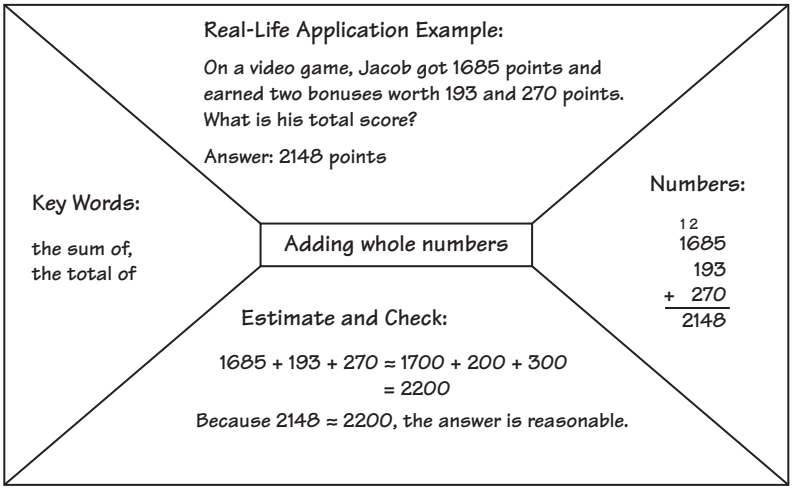
Chapter 1 Numerical Expressions and Factors

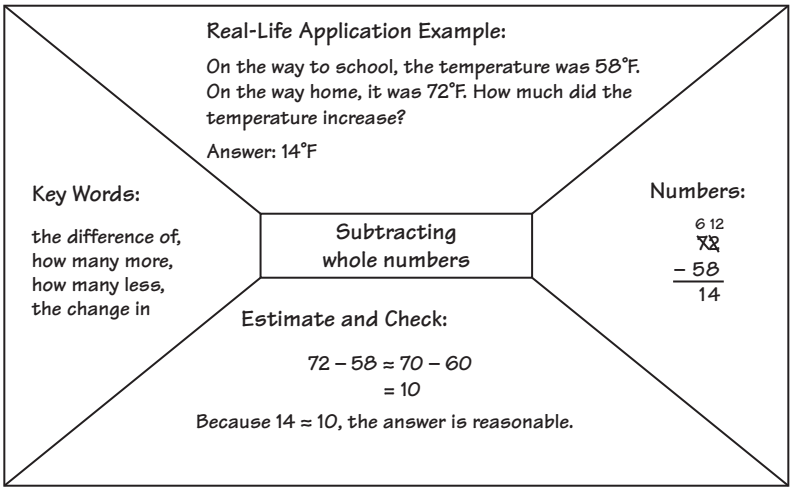
Information Frame

1–9. Sample answers are given.

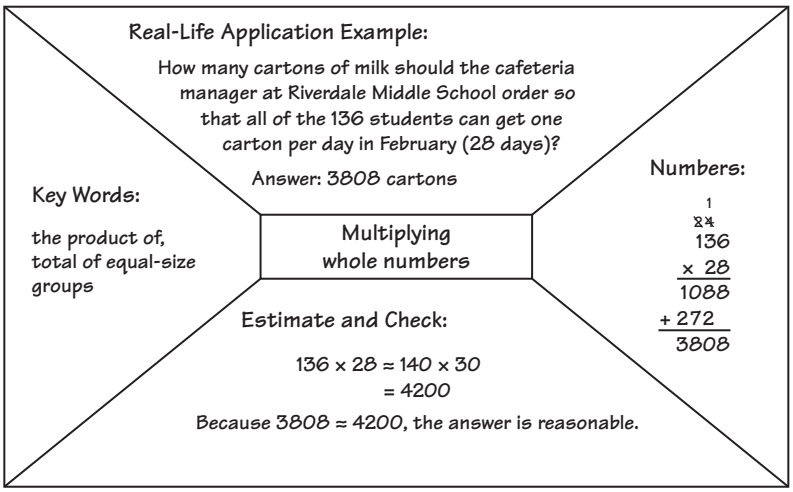
1.



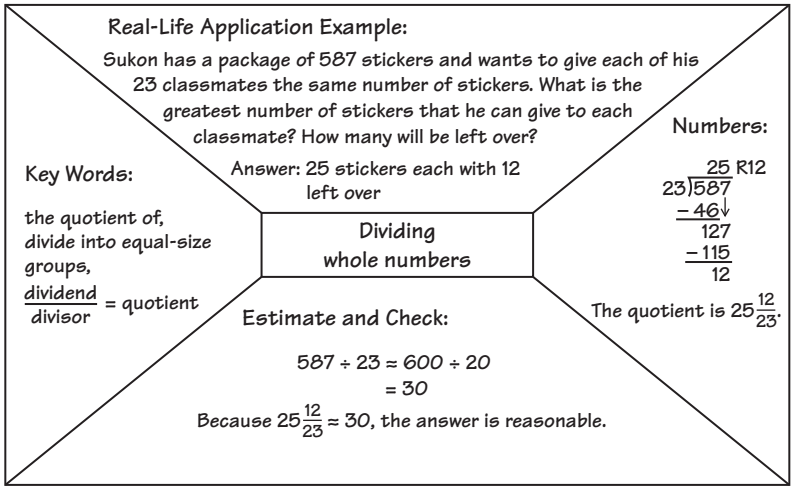
2.



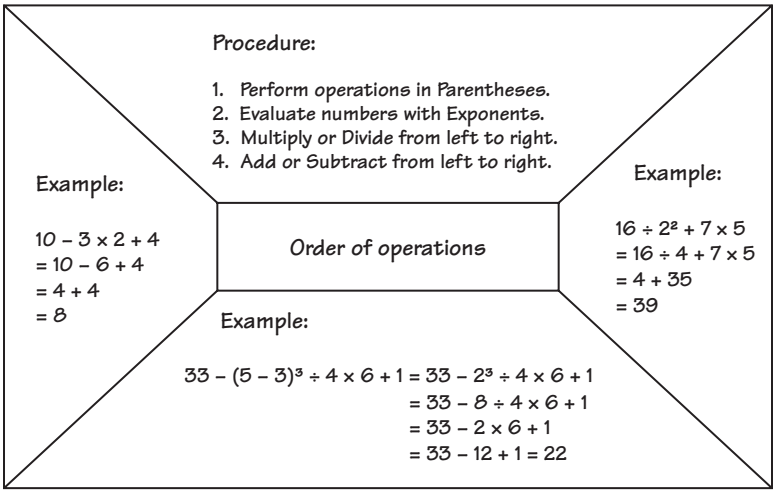
3.



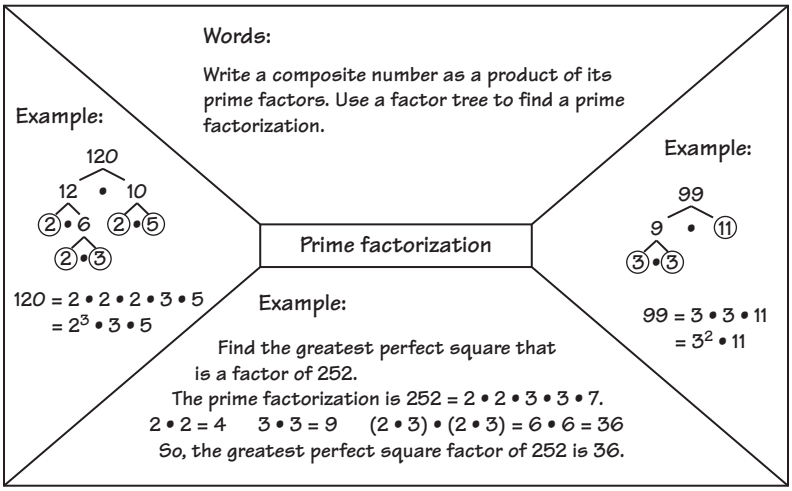
4.



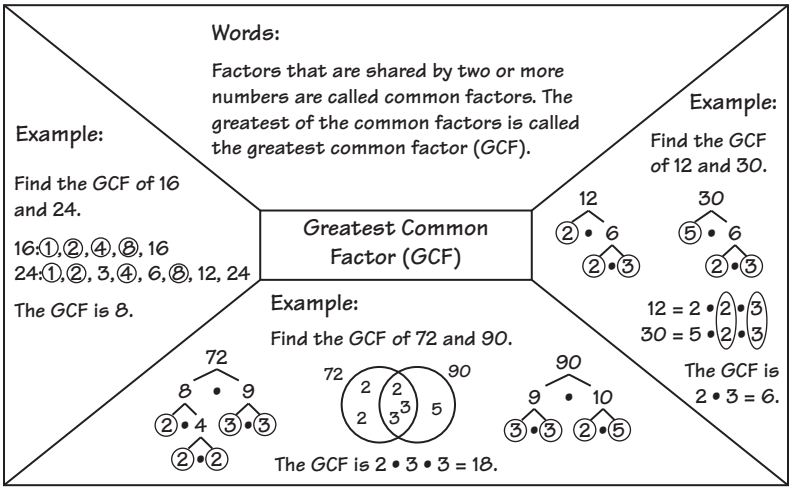
5.



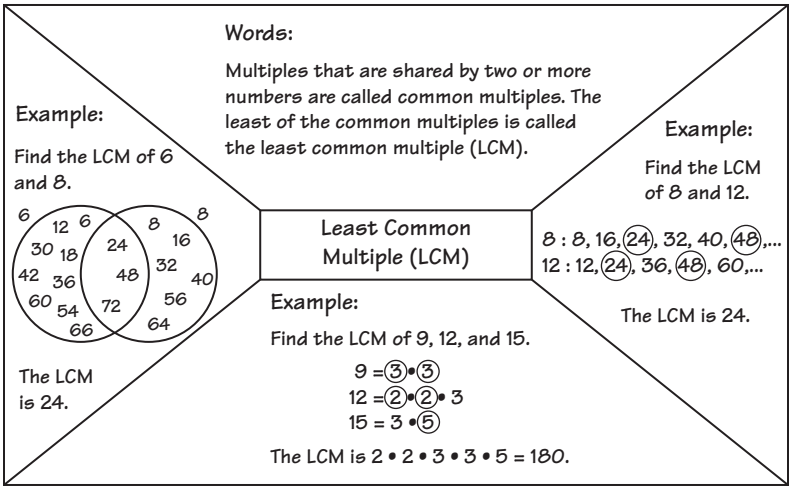
6.



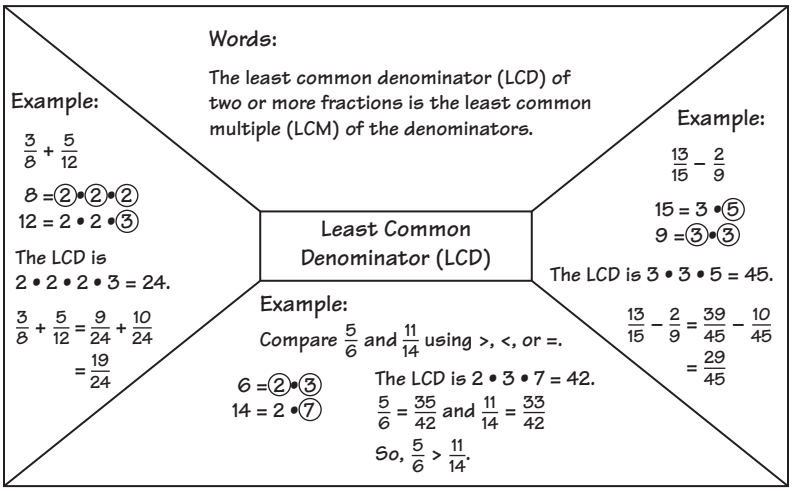
7.



8.



9.



Chapter 2 Fractions and Decimals

Notetaking Organizer

1–8. Sample answers are given.

1.

$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ $(b \neq 0$ and $d \neq 0)$	Multiplying fractions Multiply the numerators and multiply the denominators. The denominators cannot be 0. Example: $\frac{3}{4} \times \frac{5}{8} = \frac{3 \times 5}{4 \times 8} = \frac{15}{32}$
How do you multiply two mixed numbers?	

2.

$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ $(b \neq 0$ and $d \neq 0)$	Multiplying mixed numbers 1. Write each mixed number as an improper fraction. 2. Multiply as you would with fractions. Example: $1\frac{2}{3} \times 2\frac{3}{4} = \frac{5}{3} \times \frac{11}{4}$ $= \frac{5 \times 11}{3 \times 4}$ $= \frac{55}{12}$, or $4\frac{7}{12}$
Why can't b and d equal 0? How do you divide mixed numbers?	

3.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

(b, c, and $d \neq 0$)

Dividing mixed numbers

1. Write each mixed number as an improper fraction.
2. Divide as you would with fractions.

Example:

$4\frac{3}{4} \div 2\frac{1}{2} = \frac{19}{4} \div \frac{5}{2}$

$= \frac{19}{4} \times \frac{2}{5}$

$= \frac{19 \times \cancel{2}^{1}}{\cancel{4} \times 5}$

$= \frac{19}{10}$, or $1\frac{9}{10}$

How do you divide decimals?

4.

Be sure to line up the decimal points so that you add or subtract only the digits that have the same place value. You may have to insert zeros.

addend
+ addend
sum

Adding and subtracting decimals

1. Line up the decimal points.
2. Bring down the decimal point.
3. Add or subtract as you would with whole numbers.

Example: 17.625 + 108.3

1
17.625
+ 108.300 ← Insert zeros.
125.925

How do you add or subtract a decimal and a fraction?

5.

Remember to check that your product has the same number of decimal places as the decimal factor. You may have to insert zeros in your product.

You do not have to line up the decimal points when multiplying.

factor
× factor
product

Multiplying decimals by whole numbers

1. Multiply as you would with whole numbers.
2. Count the number of decimal places in the decimal factor.
3. The prduct has the same number of decimal places.

Example: 16.64 × 3

1 1 1
16.64
× 3
49.92

two decimal places

How do you multiply two decimals?
Why don't you line up the decimal points when multiplying?

6.

Remember to check that your product has the same number of decimal places as the sum of the decimal places in the factors. You may have to insert zeros in your product.

You do not have to line up the decimal points when multiplying.

factor
× factor
product

Multiplying decimals by decimals

1. Multiply as you would with whole numbers.
2. Add the number of decimal places in the factors.
3. The sum is the number of decimal places in the product.

Example: 3.6 × 4.07

4.07 ← 2 decimal places
× 3.6 ← 1 decimal places
2442
1221
14.652 ← 3 decimal places

Why don't you line up the decimal points when multiplying?
How do you multiply a decimal by a fracton?

7.

$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$

$\text{divisor}\overline{)\text{dividend}}\ \text{quotient}$

Remember to place the decimal point in the quotient first.

Dividing decimals by whole numbers

1. Place the decimal point in the quotient above the decimal point in the dividend.
2. Divide as you would with whole numbers.
3. If needed, insert a zero in the dividend, and continue to divide.

Example: $11.7 \div 6$

$$\begin{array}{r} 1.95 \\ 6\overline{)11.70} \\ -\,6 \\ \hline 57 \\ -\,54 \\ \hline 30 \\ -\,30 \\ \hline 0 \end{array}$$

Why can you insert a zero after a decimal?
How do you divide a decimal by another decimal?

8.

$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$

$\text{divisor}\overline{)\text{dividend}}\ \text{quotient}$

Multiplying the divisor and dividend by a power of 10 does not change the quotient.

Dividing decimals by decimals

1. Multiply the divisor and the dividend by a power of 10 to make the divisor a whole number.
2. Place the decimal point in the quotient.
3. Divide as you would with whole numbers.
4. If needed, insert a zero in the dividend and continue to divide.

Example: $0.259 \div 0.14$

$$0.14\overline{)0.259} \rightarrow 14\overline{)25.90}$$

$$\begin{array}{r} 1.85 \\ 14\overline{)25.90} \\ -\,14 \\ \hline 119 \\ -\,112 \\ \hline 70 \\ -\,70 \\ \hline 0 \end{array}$$

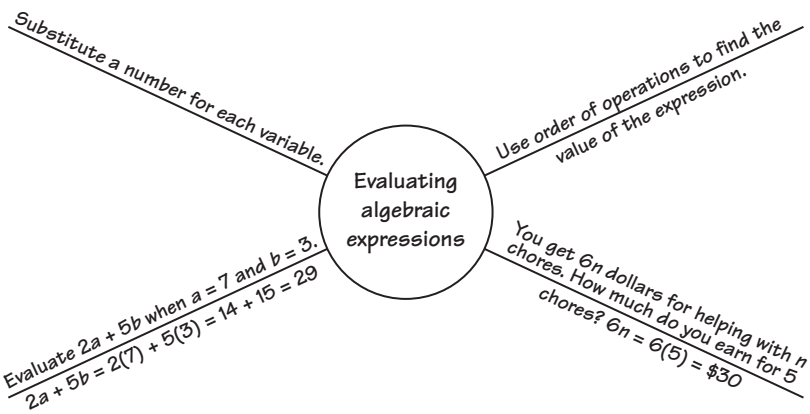
Why can you insert a zero after a decimal?
How do you divide a fraction by a decimal?

Chapter 3 Algebraic Expressions and Properties

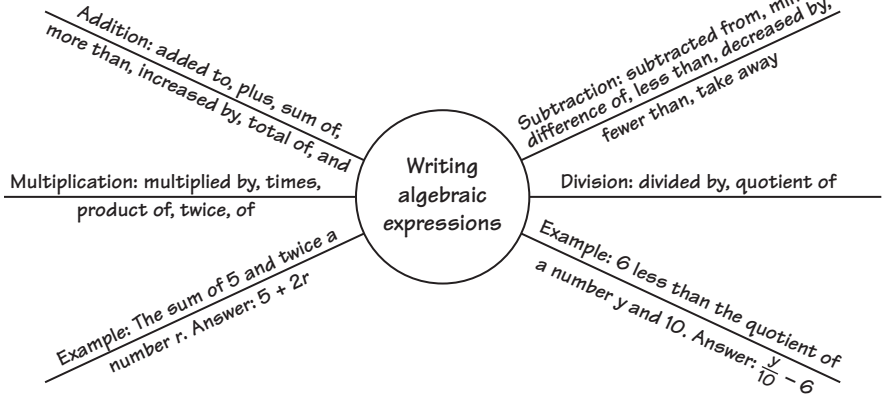
Information Wheel

1–8. Sample answers are given.

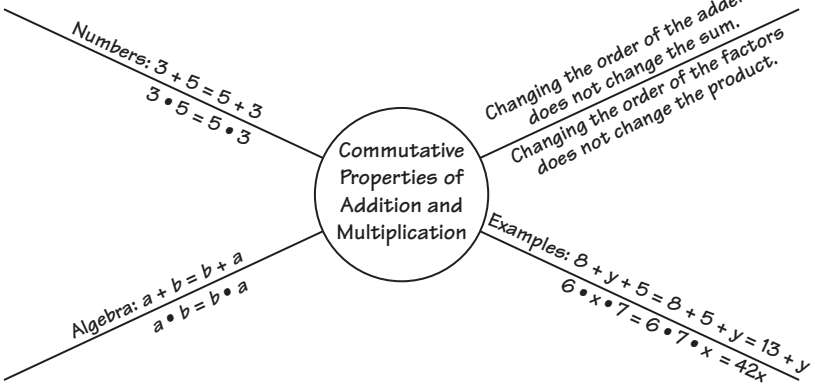
1.



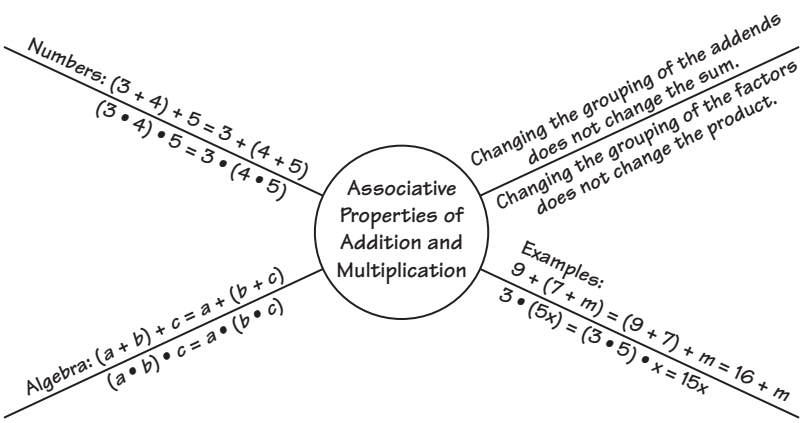
2.



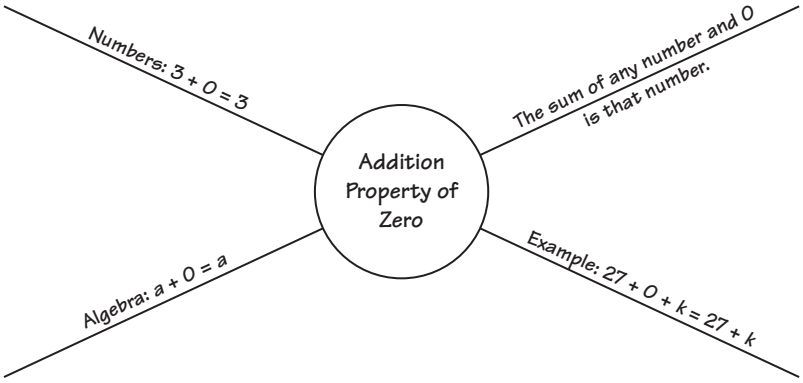
3.



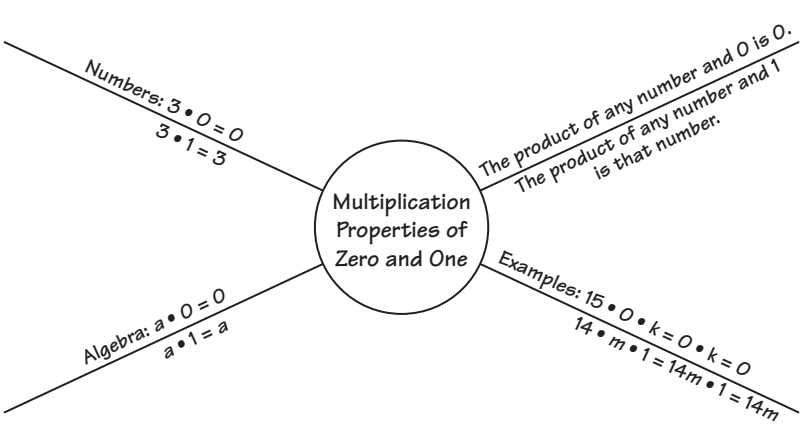
4.



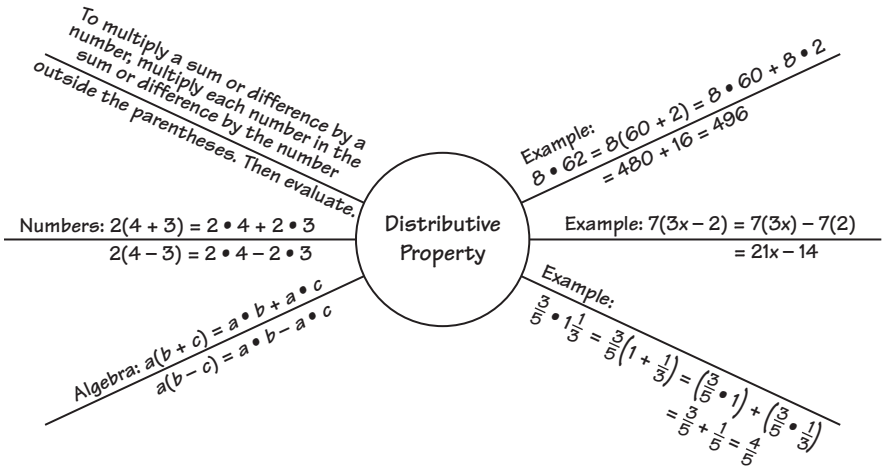
5.



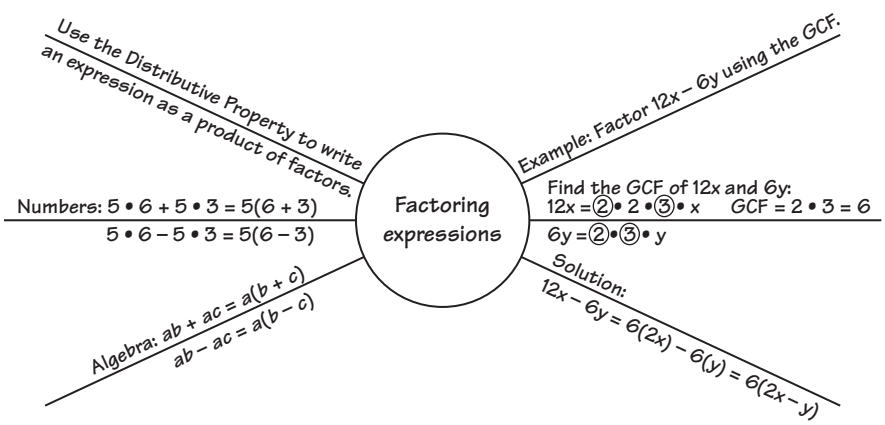
6.



7.



8.



Chapter 4 Areas of Polygons

Four Square

1–5. Sample answers are given.

1.

Area of a triangle

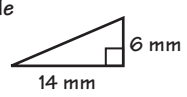
Words

The area A of a triangle is one-half the product of its base b and its height h.

Algebra

$$A = \frac{1}{2}bh$$

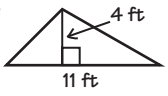
Example



$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(14)(6)$$
$$= 42$$

The area of the triangle is 42 square millimeters.

Example



$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(11)(4)$$
$$= 22$$

The area of the triangle is 22 square feet.

2.

Area of a trapezoid

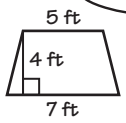
Words

The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2.

Algebra

$$A = \frac{1}{2}h(b_1 + b_2)$$

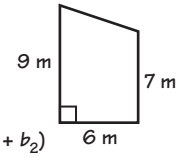
Example



$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(4)(5 + 7)$$
$$= 24$$

The area of the trapezoid is 24 square feet.

Example



$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(6)(7 + 9)$$
$$= 48$$

The area of the trapezoid is 48 square meters.

3.

Area of a composite figure

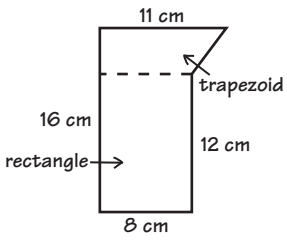
Definition

A composite figure is made up of triangles, squares, rectangles, and other two-dimensional figures.

Words

To find the area of a composite figure, separate it into figures with areas you know how to find. Then find the sum of the areas of those figures.

Example



Solution

Area of trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(4)(8 + 11)$$
$$= 38$$

Area of rectangle

$$A = lw$$
$$= 12(8)$$
$$= 96$$

So the area of the composite figure is $38 + 96 = 134$ square centimeters.

4.

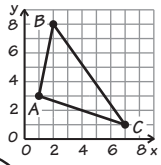
Drawing a polygon in a coordinate plane

Words

You can use ordered pairs to represent vertices of polygons. To draw a polygon in a coordinate plane, plot and connect the ordered pairs.

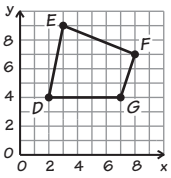
Triangle

$A(1, 3)$, $B(2, 8)$, $C(7, 1)$



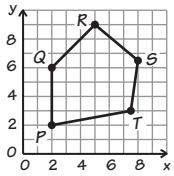
Quadrilateral

$D(2, 4)$, $E(3, 9)$, $F(8, 7)$, $G(7, 4)$



Pentagon

$P(2, 2)$, $Q(2, 6)$, $R(5,9)$, $S\left(8, 6\frac{1}{2}\right)$, $T\left(7\frac{1}{2}, 3\right)$



5.

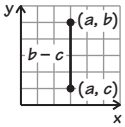
Finding distances in the first quadrant

Words

You can find the length of a horizontal or vertical line segment in a coordinate plane using the coordinates of the endpoints.

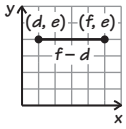
Vertical Lines

When the x-coordinates are the same, find the difference of the y-coordinates.



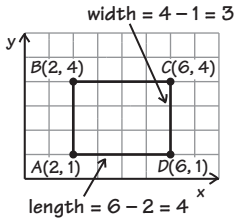
Horizontal Lines

When the y-coordinates are the same, find the difference of the x-coordinates.



Example

Find the perimeter.



The perimeter of the rectangle is $2(4) + 2(3) = 14$ units.

Chapter 5 Ratios and Rates

Definition and Example Chart

1–10. Sample answers are given.

1.

Equivalent ratios: two ratios that describe the same relationship.

Example

3 : 4 and 9 : 12

Example

50 to 40 and 5 to 4

Example

1 : 12 and 5 to 60

2.

Ratio table: a table used to find and organize equivalent ratios.

Example

Girls	5	10	20
Boys	3	6	12

Equivalent ratios: 5 : 3, 10 : 6, 20 : 12

Example

Flour (cups)	3	6	9	12
Water (cups)	1	2	3	4

Equivalent ratios: 3 : 1, 6 : 2, 9 : 3, 12 : 4

Example

Red	3	18
Yellow	7	42

Equivalent ratios: 3 : 7, 18 : 42

3.

Rate: a ratio of two quantities using different units.

Example

$\frac{\$135}{3 \text{ hours}}$

Example

$\frac{63 \text{ feet}}{12 \text{ seconds}}$

Example

$\frac{8 \text{ hits}}{25 \text{ at bats}}$

4.

Unit rate: a rate that compares a quantity to one unit of another quantity.

Example

$\frac{60 \text{ miles}}{1 \text{ hour}}$

Example

$\frac{5.25 \text{ feet}}{1 \text{ second}}$

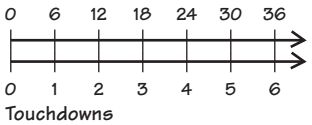
Example

$\frac{72 \text{ words}}{1 \text{ minute}}$

5. Equivalent rates: rates that have the same unit rate.

Example

Points



Touchdowns

Example

Distance (feet)	36	12	96
Time (seconds)	3	1	8

Equivalent rates: $\frac{36 \text{ ft}}{3 \text{ sec}} = \frac{12 \text{ ft}}{1 \text{ sec}} = \frac{96 \text{ ft}}{8 \text{ sec}}$

Example

Cost (dollars)	17.5	3.5	7
Apples (pounds)	5	1	2

Equivalent rates: $\frac{\$17.50}{5 \text{ lb}} = \frac{\$3.50}{1 \text{ lb}} = \frac{\$7.00}{2 \text{ lb}}$

6. Percent: a part-to-whole ratio where the whole is 100. So, you can write a percent as a fraction with a denominator of 100.

Example

$40\% = 40 \text{ out of } 100 = \frac{40}{100} = \frac{2}{5}$

Example

$112\% = \frac{112}{100} = 1\frac{12}{100} = 1\frac{3}{25}$

Example

$\frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 45\%$

7. U.S. customary system: a system of measurement that contains units for length, capacity, and weight.

Example

Unit of length: foot

Example

Unit of capacity: quart

Example

Unit of weight: pound

8. Metric system: a decimal system of measurement, based on powers of 10, that contains units for length, capacity, and mass.

Example

Unit of length: meter

Example

Unit of capacity: liter

Example

Unit of mass: gram

9. Conversion factor: a rate that equals 1.

Example

Length:

$\frac{1 \text{ mi}}{5280 \text{ ft}}$ and $\frac{5280 \text{ ft}}{1 \text{ mi}}$, $\frac{1 \text{ m}}{3.28 \text{ ft}}$ and $\frac{3.28 \text{ ft}}{1 \text{ m}}$, $\frac{1 \text{ cm}}{10 \text{ mm}}$ and $\frac{10 \text{ mm}}{1 \text{ cm}}$

Example

Capacity:

$\frac{1 \text{ gal}}{4 \text{ qt}}$ and $\frac{4 \text{ qt}}{1 \text{ gal}}$, $\frac{1 \text{ qt}}{0.95 \text{ L}}$ and $\frac{0.95 \text{ L}}{1 \text{ qt}}$, $\frac{1 \text{ L}}{1000 \text{ ml}}$ and $\frac{1000 \text{ ml}}{1 \text{ L}}$

Example

Mass:

$\frac{1 \text{ lb}}{16 \text{ oz}}$ and $\frac{16 \text{ oz}}{1 \text{ lb}}$, $\frac{1 \text{ lb}}{0.45 \text{ kg}}$ and $\frac{0.45 \text{ kg}}{1 \text{ lb}}$, $\frac{1 \text{ kg}}{1000 \text{ g}}$ and $\frac{1000 \text{ g}}{1 \text{ kg}}$

Example

Time:

$\frac{1 \text{ h}}{60 \text{ min}}$ and $\frac{60 \text{ min}}{1 \text{ h}}$, $\frac{1 \text{ day}}{24 \text{ h}}$ and $\frac{24 \text{ h}}{1 \text{ day}}$, $\frac{1 \text{ h}}{3600 \text{ sec}}$ and $\frac{3600 \text{ sec}}{1 \text{ h}}$

10. Unit analysis: a process you can use to decide which conversion factor will produce the appropriate units.

Example

12 lb = ___ oz

$12 \text{ lb} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} = 192 \text{ oz}$

Example

25 ml = ___ L

$25 \text{ ml} \cdot \frac{1 \text{ L}}{1000 \text{ ml}} = 0.025 \text{ L}$

Example

35 mph = ___ ft/sec

$\frac{35 \text{ mi}}{1 \text{ h}}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ sec}}\right) = \frac{184{,}800 \text{ ft}}{3600 \text{ sec}} \approx 51.3 \text{ ft/sec}$

Chapter 6 Integers and the Coordinate Plane

Summary Triangle

1–7. Sample answers are given.

1.

Positive integers

Definition: The whole numbers that are greater than 0; They can be written with or without a positive sign (+).

Key words: gain, up, earn, climb, rise, ascend, receive, deposit, above zero, above sea level

Examples: +2; 9; +36; 500,000

2.

Negative integers

Definition: The opposites of the positive integers; They are written with a negative sign (−).

Key words: lose, down, fall, debt, withdraw, below zero, below sea level, descend

Examples: −2; −9; −36; −500,000

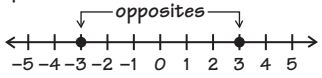
3.

Opposites

Definition: Two numbers that are the same distance from 0 on a number line, but on opposite sides of 0

The opposite of 0 is 0.

Example:



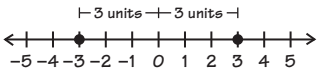
4.

Absolute value

Definition: The distance between a number and 0 on a number line

The absolute value of a number a is written as $|a|$.

Examples: $|-3| = 3$ $|3| = 3$



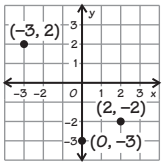
5.

Coordinate plane

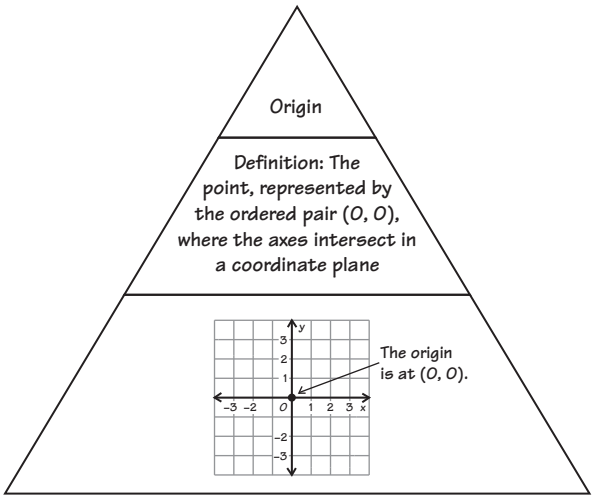
Formed by the intersection of a horizontal number line and a vertical number line

x-axis: the horizontal number line
y-axis: the vertical number line
Ordered pairs (x, y) are plotted in a coordinate plane.

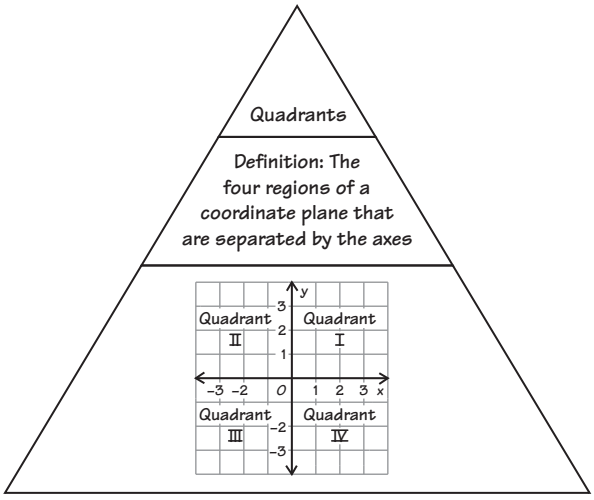
Example:



6.



7.



Chapter 7 Equations and Inequalities

Example and Non-Example Chart

1–8. Sample answers are given.

1. Inverse Operations

Examples	Non-Examples
addition and subtraction multiplication and division	addition and multiplication addition and division subtraction and multiplication subtraction and division

2. Equations Solved Using Addition or Subtraction

Examples	Non-Examples
$y + 9 = 12$ $a - 11 = 3 - 2$ $10 = x - 19$ $25 = r + 14$	$2a = 16$ $0.4b = 4$ $\frac{3}{4}y = 6$ $\frac{y}{6} = 0$

3. Equations Solved Using Multiplication or Division

Examples	Non-Examples
$2a = 16$ $\frac{y}{4} = 6$ $0.4b = 4$ $\frac{6}{7} = \frac{x}{14}$	$x - 1 = 5$ $y + 9 = 12$ $a - 11 = 3 - 2$ $10 = x - 19$

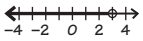
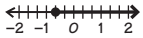
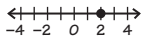
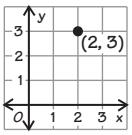
4. Equations in Two Variables

Examples	Non-Examples
$y = 5x$ $y = 7 - x$ $C = 50 + 3w$ $d = 25t$ $y = 4x - 3$	$12x = 3$ $\frac{y}{4} = 6$ $x - 1 = 5$ $a + 11 = 30$ $0.4b = 4$

5. Inequalities

Examples	Non-Examples
$x - 5 \geq 8$ $4 + 13 < n + 2$ $r > -\frac{3}{4}$ $0 \leq k$	$6y = 12$ $x + 9 = 17$ $a = 5$ $10 = c - 1$

6. Graphs of Inequalities

Examples	Non-Examples
$n < 3$  $-\frac{2}{3} \leq x$  $C \leq 500$ 	$x = 2$  

7. Inequalities Solved Using Addition or Subtraction

Examples	Non-Examples
$x + 3 < 9$ $8 \geq n - 5$ $a - 11 > 3 - 2$ $m + \frac{1}{4} \leq \frac{1}{2}$	$x + 5 = 7$ $8x > 16$ $0.4b \leq 4$ $\frac{n}{3} > 4$

8. Inequalities Solved Using Multiplication or Division

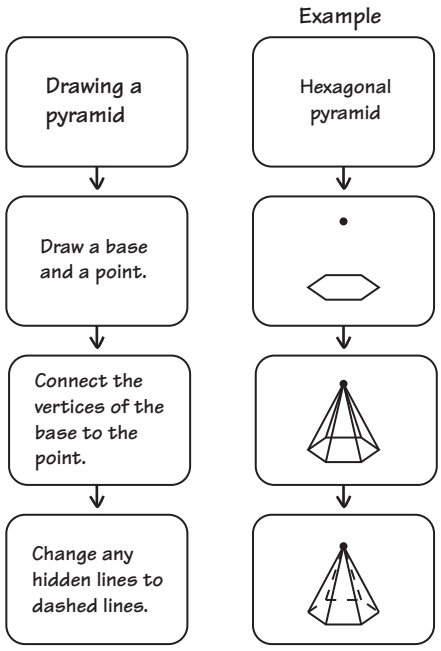
Examples	Non-Examples
$5a < 15$ $\frac{y}{4} \geq 6$ $0.4b > 4$ $\frac{6}{7} \leq \frac{x}{14}$	$4a = 12$ $\frac{x}{3} = 7$ $n - 7 > 10$ $12 \geq y + 9$

Chapter 8 Surface Area and Volume

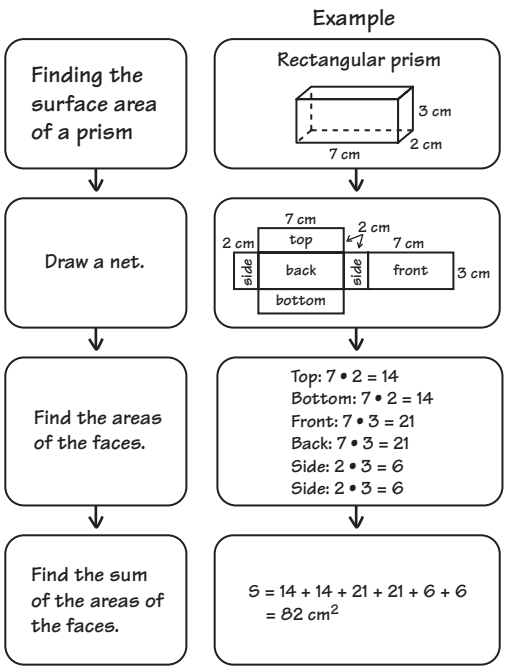
Process Diagram

1–4. Sample answers are given.

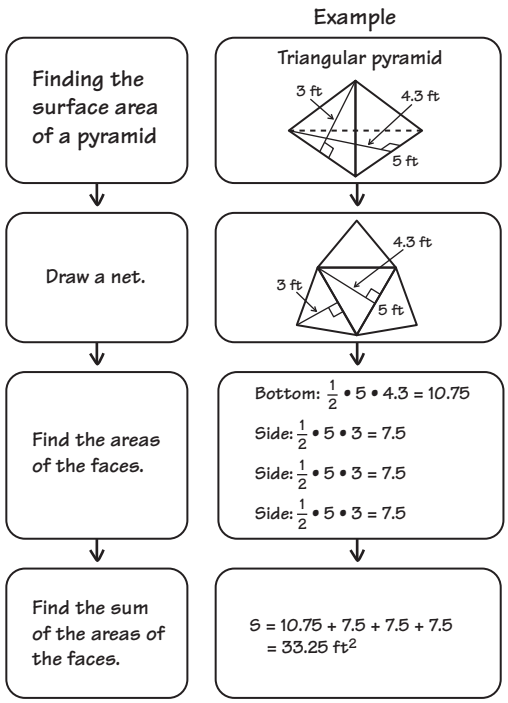
1.



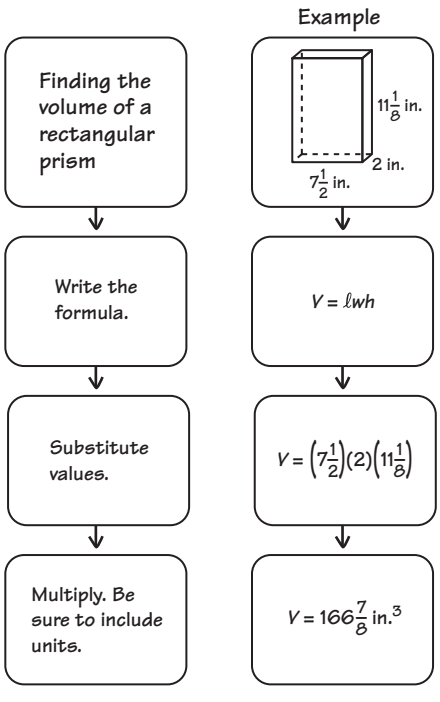
2.



3.



4.

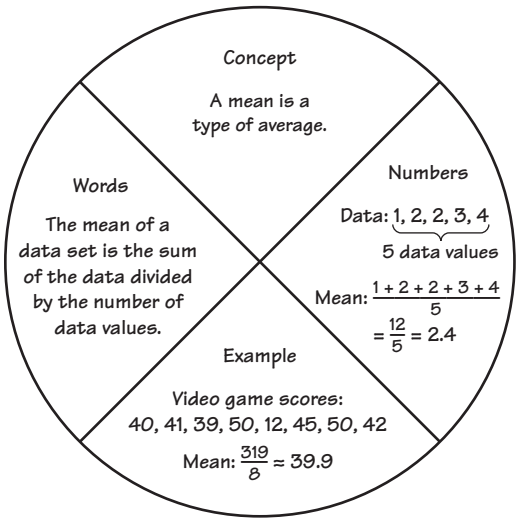


Chapter 9 Statistical Measures

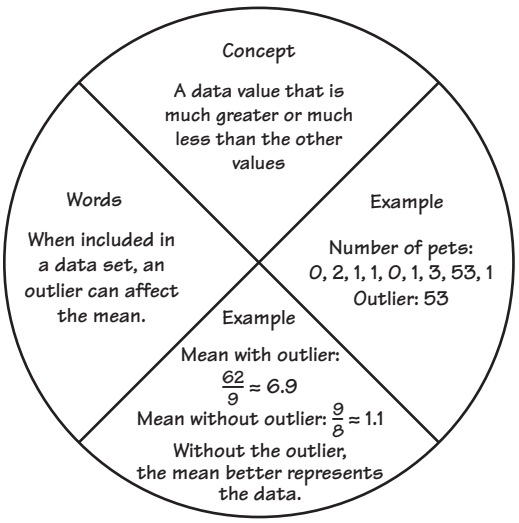
Concept Circle

1–10. Sample answers are given.

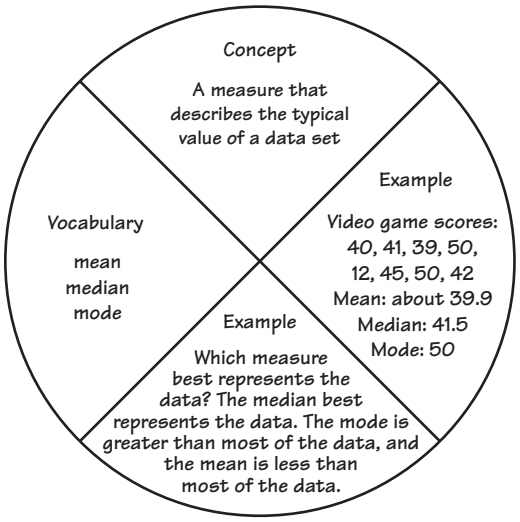
1. Mean



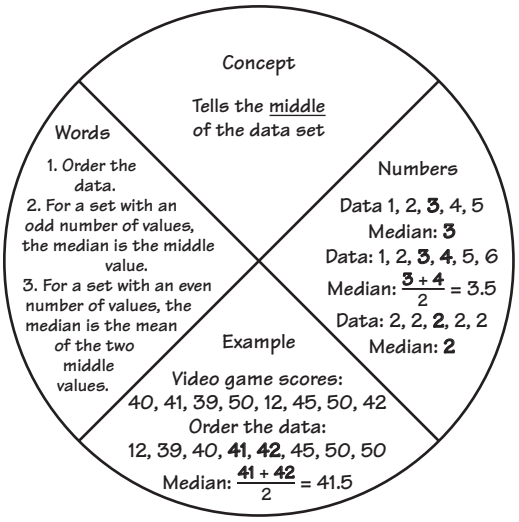
2. Outlier



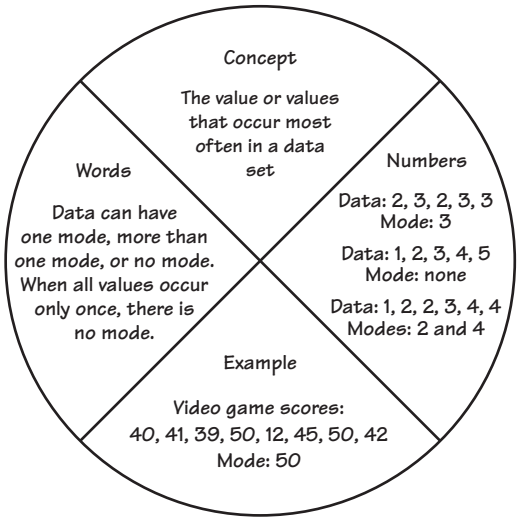
3. Measures of Center



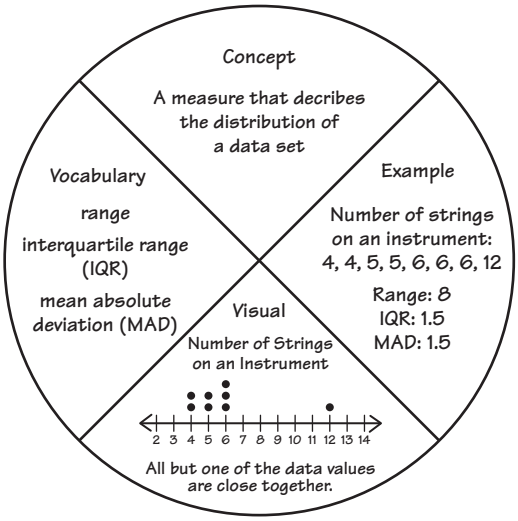
4. Median



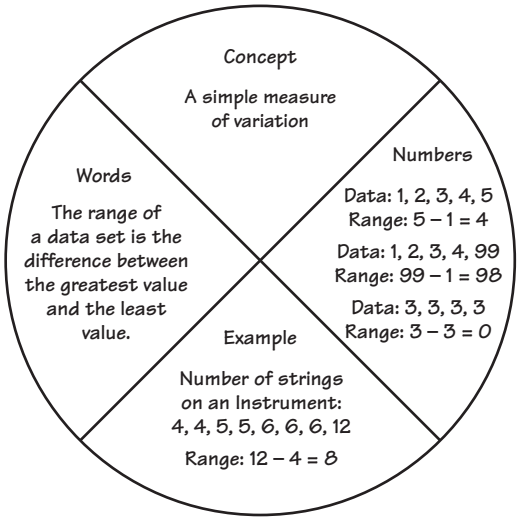
5. Mode



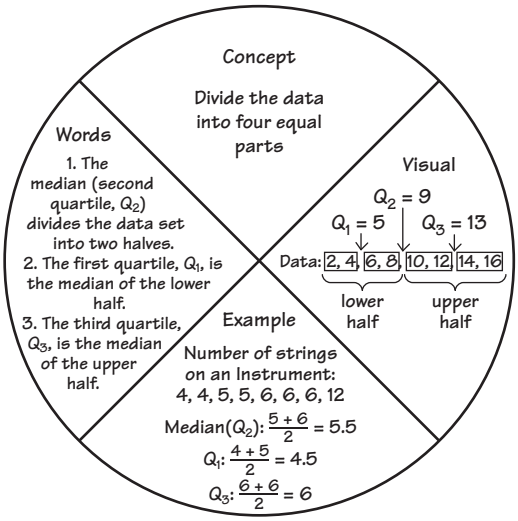
6. Measures of Variation



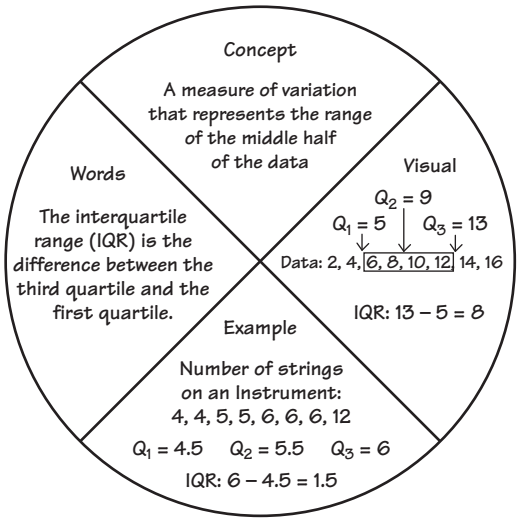
7. Range



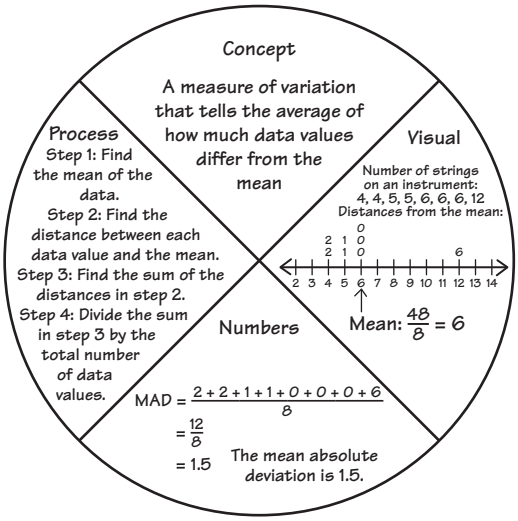
8. Quartiles



9. Interquartile Range (IQR)



10. Mean Absolute Deviation (MAD)



Chapter 10 Data Displays

Word Magnet

1–4. Sample answers are given.

1.

Stem-and-leaf plot

Uses the digits of data values to organize a data set

The stem is the digit or digits on the left.

The leaf is the digit or digits on the right.

Shows how data are distributed

Example
Tickets sold per day:
32, 21, 5, 9, 7, 28, 32, 39, 48, 47

Stem	Leaf
0	5 7 9
1	
2	1 8
3	2 2 9
4	7 8

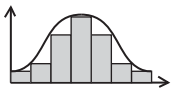
Key: 2|8 = 28 tickets

2.

Shapes of distributions

You can use dot plots and histograms to identify shapes of distributions.

Symmetric:



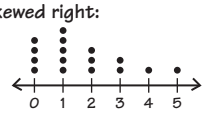
The left side is a mirror image of the right side.

When a data distribution is symmetric, use the mean to describe the center and use the MAD to describe the variation.

Skewed left:

- The "tail" extends to the left.
- Most data are on the right.

Skewed right:



- The "tail" extends to the right.
- Most data are on the left.

When a data distribution is skewed, use the median to describe the center and use the IQR to describe the variation.

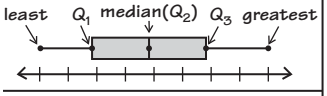
3.

Box-and-whisker plot

Represents a data set along a number line by using the least value, the greatest value, and the quartiles of the data

The five numbers that make up the box-and-whisker plot are called the five-number summary of the data set.

Shows the variability of a data set

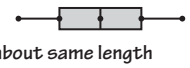


About $\frac{1}{4}$ of the data are in each whisker.

About $\frac{1}{2}$ of the data are in the box.

A long whisker or box indicates that the data are more spread out.

Symmetric:



- Whiskers about same length
- Median in the middle of the box

Skewed left:

- Left whisker longer than right
- Most data on the right

Skewed right:

- Right whisker longer than left
- Most data on the left

4.

Absolute value

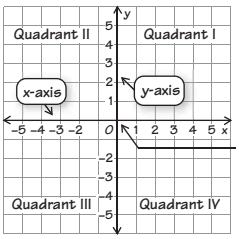
Distance between a number and 0 on a number line	The absolute value of a number a is written as $\lvert a \rvert$.
The absolute value of a positive number is positive.	$\lvert 5.3 \rvert = 5.3$
The absolute value of a negative number is positive.	$\lvert -6 \rvert = 6$
The absolute value of 0 is 0.	$\lvert 0 \rvert = 0$

Properties of Equality

Words	Algebra
Addition: When you add the same number to each side of an equation, the two sides remain equal.	**Addition:** $x - 5 = 6$, $+5 \ +5$, $x = 11$
Subtraction: When you subtract the same number from each side of an equation, the two sides remain equal.	**Subtraction:** $x + 5 = 6$, $-5 \ -5$, $x = 1$
Multiplication: When you multiply each side of an equation by the same nonzero number, the two sides remain equal.	**Multiplication:** $\frac{x}{3} = 4$, $\frac{x}{3} \cdot 3 = 4 \cdot 3$, $x = 12$
Division: When you divide each side of an equation by the same nonzero number, the two sides remain equal.	**Division:** $3x = 21$, $\frac{3x}{3} = \frac{21}{3}$, $x = 7$

Coordinate plane

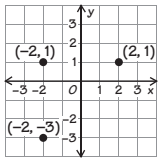
Formed by the intersection of a horizontal number line and a vertical number line



The origin is at $(0, 0)$.

Ordered pairs (x, y) are plotted in a coordinate plane.

Example



Chapter 11 Integers

Idea and Examples Chart

1–6. Sample answers are given.

1.

Integers: … , –3, –2, –1, 0, 1, 2, 3, …

- Example: –586
- Example: 0
- Example: 16

2a.

Adding integers with the same sign: Add the absolute values of the integers. Then use the common sign.

- Example: 16 + 17 = 33
- Example: –5 + (–4) = –9
- Example: –55 + (–45) = –100

2b.

Adding integers with different signs: Subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.

- Example: 8 + (–2) = 6
- Example: –8 + 2 = –6
- Example: –97 + 19 = –78

3.

Additive Inverse Property: The sum of an integer and its *additive inverse*, or opposite, is 0.

- Example: 5 + (–5) = 0
- Example: –100 + 100 = 0
- Example: 16 + (–16) = 0

4.

Subtracting integers: To subtract an integer, add its opposite.

- Example: 1 – 4 = 1 + (–4) = –3
- Example: –1 – (–4) = –1 + 4 = 3
- Example: 1 – (–4) = 1 + 4 = 5

5a.

Multiplying integers with the same sign: The product of two integers with the same sign is positive.

Example: $6 \cdot 3 = 18$

Example: $-6 \cdot (-3) = 18$

Example: $-18(-16) = 288$

5b.

Multiplying integers with different signs: The product of two integers with different signs is negative.

Example: $6 \cdot (-3) = -18$

Example: $-6 \cdot 3 = -18$

Example: $18(-16) = -288$

6a.

Dividing integers with the same sign: The quotient of two integers with the same sign is positive.

Example: $100 \div 4 = 25$

Example: $-100 \div (-4) = 25$

Example: $\frac{-98}{-7} = 14$

6b.

Dividing integers with different signs: The quotient of two integers with different signs is negative.

Example: $100 \div (-4) = -25$

Example: $-100 \div 4 = -25$

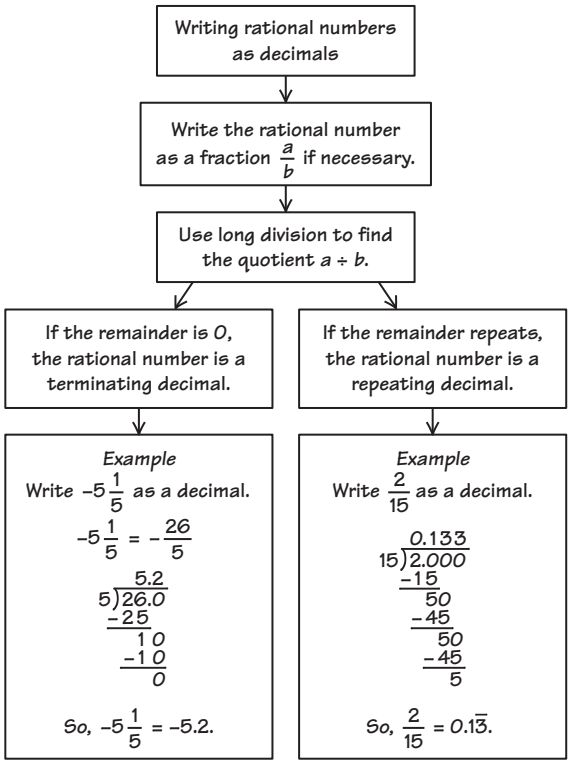
Example: $\frac{-98}{7} = -14$

Chapter 12 Rational Numbers

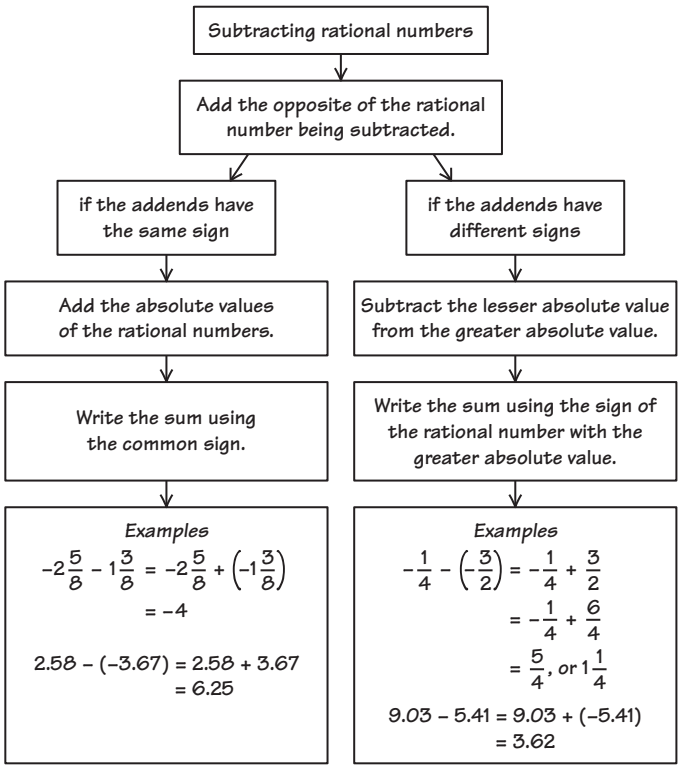
Process Diagram

1–4. Sample answers are given.

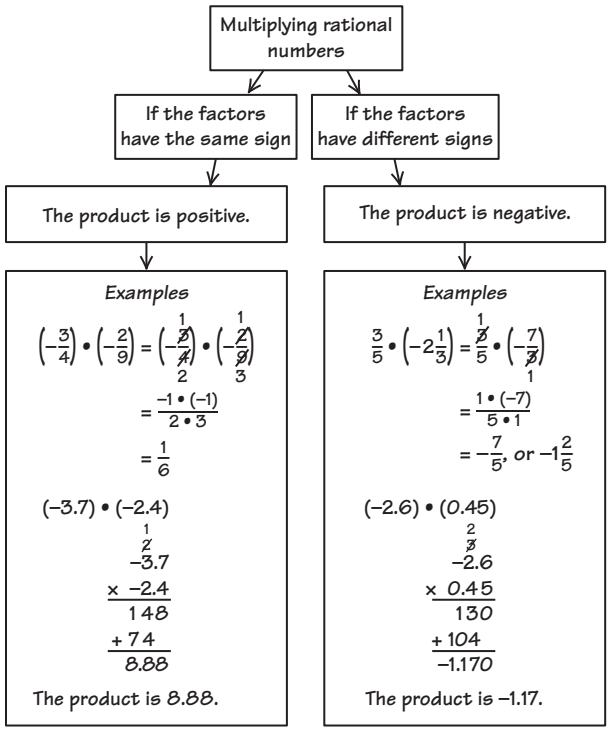
1.



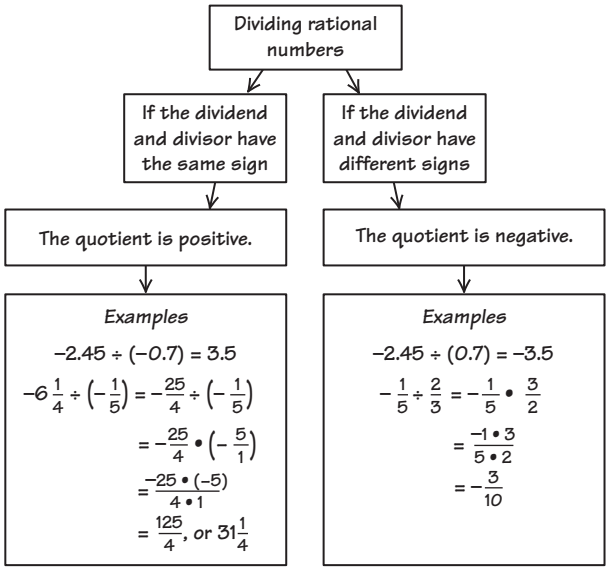
2.



3.



4.



Chapter 13 Expressions and Equations

Four Square

1–7. Sample answers are given.

1.

Definition

An algebraic expression is in *simplest form* when it has:
1. no like terms and
2. no parentheses.

Words

To write an algebraic expression in simplest form:
Step 1: Rewrite as a sum.
Step 2: Use the Distributive Property on parentheses, if necessary.
Step 3: Rearrange terms.
Step 4: Combine like terms.

Simplest form

Example

$5x^2 + 6x - 3x^2 + 8 - x$
$= 5x^2 + 6x + (-3x^2) + 8 + (-1x)$
$= 5x^2 + (-3x^2) + 6x + (-1x) + 8$
$= [5 + (-3)]x^2 + [6 + (-1)]x + 8$
$= 2x^2 + 5x + 8$

Example

$9 - 3\left(\frac{2}{3}m - \frac{1}{3}\right) + 3m$
$= 9 + (-3)\left(\frac{2}{3}m + \left(-\frac{1}{3}\right)\right) + 3m$
$= 9 + (-3)\left(\frac{2}{3}m\right) + (-3)\left(-\frac{1}{3}\right) + 3m$
$= 9 + (-2m) + 1 + 3m$
$= (-2m) + 3m + 9 + 1$
$= (-2 + 3)m + (9 + 1)$
$= m + 10$

2.

Definition

An algebraic expression in which the exponent of the variable is 1

Examples

$-7x$, $2x + 3$, $8 - \frac{1}{4}x$

Non-examples:

x^3, $-5x^2 + x$, $x^7 - 9$

Linear expression

Example

Adding linear expressions:

$(7 - w) + 3(-2w + 4)$
$= 7 + (-1w) + 3(-2w) + 3(4)$
$= 7 + (-1w) + (-6w) + 12$
$= (-1w) + (-6w) + 7 + 12$
$= -7w + 19$

Example

Subtracting linear expressions:

$(4y + 7) - (y - 8)$

$$\begin{array}{r} 4y + 7 \\ -(1y - 8) \end{array} \Rightarrow \begin{array}{r} 4y + 7 \\ +(-1y) + 8 \\ \hline 3y + 15 \end{array}$$

3.

Words

Write the expression as a product of factors. You can use the Distributive Property.

Example

Factor $12a - 30$ using the GCF.
$12a = ② \cdot 2 \cdot ③ \cdot a$
$30 = ② \cdot ③ \cdot 5$
$GCF = 2 \cdot 3 = 6$
$12a - 30 = 6(2a) - 6(5)$
$= 6(2a - 5)$

Factoring expressions

Example

Factor $\frac{1}{4}$ out of $\frac{1}{4}r + \frac{3}{4}$.

$\frac{1}{4}r = \frac{1}{4} \cdot r$
$\frac{3}{4} = \frac{1}{4} \cdot 3$
$\frac{1}{4}r + \frac{3}{4} = \frac{1}{4} \cdot r + \frac{1}{4} \cdot 3$
$= \frac{1}{4}(r + 3)$

Example

Factor -7 out of $-21p + 28$.

$-21p = -7 \cdot 3p$
$28 = -7 \cdot (-4)$
$-21p + 28$
$= -7(3p) + (-7)(-4)$
$= -7(3p - 4)$

4.

Words

Two equations are equivalent equations if they have the same solutions. You can use the Addition, Subtraction, Multiplication, and Division Properties of Equality to write equivalent equations.

Algebra

$a = b$ and $a + c = b + c$
$a = b$ and $a - c = b - c$
$a = b$ and $a \cdot c = b \cdot c$
$a = b$ and $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$

Equivalent equations

Examples

$x - 7 = 2$ and
$x - 7 + 7 = 2 + 7$
$2d + 5 = -7$ and
$2d + 5 - 5 = -7 - 5$
$24 = \frac{y}{-4}$ and
$-4 \cdot 24 = -4 \cdot \frac{y}{-4}$
$3c = -12$ and $\frac{3c}{3} = \frac{-12}{3}$

Non-Examples

$x + 7 = 2$ and
$x + 7 - 7 = 2 + 7$
$3c = -4$ and $\frac{3c}{3} = 3 \cdot (-4)$
$7 = m + 3$ and
$7 - 7 = m + 3 - 3$
$3x + 7 = 3$ and $3x = 7 + 3$

5.

Solving equations using addition or subtraction

Words

To undo addition, use the Subtraction Property of Equality: subtracting the same number from each side of an equation produces an equivalent equation. To undo subtraction, use the Addition Property of Equality: adding the same number to each side of an equation produces an equivalent equation.

Algebra

If $a + b = c$, then $a + b - b = c - b$.

If $a - b = c$, then $a - b + b = c + b$.

Example

$x - 4 = 12$

$+4 \quad +4$

$x = 16$

Check

$x - 4 = 12$

$16 - 4 \stackrel{?}{=} 12$

$12 = 12$ ✓

Example

$x + 4.1 = 12$

$-4.1 \quad -4.1$

$x = 7.9$

Check

$x + 4.1 = 12$

$7.9 + 4.1 \stackrel{?}{=} 12$

$12 = 12$ ✓

6.

Solving equations using multiplication or division

Words

To undo multiplication, use the Division Property of Equality: dividing each side of an equation by the same number produces an equivalent equation. To undo division, use the Multiplication Property of Equality: multiplying each side of an equation by the same number produces an equivalent equation.

Algebra

If $ab = c$, then $\frac{ab}{b} = \frac{c}{b}$, $b \neq 0$.

If $\frac{a}{b} = c$, then $b \cdot \frac{a}{b} = b \cdot c$, $b \neq 0$.

Example

$-2x = 3$

$\frac{-2x}{-2} = \frac{3}{-2}$

$x = -\frac{3}{2}$, or $-1\frac{1}{2}$

Check

$-2x = 3$

$-2\left(-\frac{3}{2}\right) \stackrel{?}{=} 3$

$3 = 3$ ✓

Example

$\frac{x}{2} = -3$

$2 \cdot \frac{x}{2} = 2 \cdot (-3)$

$x = -6$

Check

$\frac{x}{2} = -3$

$\frac{-6}{2} \stackrel{?}{=} -3$

$-3 = -3$ ✓

7.

Solving two-step equations

Words

Undo the operations in the reverse order of the order of operations:

1. Undo addition or subtraction.
2. Undo multiplication or division.

After solving for the variable, check your solution.

Example

$3x + 4 = 1$

$-4 \quad -4$

$3x = -3$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

Check

$3x + 4 = 1$

$3(-1) + 4 \stackrel{?}{=} 1$

$-3 + 4 \stackrel{?}{=} 1$

$1 = 1$ ✓

Example

$\frac{3}{7} - \frac{a}{3} = -\frac{4}{7}$

$-\frac{3}{7} \quad -\frac{3}{7}$

$-\frac{a}{3} = -1$

$-3 \cdot \left(-\frac{a}{3}\right) = -3 \cdot (-1)$

$a = 3$

Check

$\frac{3}{7} - \frac{a}{3} = -\frac{4}{7}$

$\frac{3}{7} - \frac{3}{3} \stackrel{?}{=} -\frac{4}{7}$

$\frac{3}{7} + (-1) \stackrel{?}{=} -\frac{4}{7}$

$\frac{3}{7} + \left(-\frac{7}{7}\right) \stackrel{?}{=} -\frac{4}{7}$

$-\frac{4}{7} = -\frac{4}{7}$ ✓

Example

$4(x - 2) = 12$

$4x + (-8) = 12$

$+8 \quad +8$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

Check

$4(x - 2) = 12$

$4(5 - 2) \stackrel{?}{=} 12$

$4(3) \stackrel{?}{=} 12$

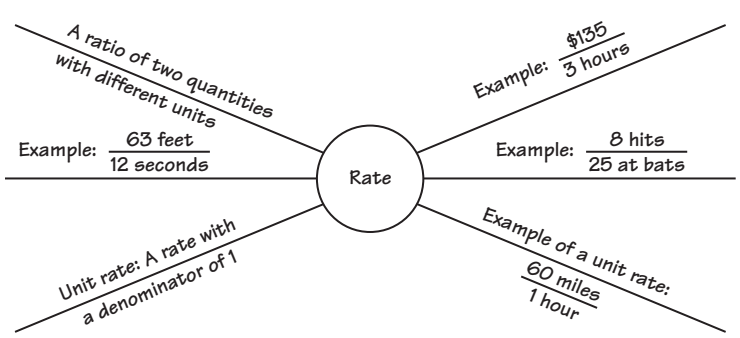
$12 = 12$ ✓

Chapter 14 Ratios and Proportions

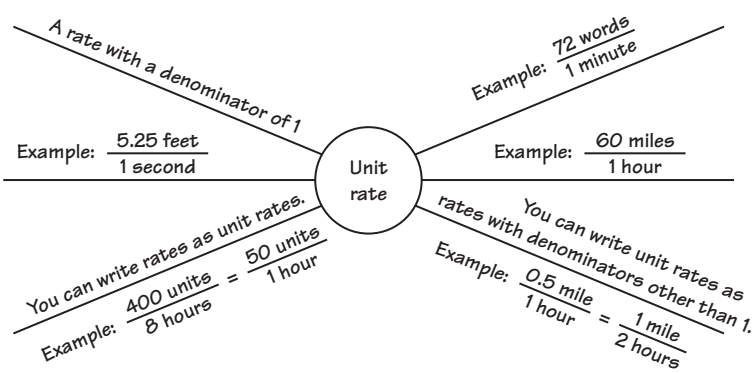
Information Wheel

1–8. Sample answers are given.

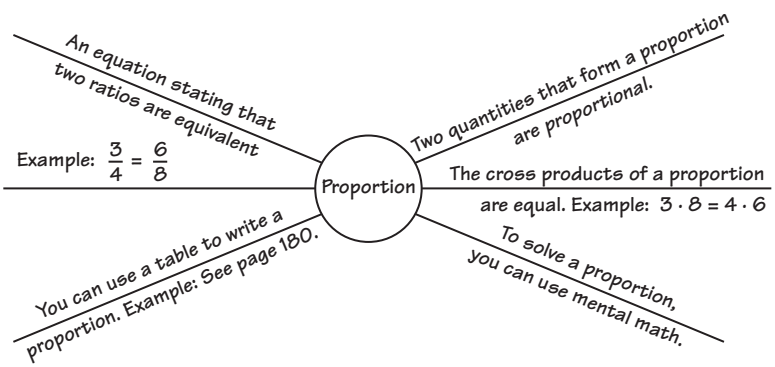
1.



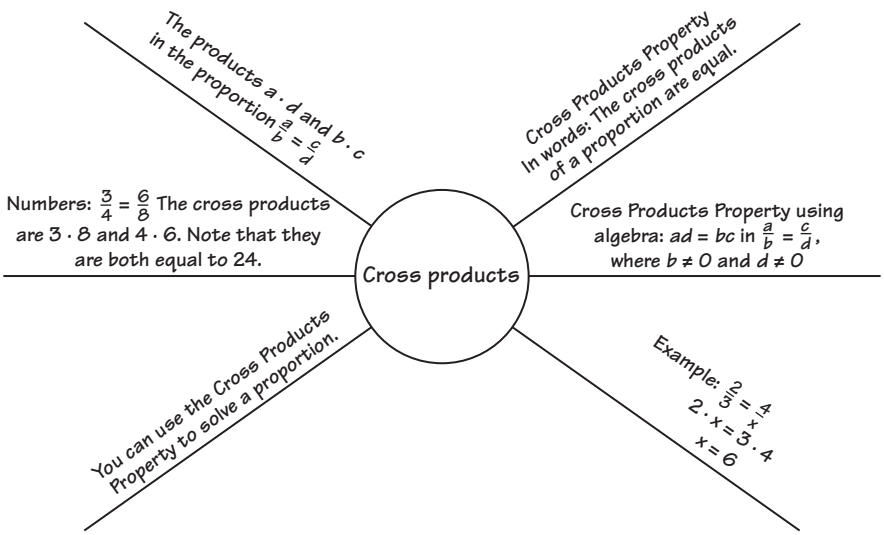
2.



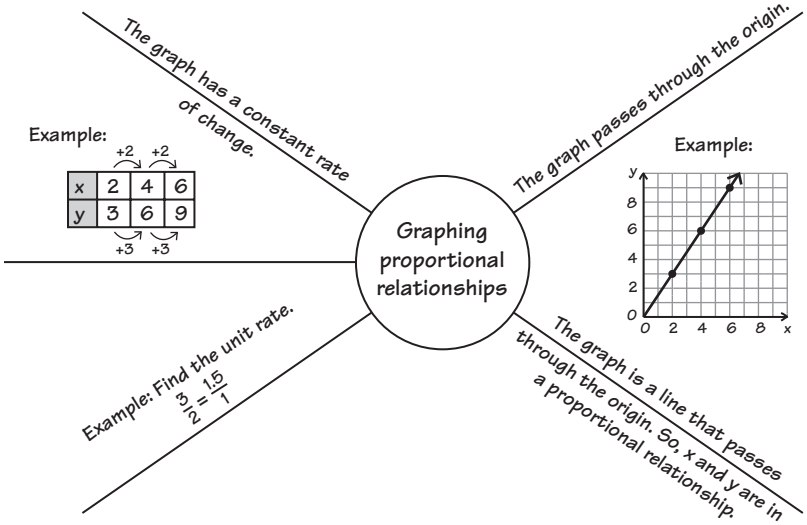
3.



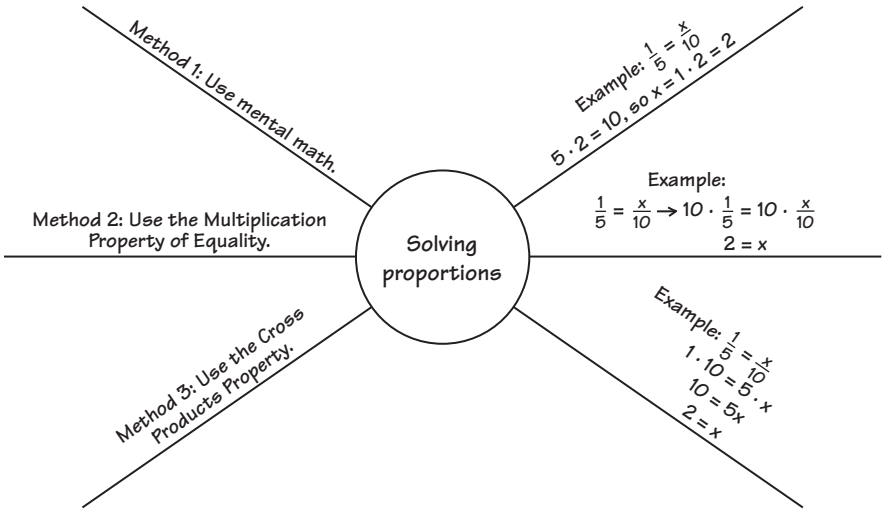
4.



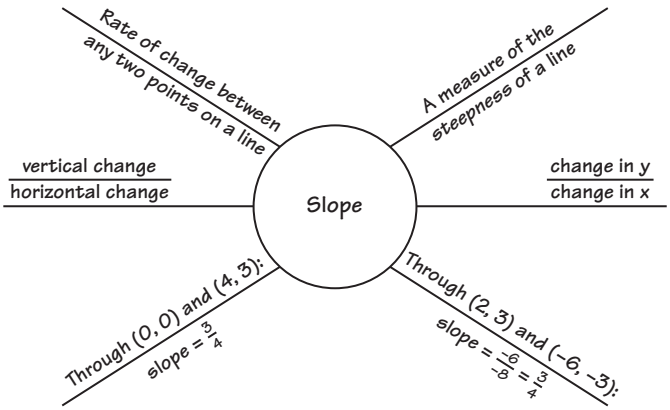
5.



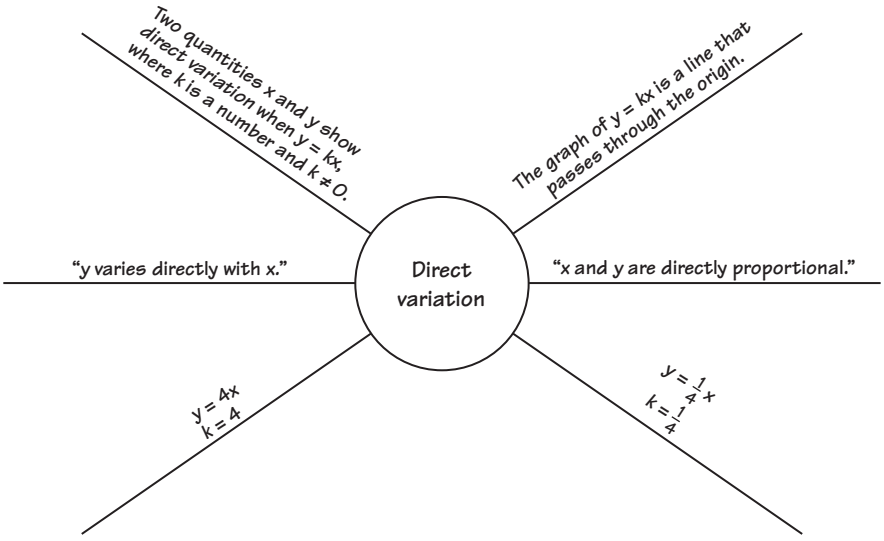
6.



7.



8.

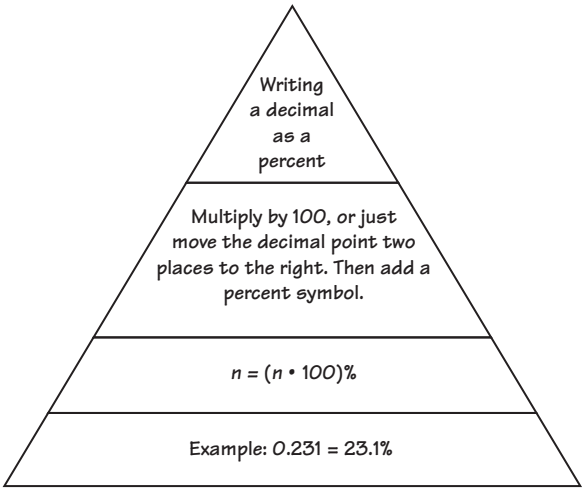


Chapter 15 Percents

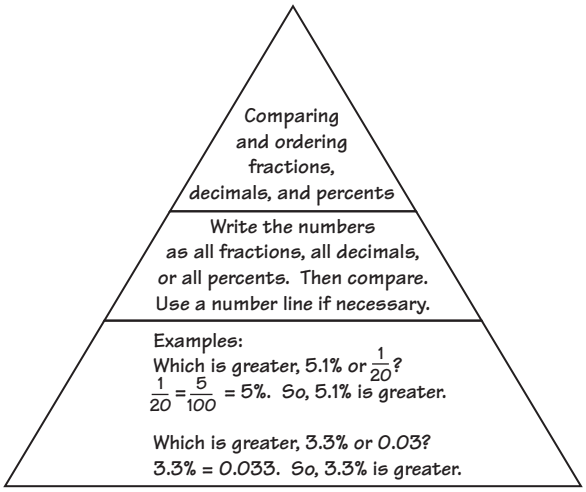
Summary Triangle

1–8. Sample answers are given.

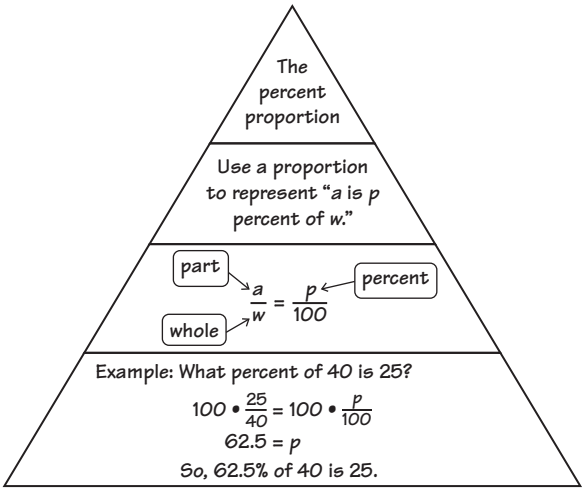
1.



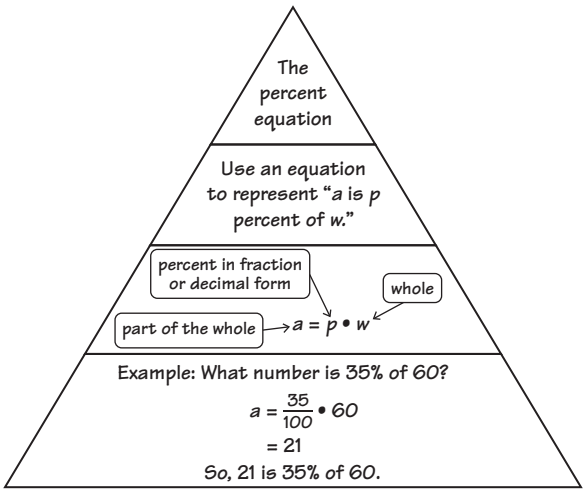
2.



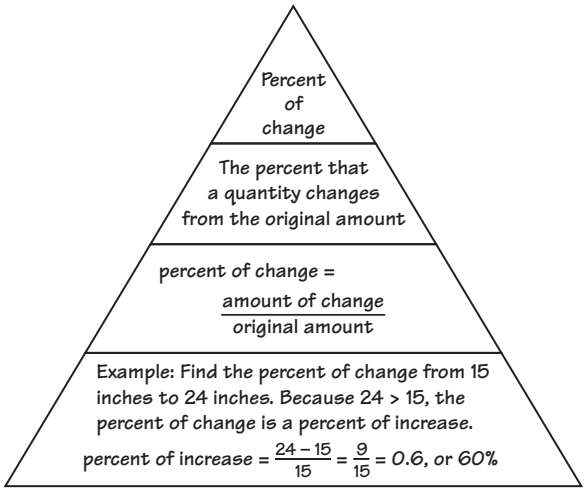
3.



4.

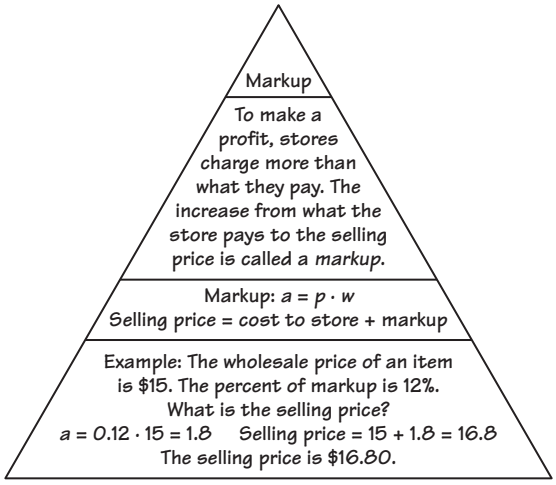


5.



6.

7.



8.