Algebra I
Unit 3: Linear Functions and Their Graphs, Rates of Change, and Applications

Time Frame: Approximately five weeks

Unit Description

This unit leads to the investigation of the role of functions in the development of algebraic thinking and modeling. Heavy emphasis is given in this unit to understanding rates of change (intuitive slope) and graphing input-output relationships on the coordinate graph. In this unit, emphasis is given to determining inputs and output for equations in the form $y = mx + b$.

Student Understandings

Students recognize functions as input-output relationships that have exactly one output for any given input. They can apply various strategies for determining if a relation is a function. Additionally, students note that the rate of change in graphs and tables is constant for linear relationships (one-differences are constant in tables), and for each change of 1 in *x* (the input), there is a constant amount of growth in *y* (the output). They can determine whether a relationship is linear (or not) by examining its equation and/or its graph.

Guiding Questions

1. Can students understand and apply the definition of a function in evaluating expressions (output rules) as to whether they are functions or not?
2. Can students apply the vertical line test to a graph to determine if it is a function or not?
3. Can students identify the matched elements in the domain and range for a given function?
4. Can students describe the constant growth rate for a linear function in tables and graphs, as well as connect it to the coefficient on the *x* term in the expression leading to the linear graph?
5. Can students intuitively relate slope (rate of change) to *m* and the *y*-intercept in graphs to *b* for linear relationships $mx + b$?

Unit 3 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Geometry	
25.	Explain slope as a representation of "rate of change" (G-3-H) (A-1-H)
Patterns, Relations, and Functions	
35.	Determine if a relation is a function and use appropriate function notation (P-1-H)
36.	Identify the domain and range of functions (P-1-H)
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
CCSS for Mathematical Content	
Seeing Structure in Expressions	
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.
Creating Equations	
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
Reasoning with Equations and Inequalities	
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Linear, Quadratic, and Exponential Models	
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.
Investigating Categorical and Quantitative Data	
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
ELA CCSS	
CCSS#	**Reading Standards in Science and Technical Subjects 6 - 12**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., table or chart) and translate information expressed in visually or mathematically (e.g., in an equation) into words.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6 - 12	
WHST.9-10.6	Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Sample Activities

Activity 1: What's a Function? (GLEs: 35, 36)

Materials List: paper, pencil, Vocabulary Self-Awareness Chart BLM, What is a Function? BLM, calculator (optional)

Have students maintain a *vocabulary self-awareness chart* (view literacy strategy descriptions) for this unit. *Vocabulary self-awareness* is valuable because it highlights students' understanding of what they know, as well as what they still need to learn, in order to fully comprehend the concept. Students indicate their understanding of a term/concept, but then adjust or change the marking to reflect their change in understanding. The objective is to have all terms marked with a + at the end. A sample chart is included in the Blackline Masters. Be sure to allow students to revisit their self-awareness charts often to monitor their developing knowledge about important concepts.

Have students use the What is a Function? BLM to complete this activity.

The BLM first provides examples of relations that are and are not functions (that are labeled as such) including real-life examples, input/output tables, mapping diagrams, and equations. Pose the question: "What is a function?" and then have students use a Think Pair Share process to help them determine what is significant in the tables. After giving the students time to complete

page 1, lead a discussion which results in the definition of a function (for every input there is exactly one output) and have students write the definition in the blank at the top of page 2.

The next section of the BLM repeats the activity with graphs that are and are not functions. Introduce the vertical line test. Ask students to explain why this vertical line test for functions is the same as the definition they used to see if the set of ordered pairs were a function.

The third section of the BLM can be used to help students define the domain and range of a function. After students have looked at the first example, have them discuss with a partner what they believe are the definitions of domain and range. Discuss with the class the correct definitions of domain and range. The BLM then provides examples in which students write the domain and range for three different relations.

Introduce function notation ($f(x)$), The function $f(x) = 2x + 3$ is provided and students are asked to determine $f(-2), f(-1), f(0)$. Give students additional input-output rules in the form of two-variable equations for more practice as needed.

The last section of the BLM asks students to determine if the set of ordered pairs in the input/output tables generated using $f(x) = 2x + 3$ satisfies the definition of a function (i.e., for each element in the domain, there is exactly one element in the range). Tell students to plot the ordered pairs and connect them and determine the domain and range. Now, have students draw several vertical lines through the input values to illustrate the idea that for a function, a vertical line intersects the graph of a function at exactly one point.

Provide closure to the activity by summarizing and reviewing the major concepts presented in the activity.

Activity 2: Identify! (GLEs: 8, 35, 36)

Materials List: paper, pencil, Identify BLM, calculator

Give students the Identify BLM. One page contains a set of linear equations and the other contains a set of ordered pairs. Have students identify the domain and range of each relation. Have students work in pairs to determine which domain-range pairs on the second page of the BLM match the given equation. The set of linear equations includes some that depict real-world scenarios. These linear equations also include some that are in "unsimplified" form (e.g., $3y - 3(4x + 2) = 2y + 3$), so that students can have practice in using order of operations when they substitute a value in for one of the variables and solve for the other.

Have students determine which relations are also functions. For those relations they determine to be functions, have students identify the independent and dependent variables and rewrite the linear function using function notation. For example, if students determine that $3x + y = 8$ is a linear function, then they could rewrite it as $h(x) = -3x + 8$.

Activity 3: Functions of Time (GLEs: 15, 36; CCSS: F-LE.1, F-LE.5; ELA: WHST.9 – 10.6)

Materials List: paper, pencil, computer with spreadsheet program or a poster board, supplies needed for time functions chosen for this activity

Have students collect and graph data about something that changes over time. (Examples include the temperature at each hour of the day, the height of a pedal on a bicycle when being ridden, the number of cars in a fast food parking lot at different times of the day, or the length of a plastic grow creature as it sits in water.) It may be beneficial to have some ideas ready for students to choose from rather than have students determine what they will collect data about on their own. Have students organize the data in a spreadsheet and make a graph of the data. Have students construct a *PowerPoint*® presentation to present their data and findings to the class. In the presentation, students should discuss whether the data represents a linear or non-linear function (including how they arrived at their decision), identify the domain and range of the function, and describe what the domain and range means in the context of the situation (data). When discussing the domain and range in terms of the context, students should realize that the domain will not consist of all real numbers as time would not be represented by a negative value. In many situations the students will collect data about, the range will also be confined to numbers greater than or equal to zero. Be sure students understand the reason the domain and range of their functions are restricted. When students present their findings to the class, perhaps they should first show their graphs to the class without labels to see if other students can guess what they observed. Then, throughout the presentations, allow students to ask questions to clarify their understanding and to critique the reasoning of those presenting. If technology is not available, have students construct the table and graph by hand on a poster board.

Activity 4: Patterns and Slope (GLEs: 13, 25; CCSS: A-REI.10, F-LE.2, S-ID.7; ELA: RST.9-10.7)

Materials List: math learning log, paper, pencil, square algebra tiles, Patterns and Slope BLM, graph paper

This activity has not changed because it already incorporates the CCSS.

Have students use the Patterns and Slope BLM to complete this activity. Divide students into groups and provide them with square algebra tiles. Have the students arrange 3 tiles in a rectangle and record the width (x) and the perimeter (y) on the BLM. Have the students fit 3 more tiles under the previous tiles and continue adding tiles, putting the values in a table. Students should continue working with their groups to complete the BLM through the completion of the table. Guide students as they complete the remainder of the BLM.

Have students notice that the change in the y-values is the same. Have them graph the data and decide if it is linear. Ask students what changed in the pattern *(the widths that keep increasing)*

and what remained constant *(the length of the sides added together (3+3))*. Have the students write a formula to describe the pattern *(*$y = 6 + 2x$*)*. Guide students to conclude that what remained constant in the pattern will be the constant in the formula, and the rate of change in the pattern will be the slope. Guide students to make a connection between the tabular, graphical and algebraic representation of the slope.

In their math *learning logs* (view literacy strategy descriptions) have students respond to the following prompt:

> A child's height is an example of a variable showing a positive rate of change over time. Give two examples of a variable showing a negative rate of change over time. Explain your answer.

Have students share their answers with the class and combine a class list of all student answers. Discuss the answers and have students determine whether the examples are indeed negative rates of change.

Activity 5: Recognizing Linear Relationships (GLEs: 9, 39; CCSS: F-LE.2, S-ID.7; ELA: RST.9 -10.7)

Materials List: paper, pencil

This activity has not changed because it already incorporates the CCSS.

Provide students with several input-output tables (linear) paired with a graph of that same data. Include examples of real-life linear relationships. (Examples of linear data sets can be found in any algebra textbook.) Introduce slope as the concept of $\frac{rise}{run}$. Have students determine the slope of the line and then investigate the change in the *x*-coordinates and the accompanying change in the *y*-coordinates. Ask if a common difference were found. How does this common difference in the *y*-coordinates compare to the slope (rate of change) found for the line? Using this information, have students conjecture how to determine if an input-output table defines a linear relationship. *(There is a common difference in the change in y over the change in x.)* Have students write a linear equation for each of the graphs. Have students compare the input-output tables, the graphs, and the equations to see how the slope and *y*-intercepts affect each. Also have students discuss the slope and the y-intercept in terms of the context of each real-life data set.

Activity 6: Rate of Change (GLEs: 13, 15, 25, 39; CCSS: A-SSE.1, A-REI.10, F-LE.2, F-LE.5, S-ID.7; ELA: RST 9 -10.7)

Materials List: paper, pencil, Rate of Change BLM, graph paper, straightedge

Use the Rate of Change BLM to introduce the following problem:

> David owns a farm market. The amount a customer pays for sweet corn depends on the number of ears that are purchased. David sells a dozen ears of corn for $3.00.

Place the students in groups and ask each group to make a table reflecting prices for purchases of 6, 12, 18, and 24 ears of corn (question 1a on the BLM). Have each group share their table, then have groups complete question 1 on the Rate of Change BLM. Be sure to pause at different points while students work on question 1 to check for understanding. Students will write and graph four ordered pairs that represent the number of ears of corn and the price of the purchase. They will write an explanation of how the table was developed, how the ordered pairs were determined, and how the graph was constructed. After ensuring that each group has a valid product, ask the students to use a straightedge to construct the line passing through the points on the graph. Each group will find the slope of the line. Review with students the idea that slope is an expression of a rate of change. Ask students to explain the real-life meaning of the slope. *(For every ear of corn purchased, the price goes up $.25).*

Introduce the slope-intercept form of an equation. Have groups determine the equation of the lines by examining the graph for the slope and *y*-intercept. Point out to the students that the value of *y* (the price of the purchase) is determined by the value of *x* (the number of ears purchased). Therefore, *y* is the dependent variable (the set of range values) and *x* is the independent variable and represents the domain values. Point out to the student that in the situation described in question 1 the value of *y* will always increase as the value of *x* increases. This is indicated by the fact that there is a positive slope. Also, point out that the *y*-intercept is at the origin because no purchase would involve a zero price. Ask the students to use the equation to find the price of a purchase of four ears of corn. Ask students what are the types of numbers which can logically be used in a problem of this type. (*Only positive whole numbers are acceptable values of x; only multiples of $.25 are acceptable numbers for the value of y.)*

Have students work with their groups to complete the second problem on the Rate of Change BLM.

Have students participate in a math *text chain* (view literacy strategy descriptions) activity to create word problems using real-life applications that are linear relationships. Students should now be familiar with *text chains* after the activities in Units 1 and 2. A sample *text chain* might be:

Student 1: Jimi wants to save money to buy a car.
Student 2: He has been mowing lawns to earn money
Student 3: He charges $30 per lawn.
Student 4: What is the rate of change of this linear relationship?

Students should also explain what types of numbers (*positive whole numbers*) are acceptable to use for *x* and positive numbers that are multiples of 30 must be values of *f(x)*.

Have groups share their math *text chains* with the entire class and have the other groups solve and critique the problems. Students should be able to explain the meaning of slope in each of the real-world application problems; they should explain why the graph of the equation crosses the origin in their examples. Students should also explain what types of numbers are acceptable to use for *x* and *y*; they should be able to explain the logical parameters of the problems that they develop.

Activity 7: Make that Connection! (GLEs: 13, 15, 25, 36; CCSS S-ID.7; ELA: RST.9-10.7)

Materials List: paper, pencil, calculator, graph paper

Have students generate a table of values for a given linear function expressed as $f(x) = mx + b$. An example would be the cost of renting a car is $25 plus $0.35 per mile. Have students label the input value column of the table "Domain" and the output value column "Range." Have students select their own domain values for the input variable and generate the range values for the output variable. Next, have students calculate the differences in successive values of the output, and find a constant difference. Then have them relate this constant difference to the slope of the linear function. Next, have students graph the ordered pairs and connect them with a straight line. Finally, discuss with the students the connections between the table of values, the constant difference found, the graph, and the function notation. Last, have students do the same activity using a linear function that models a real-world application. For example, students could investigate the connections between the algebraic representation of a cost function, the table of values, and the graph. Students should explain the meaning of the slope and the *y*-intercept in each of the real-world application models. The students should also graph the linear functions that they work with. An interesting way for students to describe the meaning of slope, y-intercept, the variables x and y is to write a *RAFT* (view literacy strategy descriptions). Since a *RAFT* was used in Unit 1 students should be familiar with it.
This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. *RAFT is* an acronym that stands for Role, Audience, Format, and Topic.

To connect with this activity the parts are defined as:

Role – slope (*m*)
Audience – the variable *y*
Format – narrative (mystery short story)
Topic – The Perils of the Variable *X*

Help students to understand that they are going to take the **R**ole of the slope (*m*) and to write a short story for the variable *y*, the **A**udience. The **F**ormat of the writing should be a short story with the **T**opic entitled, "The Perils of the Variable *X*." The story should include Mr. B, the evil y–intercept, as a main character. Once *RAFT* writing is completed, have students share with a partner, in small groups, or with the whole class. Students should listen for accurate information

and sound logic in the *RAFT*s. Listen to the stories for student understanding to determine whether more practice using input-output tables and function form of a line is needed.

Activity 8: Graph Families (GLEs: 37, 38, 39; CCSS ELA: RST.9-10.7)

Materials List: paper, pencil, Graph Families BLM, graphing calculator

This activity has not changed because it already incorporates the CCSS.

Activities 8 and 9 are a study of families of lines. A family of lines is defined as a group of lines that share at least one common characteristic. For example, these lines may have different slopes and the same *y*-intercept or different *y*-intercepts and the same slope. Parallel and perpendicular lines are also examples of families of lines and will be studied in Unit 4.

Use the Graph Families BLM to complete this activity. First, generate a discussion on families of linear graphs by describing the following situation.

> Suppose you go to a gourmet coffee shop to buy coffee beans. At the store, you find that one type of beans costs $6.00 per pound and another type costs $8.00 per pound.

Place the students in groups and have them complete the BLM through question 4. Ask each group to share its findings, and ensure that each group finds the correct equations, slopes, and *y*-intercepts. Have students complete questions 5 and 6 and discuss the students' conclusions.

Have students use a graphing calculator to complete the remainder of the BLM. If a graphing calculator is not available, have the students graph the equations by hand. The BLM will lead students to discover that a line will get steeper as the absolute value of the slope is increased and flatter as the absolute value of the slope is decreased. They will also observe the difference in lines with positive and negative slopes. Examples of graphs of horizontal and vertical lines are also included on the BLM.

Conclude the lesson by clarifying what is meant by the term *family of lines* and discussing similarities and differences of the types of families.

Activity 9: Slopes and *Y*-Intercepts (GLEs: 38)

Materials List: paper, pencil, Slopes and Y-intercepts BLM, graphing calculator

Have students use the Slopes and Y-intercepts BLM to complete this activity. After students have completed the BLM, have a class discussion of their findings. Have students explain how the changes in the *y*-intercepts affect the graphs. Have students explain the effects of the change in the slope on the graphs. Have students make conjectures about positive and negative slopes. Discuss the slopes of horizontal and vertical lines and the lines $y = x$ and $y = -x$. Help students

intuitively relate slope (rate of change) to *m* and the *y*-intercept in graphs to *b* for each of these linear functions expressed as $f(x) = mx + b$.

After activities 7, 8, and 9, have students participate in a *professor know-it-all* activity (view literacy strategy descriptions) in which they assume roles of professors or experts who are to provide answers to questions posed by their classmates. Form groups of three or four students. Give them time to review the content covered in activities 7, 8, and 9. Have the groups generate three to five questions about the content. Call a group to the front of the class. These are the "professors." Invite questions from the other groups. Have the chosen group huddle, discuss, and then answer the questions. After about 5 minutes, ask a new group to come up and repeat the process. The class should make sure the "professors" respond accurately and logically to their questions. The activity should continue until the instructor believes that all students are proficient with function form of a line.

Activity 10: Rate of Growth (GLEs: 11, 25, 38; CCSS: A-REI.10, F-LE.1, F-LE.2; ELA: RST.9–10.7)

Materials List: paper, pencil

This activity has not changed because it already incorporates the CCSS.

Provide students with two similar triangles, quadrilaterals, or other polygons. Have students measure the corresponding side lengths in these two similar figures and plot them as ordered pairs (i.e., students would plot the ordered pair [original side length, corresponding side length] for each pair of corresponding sides). Have students first determine the ratio between the side lengths and then compare that ratio to the slope of the line. Have them determine whether the relationship is linear or non-linear. Ask students to write the equation in slope-intercept form to find that $y = kx$, where *k* is the ratio they found between the corresponding parts of the two similar figures. Have students describe the slopes of these linear functions as they relate to describing the proportional relationship between two similar figures. Next, have the students switch the order of the ordered pairs that were plotted and plot them (i.e., corresponding side length, original side length). Determine the ratio between two corresponding sides and compare to the ratio to the slope of the new line. Ask, "How do the ratios compare to one another?" (*The ratios are reciprocals.*) What do the equations mean in a real-life setting? (*The two equations indicate how to convert between lengths in the two figures. One says to multiply the values in the smaller figure by some number to get the corresponding values in the larger figure. The other indicates how to find the lengths in the smaller figure from the values in the larger figure.*) Repeat this activity several times until students understand that the equation of the line describes the proportional relationship between the side lengths of the two figures and that the proportional relationship represents a rate of growth from the small figure to the large (or vice versa).

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- The students will submit a portfolio with artifacts such as these:
  - daily student journal
  - teacher observation checklists or notes
  - examples of student products
  - scored tests and quizzes
  - teacher observations of group presentations
- The students will use the definition of a function and/or the vertical line test to determine whether relations are functions.
- The students will generate the functional notation for a linear function expressed in *x* and *y*.
- The students will generate a function's graph from an input-output table.
- The students will make a poster of a function represented in three different ways and describe the domain and range of the function.
- Given a graph that is a function of time, the students will write a story that relates to the graph.
- The students will answer open-ended questions such as these:
  Maria is hiking up a mountain. She monitors and records her distance every half hour. Do you think the rates of change for every half hour are constant? Explain your answer.
- The students will solve constructed response items such as these:
  Signature Office Supplies is a regional distributor of graphing calculators. When an order is received, a shipping company packs the calculators in a box. They place the box on a scale which automatically finds the shipping cost. The cost *C* depends on the number *N* of the calculators in the box, with the rule C = 1.25N + 4.95.
  - Make a table showing the cost for 0 to 20 calculators.
  - How much would it cost to ship an empty box? (4.95) How is that information shown in the table and the cost rule?
  - How much does a single calculator add to the cost of shipping a box? (1.25) How is that information shown in the table and the cost rule?
  - Write and solve equations and inequalities to answer the following questions.
    a. If the shipping cost is $17.45, how many calculators are in the box? (*10 calculators)*
    b. How many calculators can be shipped if the cost is to be held below $25? (*16 calculators)*
    c. What is the cost of shipping eight calculators? (*$14.95)*

  - Graph the equation on a coordinate plane. What is the meaning of the slope of the line in terms of the problem? What is the meaning of the y intercept in terms of the problem?
  - What questions about shipping costs could be answered using the following equation and inequality?

    $27.45 = 4.95 + 1.25N$

    $4.95 + 1.25N \leq 10$
- The students will complete entries in their math *learning logs* using such topics as these:
  - Sketch the graph of a relation that is not a function and explain why it is not a function.
  - Explain algebraically and graphically why $y = 2x^2 - 7$ is a function.
  - Explain why the vertical line test works.
  - Explain the meaning of *m* and *b* in an equation in the form $y = mx + b$. What is slope? What is *y*-intercept?
  - Explain how you can tell if the relationship between two sets of data is linear.

Activity-Specific Assessments

- Activity 1: The students will decide if the following relations are functions:
  a. number of tickets sold for a benefit play and amount of money made *(yes)*
  b. students' height and grade point averages (*no)*
  c. amount of your monthly loan payment and the number of years you pay back the loan (*no)*
  d. cost of electricity to run an air conditioner during peak usage hours and the number of hours it runs (*yes)*
  e. time it takes to travel 50 miles and the speed of the vehicle (*yes)*

- Activity 3: The student will write a report explaining the procedures and the conclusions of the investigation. Provide the student a rubric to use when he/she writes the report including questions that must be answered in the report such as: How did you decide on values to use for your axes? What did you and your partner learn about collecting and graphing data? Use the Functions of Time Rubric BLM.

- Activity 5: The student will find the rate of change between consecutive pairs of data.
  Example:

x	1	3	4	7
y	3	7	9	15

  Is the relationship shown by the data linear? *(Yes)* Explain your answer. (*There is a common difference between the change in y over the change in x (2.)*)

- Activity 7: The student will solve constructed response items such as these:
  Suppose a new refrigerator costs $1000. Electricity to run the refrigerator costs about $68 per year. The total cost of the refrigerator is a function of the number of years it is used.

a. Identify the input and output variables
b. State the reasonable domain and range of the function.
c. Write an equation for the function. ($C = 1000 + 68N$)
d. Make a table of values for the function.
e. Graph the function.
f. Label the constant difference (slope) on each of the representations of the function.
g. Interpret the slope and the y intercept in this example of a linear model.

- Activity 9: The student will sort a set of linear functions into families based on slope and *y*-intercept characteristics.