Summit Public Schools

Summit, New Jersey

Grade Level 8/ Content Area: Mathematics

Length of Course: Full Academic Year

Curriculum: Algebra Enriched

Developed by: Colin Breivogel

Course Description:

This course represents a comprehensive Algebra I course. It has been designed to offer a rigorous and challenging curriculum that meets the 8th Grade Algebra Common Core Standards. The course covers a wide range of topics but also investigates each topic in great depth. Students will be expected to master the skills necessary to continue successfully into higher-level mathematics courses in subsequent years. Students will learn the algebraic properties necessary to solve a variety of problems including multi-step, absolute value, and quadratic equations. They will learn to represent a variety of functions graphically and algebraically. These functions will include linear, quadratic, absolute value, and exponential. The rules and patterns of exponents will be explored and students will use exponents to model real world situations like exponential decay/growth and compound interest. Students will learn how to solve quadratic equations using both factoring and the quadratic formula. They will explore the characteristics of graphs of quadratic functions and how these graphs relate to equations and solutions. Students will explore radical expressions and fractional roots. They will learn how to simplify radical expressions and rationalize these expressions. They will also learn about polynomials and various operations using polynomials. This will include addition, subtraction, multiplication, and division of polynomial expressions. Throughout all of these topics critical thinking and problem solving skills will be emphasized. Students will be expected to apply their knowledge to real and relevant problems. Students will also learn the appropriate use of technology, such as graphing calculators and web-based utilities to model and analyze a wide range of mathematical relationships.

Texts and Resources:

Algebra 1: An Integrated Approach (McDougal, Littell & Co. ©1991) Algebra: tools for a Changing World (Prentice-Hall © 2001) NCTM-Illuminations Online Resources (http://illuminations.nctm.org/ActivityDetail.aspx?ID=146)

Standards:

Common Core State Standards for Mathematics (2012) (http://www.corestandards.org/Math)

Unit 1: Relationships between Quantities and Reasoning with Equations

Standard N.Q.1-3, A.SSE.1, A.APR.1, A.CED.1-4, A.REI.1

Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Big Ideas: Course Objectives / Content Statement(s)

- Reason quantitatively and use units to solve problems.
- Interpret the structure of expressions.
- Create equations that describe numbers or relationships.
- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?

How are verbal and algebraic models and formulas used to represent real life situations?

- How can units of measure be used to make problem solving decisions?
- How can variables be used to solve problems dealing with consecutive integers?
- What are linear equations and inequalities and how are they related to graphs?
- What are exponential functions and how are they related to graphs?
- How can the result of an equation be checked?

Enduring Understandings What will students understand about the big ideas?

Students will...

- Construct algebraic equations to represent patterns and real-world problems.
- Use algebraic techniques and properties to set up equations, translate words into symbols, and translate problems into equations.
- Recognize equivalent expressions and equations.
- Apply dimensional analysis strategies to facilitate computations.
- Differentiate between linear and non-linear (including exponential) relationships.

 How can rates, ratios, percents, and proportions be applied to problem solving?

Areas of Focus: Proficiencies (Cumulative Progress Indicators)

Students will:

N.Q.1-3. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Define appropriate quantities for the purpose of descriptive modeling. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

A.CED.1-4. Create linear, quadratic, simple rational and exponential equations and inequalities in one variable and use them to solve problems. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a

Examples, Outcomes, Assessments

Instructional Focus:

- Converting words and phrases to expressions and operations.
- Quickly assess a visual model (graph) as being linear, quadratic, absolute value or exponential.
- Apply algebraic properties to set-up and solve rate problems.
- Applying equations for exponential growth and decay and using these equations to solve real-world problems.

Sample Assessments:

- 2 unit quizzes
- 1 unit test

SCR: Identify the vertex the following quadratic: $y = 2(x - 3)^2 - 5$

ECR: One jet travels at 400mph. Another travels at 600mph. The jets start from the same point and travel in the same direction, but the slower jet leaves 2 hours earlier. How long will it be before the faster jet overtakes the slower jet? How far will they be from their departure point?

Performance Assessment Task: In your own words, describe the key features of the graph of a quadratic function and how it compares to the quadratic's equation (parabolic form).

Instructional Strategies: Interdisciplinary Connections

• Science: Construct exponential growth and decay models for experiments involving germ growth or radioactive decay.

viable argument to justify a solution method.

Technology Integration

 Using a computer model or graphing calculator, graph multiple linear equations that vary in slope but have the same y-intercept. How is slope represented on the graph of a linear equation?

Global Perspectives

• Research the different units of measure used around the world. Write an equation to convert each of the different units of measure used for length into the customary system used in the United States. Describe some of the benefits or drawbacks of the different systems of measurement.

Unit 2: Linear and Exponential Functions

Standard N.RN.1-2, 8.EE.8, A.REI.5-6, A.REI.10-12, 8.F.1-5, F.IF.1-9, F.BF.1-3, F.LE.1-3, F.LE.5

Students learn function notation and the characteristics of a function. Explore examples of functions and interpret functions graphically, numerically, symbolically, and verbally. Work will include functions that can be represented with formulas as well as those that cannot. Students will reason appropriate units when functions are given in context. Systems of equations and inequalities will be explored and solutions will be interpreted. Students will extend their understanding of integral exponents and exponential functions. They will compare and contrast linear and exponential functions and their appropriate usage.

Big Ideas: Course Objectives / Content Statement(s)

- Extend the properties of exponents to rational exponents.
- Analyze and solve linear equations and pairs of simultaneous linear equations.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.
- Define, evaluate, and compare functions.
- Understand the concept of a function and use function notation.
- Use functions to model relationships between quantities.
- Interpret functions that arise in applications in terms of a context.
- Analyze functions using different representations.
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.
- Construct and compare linear, quadratic, and exponential models and solve problems.

Essential Questions	Enduring Understandings
What provocative questions will foster	What will students understand about the

inquiry, understanding, and transfer of learning?	big ideas?
 How are rational exponents used to represent radical notation? How are the solutions to systems of equations represented graphically or algebraically? How can the solution to a system of equations be interpreted in the context of real-world problems? What are the solutions to linear equations and inequalities and how can they be represented graphically? What is a function and how can it be represented graphically, algebraically, or with other methods? How can functions be used to represent bi-variate data? How does the addition or subtract of constants to a functions change it's graph? When is it appropriate to use linear functions versus a quadratic or exponential function? How can you use linear, quadratic, and exponential functions to solve real-world problems? 	 Students will Write expressions involving radicals and rational exponents using the properties of exponents. Write and solve in a variety of manners linear equations in the context of real-world examples. Understand that the graphs of equations are representations of the equations solutions for linear, exponential, absolute value, and logarithmic functions. Understand the definition and characteristics of a function Compare the properties and characteristics of different functions including their graphs and equations. Construct functions in a variety of manners to represent the relationship between two quantities. Distinguish situations that can be represented with linear versus exponential functions. Use linear and exponential functions to solve real-world examples.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: N.RN.1-2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. 8.EE.8 Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Solve systems of two linear equations in two variables algebraically, and	 Instructional Focus: Using rational exponents to model radical equations Graph Linear equations to determine possible solutions Solve systems of equations graphically and algebraically Define functions and identify key characteristics of their graphs and equations Model real world situations with

estimate solutions by graphing the equations. Solve real-world and mathematical problems leading to two linear equations in two variables.

A.REI.5-6 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.10-12 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Explain why the *x*-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

8.F.1-5 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. Construct a function to model a linear relationship between two quantities. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. F.IF.1-9 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in

function equations

- Create and compare linear, quadratic, and exponential functions
- Applying equations for exponential growth and decay and compare the results to a similar linear relationship

Sample Assessments:

- 2 unit quizzes
- 1 unit test

SCR: Evaluate the function f(x) = 4x(x-6) for the domain $\{-2, 0, 1.5\}$.

ECR: A ball is dropped from the top of a building 145 feet high. Write a function to model the height of the ball for any time "t" seconds. Use your function to determine after how many seconds the ball hits the ground.

Performance Assessment Task: In your own words, compare and contrast the graphs and equations for linear, quadratic, and exponential functions. Be sure to discuss the relative rates of change for each type and potential uses for application.

Instructional Strategies: Interdisciplinary Connections

- Science: Use projectile motion equations to model the height of objects when affected by gravity.
- Social Studies: Write functions to determine rate of return on stock investments.

Technology Integration

 Using a computer model or graphing calculator, graph linear, quadratic, and exponential functions and compare and contrast their characteristics.

Media Literacy

 Look through the local newspaper or magazine and find 3 advertisements for local or national banks. Compare the terms of a context. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Compare properties of two functions each represented in a different way.

F.BF.1-3 Write a function that describes a relationship between two quantities. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.1-3 Distinguish between situations that can be modeled with linear functions and with exponential functions. Construct linear and exponential functions, including arithmetic and geometric sequences.
F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

advertised rates for checking or savings accounts. Use an exponential equation to compare the different offers and determine which will lead to the greatest interest after 1, 5, and 10 years.

Unit 3: Descriptive Statistics

Standard S.ID.1-3 & 5-9, 8.SP.1-4

Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Big Ideas: Course Objectives / Content Statement(s)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Investigate patterns of association in bivariate data.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of

learning?

Enduring Understandings
What will students understand about the
big ideas?

- How is a scatter plot different from graphing a linear equation?
- How is a line plot different from a histogram?
- What are outliers? How are they determined?
- What type of correlation does the plotted data suggest? How does this correlation relate to the two variables?

Areas of Focus: Proficiencies (Cumulative Progress Indicators)

Students will:

S.ID.1-3. Represent data with plots on the real number line (line plots, histograms, and scatter plots). Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread interquartile range) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

8.SP.1-4 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative

Students will...

- Apply a line-of-best-fit (trend line) for a given set of plotted data.
- Identify a linear equation for a given trend line.
- Use the trend line or related equation to extrapolate additional output values for given inputs.
- Interpret linear models (line plots, frequency tables & histograms).

Examples, Outcomes, Assessments

.Instructional Focus:

- These tasks will represent an extension of previous data modeling with emphasis on developing linear equations and extrapolating additional values (ordered pair).
- Instruction will focus on the differences, advantages and limitations of each data model.

Sample Assessments:

- 2 unit quizzes
- 1 unit test

SCR: If p varies inversely with q, and p = 6 when q = 8, find q when p = 12.

ECR: Plot a given set of data and determine the rate of change. Explain why the data's relationship is/is not linear.

Performance Assessment Task: Develop a scatter plot for a given set of data. Apply a trend line to the model and identify a linear equation that matches the correlation. Use the equation or trend line to find additional ordered pair.

Instructional Strategies: Interdisciplinary Connections

• Science: Develop a scatter plot for

frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

S.ID.5-9. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data. Recognize possible associations and trends in the data. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Compute (using technology) and interpret the correlation coefficient of a linear fit. Distinguish between correlation and causation.

an experiment (for example, the relationship between temperature and pressure for a given volume). Identify the trend line and determine if the relationship is linear.

Technology Integration

 Using online models (for example <u>NCTM-Illuminations</u>), enter data into a frequency table and examine the resulting data plot and trend line. Identify the relationship between input and output values.

Media Literacy

• Voter polls are much more accurate than they used to be. The science of random selection poll participants began after a poll conducted in 1936 inaccurately predicted who would win the Presidential election. Find a news article that uses poll data as a source of information. Research the polling company and the methods used and describe why random selection polling is an accurate way of making predictions.

Unit 4: Expressions and Equations

Standard A.SSE.1-3, A.APR.1, A.CED.1-4, A.REI.4 & 7

Students will extend the laws of exponents to rational exponents; apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions.

Big Ideas: Course Objectives / Content Statement(s)

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.
- Perform arithmetic operations on polynomials.
- Create equations that describe numbers or relationships.
- Solve equations and inequalities in one variable.
- Solve systems of equations.

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?

How can viewing the parts of a complicated expression as one entity aid in the interpretation of the expression?

- How was the quadratic formula derived?
- What methods are available to solve quadratic equations and under what conditions are each used?
- In what situations will a problem best be modeled by an exponential function? a quadratic function? a linear function?

Areas of Focus: Proficiencies (Cumulative Progress Indicators)

Students will:

A.SSE.1, A.SSE.2. Interpret parts of expressions in terms of context including those that represent square and cube roots; use the structure of an expression to identify ways to rewrite it.

A.SSE.3 Manipulate expressions using factoring, completing the square and properties of exponents to produce equivalent forms that highlight particular

Enduring Understandings What will students understand about the big ideas?

Students will...

- Interpret parts of expressions in terms of context
- Manipulate expressions using factoring, completing the square and properties of exponents.
- Write linear and exponential functions from graphs, tables, or as description of the relationship.
- Solve quadratic equations in one variable using a variety of methods.

Examples, Outcomes, Assessments

Instructional Focus:

- Adding, subtracting, multiplying and factoring polynomials and using them to solve real-world problems.
- Choosing the appropriate method for solving polynomial functions.
- Applying equations for exponential growth and decay and using these equations to solve real-world problems.

properties such as the zeros or the maximum or minimum value of the function.

A.APR.1 Perform addition, subtraction and multiplication with polynomials and relate it to arithmetic operations with integers.

F.BF.2 Write linear and exponential functions from graphs, tables, or a description of the relationship, recursively and with an explicit formula, and describe how quantities increase linearly and exponentially over equal intervals.

A.CED.1, 1 & 4 Create equations and inequalities in one variable and use them to solve problems; create linear and quadratic equations that represent a relationship between two or more variables and graph equations on the coordinate axes with labels and scale.

A.REI.4 Derive the quadratic formula by completing the square and recognize when there are no real solutions; solve quadratic equations in one variable using a variety of methods.

• Using completing the square to derive the quadratic formula.

Sample Assessments:

- 2 unit quizzes
- 1 unit test

SCR: Factor the quadratic trinomial.

$$w^2 - 2w - 24$$

ECR: Suppose you throw a baseball with an upward starting velocity of v ft/s. The equation $h = -16t^2 + vt + s$ is used to find the ball's height h in feet t seconds after it is thrown. If the ball is tossed directly upward with a starting velocity of 46 ft/s from a starting height of 6 ft, when will the ball hit the ground?

Performance Assessment Task: In your own words, explain what is accomplished by factoring. As part of your explanation, write a sample problem where you factor a monomial from a polynomial. Then write and factor another problem that is a quadratic expression. Check your solution by multiplying after you factor.

Instructional Strategies:

Interdisciplinary Connections

• Geometry: The base of a triangle is 8 ft more than twice its height. The area of the triangle is 45 ft². Find the dimensions of the triangle.

Technology Integration

 Using a graphing calculator, graph the functions on the same calculator screen.

$$Y = 2x - 5$$

$$Y = x + 4$$

$$Y = 2x^{2} + 3x - 20$$

How are the x-intercepts of the linear functions related to the x-intercepts of the quadratic function?

Media Literacy
Use print or Internet media to find a
real life situation that can be
modeled by each of the following:
exponential equation, linear equation,
and quadratic equation. Write an
equation to model each situation and
explain why your equation is an
appropriate and accurate model.

Unit 5: Quadratic Functions and Modeling

Standard N.RN.3, F.IF.4-9, F.BF.1, F.BF.3 & 4; F.LE.3.

Students will explore distinctions between rational and irrational numbers; compare key characteristics of quadratic functions to those of linear and exponential functions; identify the real solutions of a quadratic equation as the zeros of a related quadratic function; expand their experience with functions to include more specialized functions including absolute value, step and piecewise-defined.

Big Ideas: Course Objectives / Content Statement(s)

- Use properties of rational and irrational numbers.
- Interpret functions that arise in applications in terms of a context.
- Analyze functions using different representations.
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.
- Construct and compare linear, quadratic and exponential models and solve problems.

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?

Enduring Understandings What will students understand about the big ideas?

How can we determine whether the sum and product of real numbers is rational or irrational?

- How do the key features of graphs and tables enable one to interpret and model a relationship between two quantities?
- How can factoring and completing the square show the zeros, extreme values and symmetry of a graph and how can these key features be interpreted in terms of a context?
- How is the graph of a function affected by replacing f(x) by f(x)+k, kf(x), f(kx) and f(x+k)?
- How do the graph and table of a quantity increasing exponentially differ from a quantity increasing linearly?

Students will

- Use factorization, when suitable, to identify zeros of polynomials and use the zeros to construct a rough graph of the function..
- Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, or verbal description).
- Write functions in different but equivalent forms by manipulating quadratic expressions using methods such as factoring and completing the square.
- Identify the effects of translations on a function and find the value of *k* given the graphs.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)

Students will:

A.APR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

F.IF.4, 5, & 7 Sketch the graph of a function that models a relationship between two quantities showing key features including intercepts,

minimums/Maximums, domain and rate of change by hand in simple cases and using technology in more complicated cases and relate the domain of the function to its graph.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions).

F.IF.6 Calculate and interpret the average rate of change of a function.

F.IF.8 Write functions in different but equivalent forms by manipulating quadratic expressions using methods such as factoring and completing the square.

F.BF.1 Write a function that describes a linear or quadratic relationship between two quantities given in context using an explicit expression, a recursive process, or st3eps for calculation and relate these functions to the model

F.BF.3 Identify the effects of translations on a function and find the value of k given the graphs.

L.LE.3 & 5 Compare linear, quadratic, and exponential models to determine that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function (include interpretation of parameters in terms of a context).

Examples, Outcomes, Assessments

Instructional Focus:

- Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions.
- Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.
- Interpret functions that arise in application in terms of a context.

Sample Assessments:

- 2 unit quizzes
- 1 unit test

SCR:

How many x-intercepts will the equation have?

$$y = -2(x - 4)^2 + 5$$

ECR:

A parabola has an equation of the form $y = a(x - h)^2 + k$. The vertex is at (4, -3) and the graph passes through the points (6, 9). Find the equation of the parabola.

Performance Assessment Task: From 1950 to 1990, the average annual consumption, C, of cigarettes per American (18 or older) can be modeled by

$$C = 4024.5 + 51.4t - 3.1t^2$$

Where t is the year, with t = 0 corresponding to 1960. Graph the equation on a graphing calculator and answer the question below.

From 1966 on, all cigarette packages were required by law to carry a health warning. Did the warnings have any effect? Explain.

Instructional Strategies:

Interdisciplinary Connections

• From 1960 to 1990, the total government payroll in the US can be modeled by P = 35t² + 115t + 3410, where P is in millions of dollars and t = 0 corresponds to 1960. Judging from the payroll between 1960 and 1990, is it possible that the government payroll will reach 70 billion dollars a year?

Technology Integration

• On a graphing calculator, sketch the graph of $y = x^2 - 3$.

Zoom once to get a better view of the positive x-intercept. Zoom a second time to get an even better view. Use the cursor keys to determine the x-intercept. Zoom a third time. Use the trace feature to find the y-value that is closest to 0. The corresponding x-value is your approximation.

Global Perspective

• The Arecibo Observatory is a radio telescope located in Puerto Rico. The observatory's 1,000 ft radio telescope is the world's largest single–aperture telescope. The shape of the dish used can be modeled with a quadratic equation. Research the dimensions of the telescope and write an equation that closely models the telescope and describe some of the research done by radio telescopes