

## 7-4 Factoring $ax^2 + bx + c$

### *Objective*

Factor quadratic trinomials of the form  
 $ax^2 + bx + c$ .

## 7-4 Factoring $ax^2 + bx + c$

In the previous lesson you factored trinomials of the form  $x^2 + bx + c$ . Now you will factor trinomials of the form  $ax^2 + bx + c$ , where  $a \neq 0$ .

Check for GCF!

## 7-4 Factoring $ax^2 + bx + c$

To factor a trinomial like  $ax^2 + bx + c$  into its binomial factors, write two sets of parentheses

$$(\quad x + \quad)(\quad x + \quad).$$

Write two numbers that are factors of  $a$  next to the  $x$ 's and two numbers that are factors of  $c$  in the other blanks. Multiply the binomials to see if you are correct.

$$\begin{array}{c} 3 \cdot 2 = 6 \qquad \qquad 2 \cdot 5 = 10 \\ (3x + 2)(2x + 5) = 6x^2 + 19x + 10 \end{array}$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 1a

Factor each trinomial by **guess and check**.

$$6x^2 + 11x + 3$$

$$\text{GCF} = 1$$

$+bx + c \rightarrow$  same (add)

1	6
2	3

$(3x + 1)(2x + 3)$

$\frac{3}{1 \quad 3}$

$$6x^2 + 9x + 2x + 3$$

$$6x^2 + 11x + 3 \checkmark$$

**7-4**

## Check It Out! Example 1a

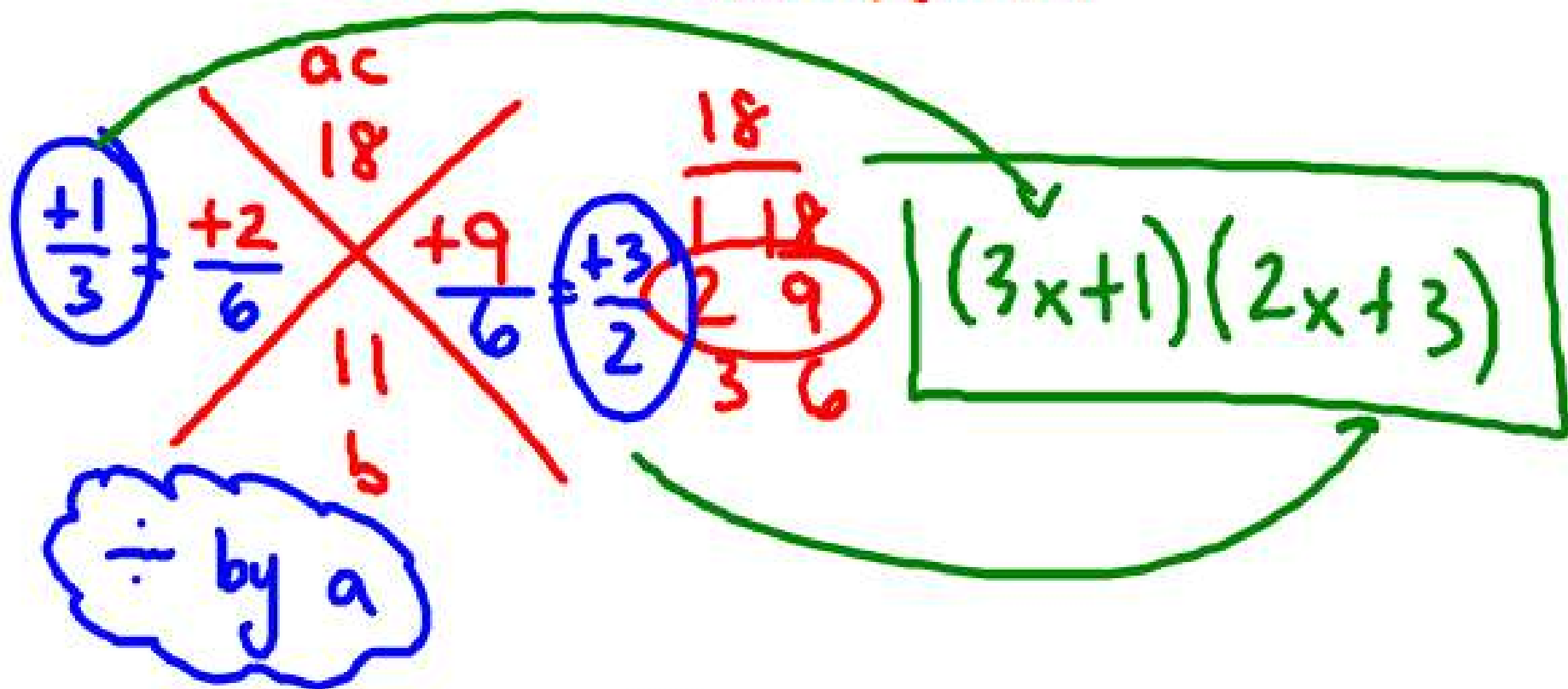
## X-Method

**Factor each trinomial by ~~guess and check~~.**

$$6x^2 + 11x + 3$$

$$GCF = 1$$

$+bx + c \rightarrow \text{same (add)}$



## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 1b

Factor each trinomial by guess and check.

$$3x^2 - 2x - 8 \quad \text{GCF} = 1$$

$-bx - c \rightarrow$  opp (subtract)  
↑ Bigger Product

$$\frac{3}{1 \ 3} \quad (1x - 2)(3x + 4) \quad \frac{8}{1 \ 8} \quad \frac{2 \ 4}{2 \ 4}$$

$$3x^2 + 4x - 6x - 8$$

$$3x^2 - 2x - 8 \checkmark$$

## 7-4 Factoring $ax^2 + bx + c$

**Check It Out! Example 1b** X-Method

Factor each trinomial by ~~guess and check~~.

$$3x^2 - 2x - 8$$

$$\text{GCF} = 1$$

$-bx - c \rightarrow$  opp (subtract)

Bigger #

$-\frac{2}{1}$   $-\frac{6}{3}$   $+\frac{4}{3}$   $2$

$\begin{array}{cc} 24 & 12 \\ 1 & 3 \\ 8 & 6 \end{array}$

$$(1x - 2)(3x + 4)$$

## 7-4 Factoring $ax^2 + bx + c$

So, to factor  $ax^2 + bx + c$ , check the factors of  $a$  and the factors of  $c$  in the binomials. The sum of the products of the outer and inner terms should be  $b$ .

The diagram illustrates the FOIL method for factoring a quadratic expression  $ax^2 + bx + c$ . It shows two binomials,  $(\square x + \square)$  and  $(\square x + \square)$ , where the squares represent unknown factors. The process is as follows:

- F (First):** A red arrow connects the first terms of the binomials, labeled "Product =  $a$ ".
- L (Last):** A green arrow connects the last terms of the binomials, labeled "Product =  $c$ ".
- O (Outer):** A blue arrow connects the first term of the first binomial to the last term of the second binomial.
- I (Inner):** A blue arrow connects the last term of the first binomial to the first term of the second binomial.
- O + I:** A red "O + I" label indicates the sum of the outer and inner products, which equals  $b$ .

The resulting expression is shown as:

$$(\square x + \square)(\square x + \square) = \overset{F}{a}x^2 + \overset{O+I}{b}x + \overset{L}{c}$$

Below the binomials, a blue arrow points to the sum of the outer and inner products, labeled "Sum of outer and inner products =  $b$ ".



## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2a

Factor each trinomial. Check your answer.

$$6x^2 + 17x + 5$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2a

Factor each trinomial. Check your answer.

$$6x^2 + 17x + 5$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2b

Factor each trinomial. Check your answer.

$$9x^2 - 15x + 4 \quad \text{GCF} = 1$$

$-bx + c \rightarrow \text{same (add)}$

$$\begin{array}{|c|c|} \hline 9 & 1 \\ \hline 3 & 3 \\ \hline \end{array} \quad (3x-1)(3x-4) \quad \begin{array}{|c|c|} \hline 4 & 1 \\ \hline 2 & 2 \\ \hline \end{array}$$

$$9x^2 - 12x - 3x + 4$$

$$9x^2 - 15x + 4 \checkmark$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2b

Factor each trinomial. Check your answer.

$$9x^2 - 15x + 4$$

$$\text{GCF} = 1$$

$-bx + c \rightarrow$  same (add)

The handwritten solution shows the factoring process for  $9x^2 - 15x + 4$  using the AC method. It starts with the trinomial and identifies  $\text{GCF} = 1$ . The coefficient  $a = 9$  is circled in blue. The constant term  $c = 4$  is also circled in blue. The product  $ac = 36$  is written above the terms. The coefficient  $b = -15$  is written below the terms. The goal is to find two numbers that multiply to 36 and add to -15. The numbers -12 and -3 are identified. The middle term  $-15x$  is split into  $-12x$  and  $-3x$ . The expression is then grouped as  $(9x^2 - 12x) + (-3x + 4)$ . Factoring out the GCF from each group gives  $3x(3x - 4) - 1(3x - 4)$ . The common binomial factor  $(3x - 4)$  is factored out, resulting in the final answer  $(3x - 4)(3x - 1)$ .

$$\begin{array}{r} 36 \\ \overline{1} \ 36 \\ 2 \ 18 \\ 3 \ 12 \\ 4 \ 9 \\ 6 \ 6 \end{array}$$
$$(3x - 1)(3x - 4)$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2c

Factor each trinomial. Check your answer.

$$3x^2 + 13x + 12$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 2c

Factor each trinomial. Check your answer.

$$3x^2 + 13x + 12$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 3a

Factor each trinomial. Check your answer.

$$6x^2 + 7x - 3 \quad \text{GCF} = 1$$

$+bx - c \rightarrow$  opp (subtract)  
↑  
Bigger Product

$$\frac{1}{6} \quad (3x - 1)(2x + 3) \quad \frac{1}{3}$$

$$6x^2 + 9x - 2x - 3$$

$$6x^2 + 7x - 3 \checkmark$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 3a

Factor each trinomial. Check your answer.

$$6x^2 + 7x - 3 \quad \text{GCF} = 1$$

$+bx - c \rightarrow$  opp (subtract)  
↑  
Bigger #

Diagram illustrating the factoring process for  $6x^2 + 7x - 3$  using the AC method.

Two trial divisions are shown:

- $\frac{2x+3}{x+\frac{7}{6}}$  (Crossed out with a red X)
- $\frac{2x-1}{x+\frac{3}{2}}$  (Correct division, with  $\frac{3}{2}$  circled in red)

Green arrows indicate the final factors:  $(2x+3)$  and  $(3x-1)$ .

Factored form:  $(2x+3)(3x-1)$



## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 3b

Factor each trinomial. Check your answer.

$$4n^2 - 1n - 3 \quad \text{GCF} = 1$$

$-bn - c \rightarrow \text{opp (subtract)}$

Bigger Product

$$\begin{array}{r} 4 \\ 1 \overline{) 4} \\ 2 \quad 2 \end{array} \quad (1n - 1)(4n + 3)$$

$$\begin{array}{r} 3 \\ 1 \overline{) 3} \\ 1 \quad 3 \end{array}$$

$$4n^2 + 3n - 4n - 3$$

$$4n^2 - 1n - 3 \checkmark$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 3b

Factor each trinomial. Check your answer.

$$4n^2 - 1n - 3 \quad \text{GCF} = 1$$

$-bn - c \rightarrow \text{opp}(\text{subtract})$

$\frac{-1}{1} = \frac{-4}{4}$   $\frac{+3}{4}$

$\frac{12}{1 \ 12}$   
 $\frac{2 \ 6}{3 \ 4}$

$(1n - 1)(4n + 3)$

*Bigger #*

## 7-4 Factoring $ax^2 + bx + c$

When the leading coefficient is negative, factor out  $-1$  from each term before using other factoring methods.

### Caution

When you factor out  $-1$  in an early step, you must carry it through the rest of the steps.

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 4a

Factor each trinomial.

$$\textcircled{-6}x^2 - 17x - 12 \quad \text{GCF} = -1$$

$$-1(6x^2 + 17x + 12)$$

$+bx + c \rightarrow$  same (add)

Factor pairs of 72:

$\frac{+4}{3} = \frac{+8}{6}$	$\frac{+9}{6} = \frac{+3}{2}$
<del>17</del>	<del>17</del>

Factor pairs of 72:

1	72
2	36
3	24
4	18
6	12
8	9

$$-1(3x + 4)(2x + 3)$$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 4a

Factor each trinomial.

$$-6x^2 - 17x - 12$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 4b

Factor each trinomial.

$-3x^2 - 17x - 10$       GCF = -1

$-1(3x^2 + 17x + 10)$

$\pm bx + c \rightarrow$  same (add)

$\frac{+2}{3}$      $\frac{+15}{3}$      $\frac{+5}{1}$      $\frac{+30}{15}$

30    30

17

$-1(3x+2)(1x+5)$

## 7-4 Factoring $ax^2 + bx + c$

### Check It Out! Example 4b

Factor each trinomial.

$$-3x^2 - 17x - 10$$

SKIP!

## 7-4 Factoring $ax^2 + bx + c$

### HOMEWORK

**BASIC:** p. 484 #25-64, 68-74, 76

**AVERAGE:** p. 484 #25-70, 75-76

### HOMEWORK HELP?

#30, 35, 48, 50, 58, 64

not #49-51, 60

Inclass