extrapolate values. After students have completed the experiments, have them prepare presentations to display and explain the results. The presentation should include data table, scatter plot, line of best fit, explanation of slope and *y*-intercept, and the reason the group determined the relationship was linear. The sample experiments listed on the BLM include:

### **Bouncing Ball**

Goal: to determine how the height of a ball's bounce is related to the height from

which it is dropped

Materials: rubber ball, measuring tape

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Procedure: Drop a ball and measure the height of the first bounce. To minimize

experimental error, drop from the same height 3 times, and use the average bounce height as the data value. Repeat using different starting heights.

Stretched Spring

Goal: to determine the relationship between the distance a spring is stretched and

the number of weights used to stretch it

Materials: spring, paper cup, pipe cleaner, weights, measuring tape

Procedure: Suspend a number of weights on a spring and measure the length of the

stretch of the spring. A slinky (cut in half) makes a good spring; one end can be stabilized by suspending the spring on a yard stick held between two chair backs. A small paper cup (with a wire or pipe cleaner handle) containing weights, such as peppermints, can be attached to the spring.

**Burning Candle** 

Goal: to determine the relationship between the time a candle burns and the

height of the candle.

Materials: birthday candle (secured to a jar lid), matches, ruler, stopwatch

Procedure: Measure the candle; mark the candle in 10 cm or 1/2 in. units. Light the

candle while starting the stopwatch. Record time burned and height of

candle.

Marbles in Water

Goal: to determine the relationship between the number of marbles in a glass of

water and the height of the water.

Materials: glass with water, marbles, ruler or measuring tape

Procedure: Measure the height of water in a glass. Drop one marble at a time into the

glass of water, measuring the height of the water after each marble is

added.

Marbles and uncooked spaghetti

Goal: to see how many pieces of spaghetti it takes to support a cup of marbles

Materials: paper cup with a hook (paper clip) attached, spaghetti, marbles

Procedure: place the hook on a piece of uncooked spaghetti supported between two

chairs, drop in one marble at a time until the spaghetti breaks, repeat with two pieces of spaghetti, and so on. (number of pieces of spaghetti is

independent and number of marbles is dependent)

2013 - 2014

**Activity 7: Correlation or Causation? (CCSS: S-ID.9)** 

Materials List: Vocabulary Self-Awareness Chart BLM, Causation or Correlation? BLM, paper, pencil

In the previous activities, several experiments involving data collection, scatterplots, and data analysis were performed. The key to these activities was that two quantitative variables are being measured about the same subject. The paired data was then listed and displayed on a scatterplot. Once the scatterplots were completed the assumption was made that all values plotted on the *x* axis were very closely "correlated" to those presented on the *y* axis. Whether the *x* value "causes" the *y* value may cause confusion among students.

Add the terms correlation and causation to the *vocabulary self-awareness chart* (view literacy strategy descriptions) that is found in Activity 1. The vocabulary self-awareness chart is more fully explained in Activity 1. Have students read the article concerning correlation and causation found at http://stats.org/in\_depth/faq/causation\_correlation.htm. The article provides a discuss about the difference between causation and correlation regarding health and science related studies. Topics such as the relationship between eating breakfast and a student's school performance, vaccinations and autism, smoking and alcoholism are addressed as examples of correlation. The relationship between lung cancer and smoking is described as causation. Ultimately, the article concludes that only through controlled study can two events be causation As students read the article, use the included BLM to complete a process guide (view literacy strategy descriptions). A process guide will scaffold students' comprehension within a unique format. The process guide is designed to stimulate students' thinking during or after reading. The guide will also enable the students to focus on important information, in this case the difference between "correlation" and "causation". After students have completed the *process guide*, discuss student answers and monitor their corrections. Have other students evaluate the examples made up by their classmates.

Go back to the experiments the students conducted in Activities 6 and 7 and have students discuss whether the experiments represent correlations or causations. Have students justify their reasoning and encourage students to question each other's logic.

## Activity 8: Inequalities (GLEs: 11, 14; CCSS: A.CED.3)

Materials List: paper, pencil

Provide students with real-life scenarios that can be described by an inequality. Have students graph the inequality and interpret the solution set in the context of the situation provided. Students are given inequalities to interpret that include both weak inequalities (i.e.,  $\le$  or  $\ge$ ) and strict inequalities (i.e.,  $\le$  or >), as well as absolute value inequalities. An example follows:

When Latoya measured Rory's height, she got 172 cm but may have made an error of as much as 1 cm. Letting x represent Rory's actual height in cm, write an inequality indicating the numbers that x lies between. Write the equivalent inequality using absolute value.  $(171 \le x \le 173, |x-172| \le 1)$ 

During discussion of the problems of this type have students determine what types of values are acceptable for x. Give students values and ask if the values given would be a viable option in the context being described.

# Activity 9: Is it Within the Area? Interpreting Absolute Value Inequalities in One Variable (GLEs: 5, 14; CCSS: A-CED.3)

Materials List: paper, pencil

This activity has not been changed because it incorporates the CCSS.

Review with students the idea of being within a certain distance of a location. For example, ask what it means to be within 25 miles of their home. Have students graph simple absolute value |x| < 25

inequalities in one variable on the number line. (Example: |x| < 25) The location point would always be the number that makes the expression inside the absolute value bars zero. For

example, if |x-3| < 5 is given, then the "location" is 3 because x-3 is zero at x=3. The "area" the inequality encompasses is from -2 to 8. This "area" is found simply by moving 5 units away from the "location" in both directions. Repeat this activity several times. Extend this idea to solving absolute value inequalities such as |ax+b| < c.

## Activity 10: Graphing Inequalities in Two Variables (GLE: 14; CCSS: A-CED.3)

Materials List: paper, pencil, chart paper, colored pencils

This has not been changed because it incorporates the CCSS. Introduce the activity by asking students if (5,3) and (3,1) are solutions to the inequality  $x-y\geq 1$ . Ask how many other points are solutions? Have students work with a partner and make a large coordinate grid on chart paper. Both axes should extend from -4 to 4. Have students write the value of x-y on each coordinate point (i.e., on the point (3,2) the student would write (3-2) or 1). Have students circle with a colored pencil several values that satisfy the inequality  $x-y\geq 1$ . Question students about points that lie between the points (ex. 2.5, 4.5). Have students shade all the solutions to the inequality. Use the students' conclusions about this inequality to guide a discussion on graphing all inequalities in two variables. Use algebra text to provide students with examples for practice.

### Activity 11: Modeling Fuel Consumption (CCSS: N-Q.2, A-REI.10)

Materials List: paper, pencil

This activity concerns itself with best mathematical practices. The activity to be completed is an NCTM directed problem solving activity found at <a href="http://www.nctm.org/uploadedFiles/Journals">http://www.nctm.org/uploadedFiles/Journals</a> and <a href="mailto:Books/Books/FHSM/RSM-">Books/Books/FHSM/RSM-</a>

<u>Task/Fuel for Thought.pdf</u> The activity provides students with an opportunity to analyze data, reason abstractly, and create and analyze graphs.

## **Sample Assessments**

#### **General Assessments**

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- The student will create a portfolio that includes student-selected and teacher-selected work.
- The student will complete constructed response items such as these:
  - Each gram of mass stretches a spring 0.025 cm. Use m = 0.025 and the ordered pair (50, 8.5) to write a linear equation that models the relationship between the length of the spring and the mass. y = 0.025x + 7.25
    - a) What does the *y*-intercept mean in this situation? (*When the spring is not stretched at all it measures 7.25 cm in length.*)
    - b) What is the length of the spring for a mass of 70 g? (9 cm)
  - o A taxicab ride that is 2 mi. long costs \$7. One that is 9 mi long costs \$24.50.
    - a) Write an equation relating cost to length of ride. (C = 2.5m + 2)
    - b) What do the slope and y-intercept mean in this situation? (Slope the cost goes up \$2.50 for each mile driven, y-intercept The cost is \$2 for 0 miles driven)
- The student will complete math *learning log* entries using topics as these:
  - O Describe two ways to find the slope of the graph of a linear equation. Which do you prefer? Why?
  - Write a few sentences to explain whether a line with a steep slope can have a negative slope.
  - Explain how you would graph the line  $y = \frac{3}{4}x + 5$ .
  - O Describe the relationship between the graph of a function and the equation that represents the same function.
  - o Explain why absolute value is always a non-negative number.

#### **Activity-Specific Assessments**

- Activity 1:
  - The student will write the equation of a linear function when given two points or one point and the *y*-intercept.
  - The student will convert one form of a linear equation into another equivalent form. All forms of a linear equation need to be addressed.
- Activity 2:
  - o Given a linear function and its graph, the student will find the slope and *y*-intercept graphically and algebraically.

The student will interpret the slope and *y*-intercept of a graph that depicts a realworld situation (i.e. state its real-life meaning).

## • Activity 5:

- The student will use any of the linear data sets from Unit 1 and complete the following tasks with and/or without the graphing calculator.
  - a. Make a scatter plot of the data
  - b. Draw and find the equation of the line of best fit
  - c. Give the real-life meaning of the slope and *y*-intercept for the particular experiment chosen.
  - d. Predict for a specific value of x.
  - e. Predict for a specific value of y.
  - f. Explain the process of making a scatter plot and the line of best fit.
  - g. (2013 2014) Determine whether the relationship between the data sets is correlation or causation.
- Activity 6: The students will construct a lab report describing materials, procedures, diagrams, and conclusion of the linear experiment. If students have access to computers, they may prepare a *PowerPoint®* presentation to present their findings to the class. In addition, if students have computer access students can prepare spreadsheets and scatter plots for their presentations.