Algebra I
Unit 2: Writing and Solving Linear Equations and Inequalities

Time Frame: Approximately four weeks

Unit Description

This unit includes an introduction to linear equations and inequalities and the symbolic transformation rules that lead to their solutions. Transformation of verbal expressions and sentences to numerical and algebraic expressions and equations and inequalities will be addressed. Order of operations, as it relates to solving equations and inequalities, is also included. Properties of real numbers will be used to justify solution methods of equations and inequalities.

Student Understandings

Students recognize linear growth patterns and write the related linear expressions and equations for specific contexts. Students can solve and justify the solution graphically and symbolically for single- and multi-step linear equations. Students can translate verbal expressions and sentences into algebraic expressions, equations and inequalities. Students can then solve the equations and inequalities and justify those solutions using real number system properties. Students will manipulate variables in formulas to solve for particular variable.

Guiding Questions

1. Can students graph data from input-output tables on a coordinate graph?
2. Can students recognize linear relationships in graphs of input-output relationships?
3. Can students perform simple algebraic manipulations of collecting like terms and simplifying expressions?
4. Can students translate verbal expressions and sentences into numeric and algebraic expressions, equations and inequalities?
5. Can students perform the algebraic manipulations on the symbols involved in a linear equation or inequality to find its solution and relate its meaning graphically?
6. Can students justify steps in the algebraic solution process by using appropriate real number properties?
7. Can students solve formulas for a particular indicated variable?
8. Can students solve traditional and nontraditional “story” problems?

Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
CCSS for Mathematical Content	
CCSS#	**CCSS Text**
Arithmetic with Polynomials and Rational Expressions	
A-APR.1	Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Creating Equations	
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A-CED.4	Rearrange formulas to highlight a quantity interest, using the same reasoning as in solving equations *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities	
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted in the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Linear, Quadratic, and Exponential Models	
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
ELA CCSS	
CCSS#	**CCSS TEXT**
Reading Standards for Literacy in Science and Technical Subjects 6–12	
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

Sample Activities

Activity 1: Think of a Number (GLEs: 5, 8, 11; CCSS: A.APR.1)

Materials List: paper, pencil, spreadsheet program (optional), calculator (optional)

Number puzzles are an interesting way to review order of operations, properties of a number, and simple algebraic manipulation. Give students the following directions, pause after each step:

1) Think of a number.
2) Add 8 to your number.
3) Multiply the result by 2.
4) Subtract 6.
5) Divide by 2.
6) Subtract the number with which you started.

Then have students share their answer. Students should have the same answer (5). Create a table like the one below using some numbers the students chose. Ask students if they know how the puzzle works. Have students visualize the puzzle by using symbols for the starting number and individual numbers. Then have students use a variable for the beginning number and write algebraic expressions for each step to complete the final column of the table. If necessary, have students simplify the expressions they write in the final column of the table to see why everyone gets 5 as the answer.

Starting number	6	13	10	24	x
Add 8	14	21	18	32	$x+8$
Multiply by 2	28	42	36	64	$2(x+8)$
Subtract 6	22	36	30	58	$2(x+8)-6$
Divide by 2	11	18	15	29	$\frac{2(x+8)-6}{2}$
Subtract starting number	5	5	5	5	$\frac{2(x+8)-6}{2}-x$

Have the students develop their own puzzles, using spreadsheets if available. Use an algebra textbook as a reference to provide other opportunities for students to review and practice order of operations and algebraic manipulations. Include expressions with various forms of rational numbers and integer exponents so that students can work to demonstrate computational fluency.

To ensure that students understand the significance of using appropriate order of operations in simplifying expressions, solving equations, and inequalities, provide students with the opportunity to imagine and write about what would happen if there were no organized structure for simplifying expressions. Using *SPAWN* writing (view literacy strategy descriptions), select the "What If" category for the prompt for lesson closure.
A sample prompt is provided.

> You have been simplifying expressions using order of operations. What would happen if there were no formal order to complete operations when simplifying expressions? When writing, think about formulas that you know and applications involving money.

After students have completed the writing, have the students share their writings with the class. Make sure that the comments are logically sound and the mathematics is accurate. Refer to algebra texts for more order of operations examples.

Activity 2: Order of Operations and Solving Equations (GLEs: 5, 8, 11)

Materials List: paper, pencil, calculator, Split-Page Notetaking Example BLM

Have students work in groups to review solving one-step and multi-step equations. Discuss with students the reason for isolating the variable in an equation and use the comparison of solving an equation to a "balance scale." Then provide students with examples of equations that require simplification using algebraic manipulations and order of operations before they can be solved. Have students cover up one side of the equation and completely simplify the other, then repeat with the other side of the equation. Use a math textbook as a reference to provide students with other opportunities to practice solving different types of linear equations including literal equations. Include equations with various forms of rational numbers so that students can work to increase computational fluency.

Have students use *split-page notetaking* (view literacy strategy descriptions) to show the steps of solving a multi-step equation. One of the purposes of *split-page notetaking* is to create a record for later recall and application. When students learn to take effective notes, they develop a greater understanding of key concepts and information. Have students show the steps of solving a multi-step equation in the left column. In the right column, students should write the operation that was performed and any note that will help them to later solve a similar equation. A good method of demonstrating the use of this strategy is to show the students an example of a poorly organized set of notes and an example of *split-page notetaking*. Students may use the *split-page notetaking* pages to help prepare for assessments. A Blackline Master of an example of *split-page notetaking* is included.

Activity 3: Using a graphic organizer to solve equations (GLE: 5, 8, 11; CCSS: A-APR.1)

Materials List: paper, pencil, Equation Graphic Organizer BLM

To provide students with additional practice needed to help them develop fluency in solving one-variable equations, allow them to develop a *graphic organizer* (view literacy strategy descriptions). A flow chart is an acceptable *graphic organizer* for the equation solving process. Have the students construct a flow chart for solving equations in one variable. Use the Equation Graphic Organizer BLM as a guide. Help students come up with other ways to make decisions about solving equations. For example, some students may choose not to eliminate fractions as the first step in solving equations. After the flow charts have been constructed, have the students use the charts to solve different equations. Once students have completed the *graphic organizer*, they may use it to prepare for quizzes and tests.

Activity 4: Balanced Means Equal: Proving Solution Methods (CCSS:A-REI.1)

Materials List: paper, pencil, *Vocabulary Self-awareness Chart* BLM, Proving Solution Methods BLM, calculator (optional)

In Activity 2, solving of equations was compared to a balance scale. In this activity, students are going to use algebraic properties to justify and prove the solution methods for equations. Use either of the two animations to illustrate the importance of maintaining equality on either side of the "=" during the solution process. The activity which completely represents the solution process for any type of equation is found at http://illuminations.nctm.org/LessonDetail.aspx?ID=L746. The Balance- Strategy Game found at http://www.fi.uu.nl/toepassingen/02018/toepassing_wisweb.en.html provides visual reinforcement of the equivalences needed to solve equations.

After students have completed viewing the animations and have solved several equations using the balance scale as a method to conceptualize the algebraic properties of operational equality, have students make *vocabulary self-awareness charts* (view literacy strategy descriptions). A

Vocabulary Self-Awareness Chart BLM is provided for students to use as they begin to study the properties of algebra. In addition to addition, subtraction, multiplication, and division, properties of equality include the distributive, commutative, and associative properties in the topics to be addressed in this activity. Provide students with samples of equation solutions similar to question 1 on the Proving Solution Methods BLM. Have students use the *vocabulary self- awareness chart* to write explanations for each step of the solutions in the samples provided. As students become proficient in justifying solutions, select student-made *vocabulary self-awareness charts* to place on the bulletin board to remind students about proofs of solutions. *Vocabulary self-awareness charts* may also be used to review for tests and quizzes.

The second part of the activity will continue using the *vocabulary self-awareness charts* to justify the solution steps used in solving a different set of equations. Use the accompanying Proving Solution Methods BLM and the Think Pair Square form of *discussion* (view literacy strategy descriptions). Have the students solve equations in questions 2 through 4 and justify each step in the solution process, emphasizing the application of the various properties of equality. Students should also answer questions 5 and 6 providing justifications for each step in their explanations. In this application of Think Pair Square Share, after students have individually justified solutions, give them the opportunity to pair up and explain the process they used to solve each equation and the properties they chose to justify their solutions. Once pairs have had a chance to discuss the solutions, have two pairs of students form a group of four and continue the discussion. To help students who have difficulty with solving equations and justifying solutions, post solution keys strategically around the classroom for students to use to seek hints. Only one student may visit the solution center at a time; students may not take pencil or paper to the solution centers. Monitor discussions to ensure that students are on topic. After all groups have completed the discussions for all five questions, have the groups share with the whole class. Review the assignment to ensure understanding. The samples completed and discussed in this activity can later be used to study for tests or quizzes.

Activity 5: Using Inequalities to Solve Problems (GLE: 11)

Materials List: paper, pencil, Linear Inequalities to Solve Problems BLM

In 7^{th} and 8^{th} grade, students learned to solve inequalities. Review the basics of solving one-step and multi-step inequalities. Refer to *split-page notetaking* (view literacy strategy descriptions) from Activity 2. The Linear Inequalities to Solve Problems BLM is repeated here with inequality symbols replacing the equal signs. The *split-page notetaking* can be used for practice, for studying, and for use in solving application problems.

Present students with the following problem for class discussion:

Trashawn wants to order some DVDs from Yomovies.com. DVDs cost $17 per DVD plus $5.50 for shipping and handling. If Trashawn wants to spend at most $75, how many DVDs can he buy? How much money will he have left over? Have students give more examples of vocabulary that may be used in solving inequalities such as *at least, not more than,* or *not to exceed*. Use a math textbook as a reference to provide students with more sample problems.

2013-14
Activity 6: Solving Formulas as Equations (CCSS: A-CED.1, A-CED.4, RST.9–10.7)

Materials List: paper, pencil, Isolating Variables in Formulas BLM

This activity takes the idea of solving an equation and expanding it in preparation for solving application problems and for analyzing linear equations. Inform students that a formula is a special equation. An equation involving two or more variables is a literal equation. A formula is a special type of literal equation which students will have to use in science, mathematics, or social studies. To solve a literal equation requires that one variable be solved in terms of the other variables in the equation. To solve a formula for a particular variable, use the distributive property, order of operations, and the four properties of equality. Sample formulas include $d = rt$ (distance = rate x time), the point-slope formula $(y - y_1) = m(x - x_1)$ where (x_1, y_1) is an ordered pair that solves the equation. Introducing the linear equation formulas for transformation will help students become more fluent in solving linear equations in all forms as included in Units 3 and 4. Measurement formulas for temperature, area, perimeter, and volume are also examples of formulas that should be used to practice isolating selected variables. After a discussion of equations and formulas, students should complete the Isolating Variables in Formulas BLM. Once students have completed the BLM, have students share their answers with the whole class to check the assignment for correctness.

2013-14
Activity 7: Solving Formulas as Equations: Applications (CCSS: A-CED.1, A-CED.4, RST.9-10.7)

Materials list: paper, pencil, calculator, Solving Real-World Application Problems Using a Formula BLM

For students to become proficient with solving problems which require reading situations, they need practice in developing the skill for solving literal problems which involve application situations. Use the Solving Real-World Application Problems Using a Formula BLM which provides a process for solving application problem. The modified *process guide* (view literacy strategy descriptions) is designed to scaffold students' comprehension of a mathematical situation using formulas. The guide will also help students focus on the important information and ideas, making the reading more efficient. This particular guide focuses the students' attention to the verbal instructions in addition to the mathematical processes involved in the solution to situation problems. After the students work through the *process guide*, discuss the correct solutions and monitor student understanding of the applications of formulas. The *process guide* may be used to help students prepare for assessments by reworking the problems and developing parallel types of problems.

Activity 8: Linear relationships – Keeping it "real" (GLE's: 9, 11, 13, 37, 39; CCSS:A-CED 1, A-CED.2, A-REI. 10, F-LE.1, RST.9-10.7)

Materials List: paper, pencil, Linear Relationships BLM, calculator

The Linear Relationships BLM provides students with several input-output data tables that depict linear relationships found in real-world applications. For example, the relationship between the number of gallons of gasoline and the total purchase price or the number of minutes on a cell phone and the total monthly bill both depict a linear function. Have students plot the ordered pairs generated by these data tables on a coordinate graph. See that students recognize that the graph is linear. Have students find the rate of change for the data in the tables and relate that rate of change to slope. Have students write the equation to model the situation. Discuss the real-life meaning of the slope and the *y*-intercept for each table of values. (Although students have not been formally introduced to the terminology of slope and y-intercept, these examples should provide for a good discussion on the real-life meaning of slope and *y*-intercept). Have students state the rate of change in terms of the context of the problem. For example: For every gallon of gasoline purchased, the total cost increases by _____. Give students values that provide opportunities for them to solve the linear equations algebraically. For example, if John wants to spend exactly $20 on gasoline, how many gallons can he purchase?

Activity 9: Linear Relationships in Science (GLEs: 9, 37; CCSS: A-CED.1, A-CED. 2, A- CED 3, F-LE.1, RST.9-10.7)

Materials List: paper, pencil, math learning log, calculator, Unit Conversion BLM

Using the Unit Conversion BLM, provide students with the following table of data:

Mountain		Height		Location
1	Mount Everest	8,850m	29,035 ft	Nepal
2	Qogir (K2)	8,611m	28,250 ft	Pakistan
3	Kangchenjunga	8,586m	28,169 ft	Nepal
4	Lhotse	8,501m	27,920 ft	Nepal
5	Makalu I	8,462m	27,765 ft	Nepal
6	Cho Oyu	8,201m	26,906 ft	Nepal
7	Dhaulagiri	8,167m	26,794 ft	Nepal
8	Manaslu I	8,156m	26,758 ft	Nepal
9	Nanga Parbat	8,125m	26,658 ft	Pakistan
10	Annapurna I	8,091m	26,545 ft	Nepal

Have the students graph the heights of the mountains using meters as the independent variable and feet as the dependent variable. Have the students use the graph to determine the rate of change of the line formed by the points. Lead them to discover that the rate of change is the conversion factor for the two units of measure. Have students write the equation of the line. Discuss with students that the rate of change is also the constant of variation. Have students explore temperature conversions Fahrenheit to Celsius, Celsius to Fahrenheit to Celsius, and Celsius to Kelvin to determine whether other conversion equations and formulas result in linear equations.

In their math *learning logs* (view literacy strategy descriptions) have students reflect on the following statement:

*All unit conversions are linear relationships.

Have students write a paragraph explaining why they agree/disagree with the statement, and include examples to justify their position.

Activity 10: Linear Relationships (GLEs: 9, 11, 37, 39; CCSS A-CED.1, A-CED.2, F-LE.1, RST.9-10.7)

Materials List: paper, pencil, Linear Situations BLM, calculator

Have students identify some relationships that are direct proportions. For example, they could state that distance traveled is linearly related to the rate of travel, or the cost of movie tickets is linearly related to the number purchased, or their total earnings are linearly related to the hours they work.

After some discussion and sharing, divide students into groups and distribute the Linear Situations BLM. Assign each group of students one of the linear situations. Have students create an input-output table, plot the ordered pairs, and draw the line connecting the ordered pairs. Have students write equations to model each linear relationship. Have students determine the rate of change for each relationship, and have each group present their graphs to the entire class. Discuss with the students that the rate of change is the slope for each of the relationships graphed. Have the students state the rate of change in real-life terms. (Other proportional data sets that could be used: The total cost for a bunch of grapes is directly proportional to the number of pounds purchased, the number of miles traveled is directly proportional to the number of kilometers traveled, or if the width of a rectangle is kept constant, then the area of the rectangle is directly proportional to the height.)

Have students participate in a math *text chain* (view literacy strategy descriptions) in their groups to create a problem for each of the linear situations discussed in class and included on the Linear Situations BLM. The first student initiates the story and passes the paper to the next student who adds a second line. The next student adds a third line, until the last student solves the problem. All group members should be prepared to revise the story based on the last student's input as to whether it was clear.

Example:

1st student writes: Katherine got paid on Friday from her job at Cheesy Joe's Pizza.
2nd student writes: Her paycheck was $35 and she wants to use half of it to bring her friends to see Spiderman XIV.
3rd student writes: If movie tickets are $6.50, how many friends can she bring to a movie?
4th student solves the problem. *(Since Katherine has to pay for herself, she can only bring one friend with her.)*

Activity 11: Solving and Graphing Linear Relationship Equations (GLEs: 8, 9; CCSS: A-CED.1)

Materials List: paper, pencil, calculator

Students were exposed to proportional reasoning and solving proportions in 7th and 8th grade. In 8th grade, students used proportions to find the missing sides of similar triangles.

Review with students the concept of solving proportions. Have students set up and solve proportions that deal with real-life scenarios. For example, many outboard motors require a 50:1 mixture of gasoline and oil to run properly. Have students set up proportions to find the amount of oil to put into various amounts of gasoline. Recipes also provide examples for the application of proportional reasoning. Finding the missing side lengths of similar figures can allow students to set up a proportion as well as find measures by indirect measurement. For example, students can set up and solve a proportion that finds the height of an object by using similar triangles. Select problems which have variables in the denominator of the fractions. Use a math textbook as a reference to provide students with more opportunities to practice solving application problems using proportional reasoning.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- Performance Task: The student will find something that can be paid for in two different ways, such as admission to an amusement park or museum (Some museums will charge for each admission or sell a year-round pass, or an amusement park will sell a pay-one-price ticket or a per-ride ticket) and compare the costs. The student will explain the circumstances under which each option is better and justify the answers with a table, graph, and an equation, using inequalities to express their findings.

- The student will find the mistake in the solution of the following equation, explain the mistake, and solve the equation correctly:

$$\begin{aligned} 2x &= 11x + 45 \\ 2x - 11x &= 11x - 11x + 45 \\ 9x &= 45 \\ \frac{9x}{9} &= \frac{45}{9} \\ x &= 5 \end{aligned}$$

- The student will solve constructed response items such as this:
  The amount of blood in a person's body is related to their body weight. Someone weighing 160 lbs. has about 5 quarts of blood.
  a. Write an equation relating quarts of blood to weight. ($\frac{1}{32}$, $b = \frac{1}{32}w$)
  b. Graph your equation.
  c. Estimate the number of quarts of blood in your body.
- The student will use proportions to solve the missing parts of similar figures.
- The student will determine if the following situations represent linear situations and explain why or why not:
  - ✓ The amount of a gas in a tank in liters and the amount in gallons (*yes*)
  - ✓ The temperature in Fahrenheit degrees and in Celsius degrees (*yes*)
  - ✓ The price per pound of carrots and the number of pounds (*no*)
  - ✓ The total price of tomatoes and the number of pounds (*yes*)
- The student will submit a portfolio containing artifacts such as these:
  - ✓ daily student journals
  - ✓ teacher observation checklists or notes
  - ✓ examples of student products
  - ✓ scored tests and quizzes
  - ✓ student work (in-class or homework)
- The student will respond to the following prompts in their math *learning logs:*
  - ✓ Write a letter to a friend explaining order of operations.
  - ✓ Explain how solving an inequality is similar to solving an equation? In what ways is it different?
  - ✓ Describe a situation from your experience in which one variable is:
    - increasing at a constant rate
    - decreasing at a constant rate
    - increasing but not at a constant rate
  - ✓ Explain why the graph of a linear equation in the form of $y = mx$ always goes through the origin.

Activity-Specific Assessments

- Activity 2: The student will solve constructed response items such as this:
  The drama club is selling tickets to their production of *Grease* for $4 each.
  - ✓ Make a table and a graph showing the amount of money they will make if 0, 5, 10, …, 100 tickets are sold.

- ✓ Identify the variables and write an equation for the total amount the club will make for each ticket sold. ($y = 4x$)
- ✓ Use your equation to show how much money the club will make if 250 people attend their production. *($1000*)
- ✓ The club spent $500 on their production. How many tickets must they sell to begin to make a profit? Justify your answer. (*125 tickets)*

- Activity 5: Given an inequality such as $3x - 15 \geq 45$, the student will write an application problem for the inequality.

- Activity 10: The student will choose one of the linear relationship situations and write at least two application problems that can be solved using a linear equation. The student will then write the equation for each application problem and solve it algebraically.