Math Concepts

http://www.actstudent.org/

Substitution

1) Abandoned mines frequently fill with water. Before an abandoned mine can be reopened, the water must be pumped out. The size of pump required depends on the depth of the mine. If pumping out a mine that is D feet deep requires a pump that pumps a minimum of $\frac{D^2}{25}+4D-250$ gallons per minute, pumping out a mine that is 150 feet deep would require a pump that pumps a minimum of how many gallons per minute?

A. 362
B. 500
C. 800
* D. 1,250
E. 1,750

Substitute 150 in for "D" & simplify with a calculator.

$\frac{150^2}{25}+4(150)-250 \rightarrow 1250$

2) The volume, V, of the right circular cone with radius r and height h, shown below, can be found using the formula $V=\frac{1}{3}\pi r^2 h$. A cone-shaped paper cup has a volume of 142 cubic centimeters and a height of 8.5 centimeters. What is the radius, to the nearest centimeter, of the paper cup?

A. 2
* B. 4
C. 8
D. 12
E. 16

Substitute 142 for V and 8.5 for h

$142=\frac{1}{3}\pi r^2(8.5)$ Multiply by 3, then divide by 8.5π. Finally, take the $\sqrt{\ }$ to find r.

Types of Numbers

1) How many irrational numbers are there between 1 and 6 ?

A. 1
B. 3
C. 4
D. 10
* E. Infinitely many

There are an infinite # of any type of # between any 2 #s.

2) If "x" and "y" are positive integers such that the greatest common factor of x^2y^2 and xy^3 is 45, then which of the following could "y" equal?

A. 45
B. 15
C. 9
D. 5
* E. 3

The greatest common factor of x^2y^2 & xy^3 is xy^2

xy^2 must = 45

y^2 must go into 45, so 3 is the only # that will work.

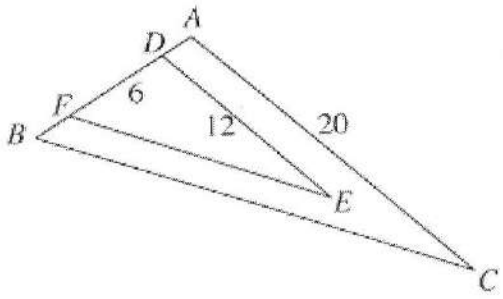
3) $\sqrt{-(-9)^2} = ?$ $(Note: i = \sqrt{-1})$

$\sqrt{-(-9)^2} \rightarrow \sqrt{-81} \rightarrow \sqrt{-1} \cdot \sqrt{81} \rightarrow 9i$

* A. $9i$
B. $9+i$
C. $9-i$
D. 9
E. -9

Proportions

1) In the figure below, $\angle ABC \cong \angle DFE$, $\angle BAC \cong \angle FDE$, D and F are on AB, $AD \cong FB$, and distances in centimeters are as shown. What is the length of AD, in centimeters?



Set up A proportion

$\frac{6}{12} = \frac{AB}{20}$ Cross mult.

$12AB = 6(20)$
$12AB = 120$
$AB = 10$
$AB - DF \rightarrow 10 - 6 = 4$
$4 \div 2 \rightarrow 2$

A. 5
B. 4
C. 3
* D. 2
E. 1

2) An industrial cleaner is manufactured using only the 3 secret ingredients A, B, and C, which are mixed in the ratio of 2:3:5, respectively, by weight. How many pounds of secret ingredient B are in a 42-pound (net weight) bucket of this cleaner?

$2x + 3x + 5x = 42$

$10x = 42$

$x = 4.2$

B is $3x \rightarrow 3(4.2) \rightarrow 12.6$

A. 4.2
* B. 12.6
C. 14.0
D. 18.0
E. 21.0

3) For all nonzero real numbers p, t, x, and y such that $\frac{x}{y}=\frac{3p}{2t}$, which of the following expressions is equivalent to t?

A. $\frac{y}{2}$
B. $\frac{3px}{2y}$
C. $\frac{6py}{x}$
D. $\frac{3py}{x}$
* E. $\frac{3py}{2x}$

$\frac{x}{y}=\frac{3p}{2t}$ Cross Multiply

$\frac{2tx}{2x}=\frac{3py}{2x}$

$t=\frac{3py}{2x}$

Arithmetic/Geometric Sequence & Series

1) What is "x", the second term in the geometric series $\frac{1}{4}+x+\frac{1}{36}+\frac{1}{108}+\ldots$? (Note: In a geometric series the ratio of any term to the next term is a constant.)

A. $\frac{1}{3}$
B. $\frac{1}{9}$
* C. $\frac{1}{12}$
D. $\frac{1}{16}$
E. $\frac{1}{18}$

$\frac{1}{36}\cdot ?=\frac{1}{108}$

$?=\frac{1}{3}$

$\frac{1}{4}\cdot\frac{1}{3}\rightarrow\frac{1}{12}$ and $\frac{1}{12}\cdot\frac{1}{3}=\frac{1}{36}$

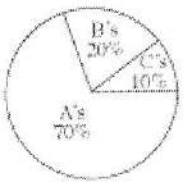
Percentage Problems

1) A DVD player with a list price of $100 is marked down 30%. If John gets an employee discount of 20% off the sale price, how much does John pay for the DVD player ?

A. $86.00
B. $77.60
* C. $56.00
D. $50.00
E. $44.00

SALE PRICE $.70(100) = \$70$
JOHN PAYS $.80(\text{SALE PRICE}) \rightarrow .8(70) = \$56$

2) The distribution of Jamal's high school grades by percentage of course credits is given in the circle graph below. What is Jamal's grade point average if each A is worth 4 points; each B, 3 points; and each C, 2 points?



$.7(4) + .2(3) + .1(2) = 3.6$

A. 3.0
B. 3.4
* C. 3.6
D. 3.7
E. Cannot be determined from the given information

Set Up & Solve/Simplify An Equation or Expression

1) Sales for a business were 3 million dollars more the second year than the first, and sales for the third year were double the sales for the second year. If sales for the third year were 38 million dollars, what were sales, in millions of dollars, for the first year?

* A. 16
B. 17.5
C. 20.5
D. 22
E. 35

first year $= x$
second year $= x+3$
third year $= 2(x+3)$

$2(x+3) = 38$
$x+3 = 19$
$x = 16$

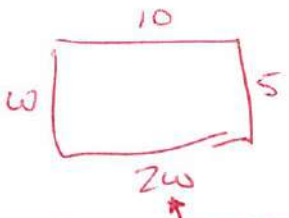
2) The length, in inches, of a box is 3 inches less than twice its width, in inches. Which of the following gives the length, l inches, in terms of the width, w inches, of the box?

A. $l = \frac{1}{2}w + 3$
B. $l = w + 3$
C. $l = w - 3$
D. $l = 2w + 3$
* E. $l = 2w - 3$

$L = 2w - 3$ ← less 3
↑ Twice the width

3) A rectangle with a perimeter of 30 centimeters is twice as long as it is wide. What is the area of the rectangle in square centimeters?

A. 15
*B. 50
C. 200
D. $3\sqrt{15}$
E. $6\sqrt{15}$



Length = 2 x width

$w + 2w + w + 2w = 30$

$6w = 30$

$w = 5$

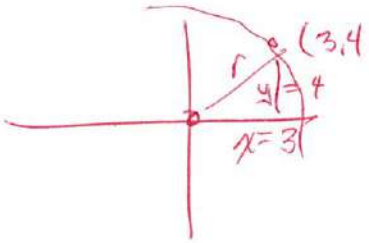
$AREA = w(2w)$

$= 5(10)$

$= 50$

4) Which of the following is an equation of the circle with its center at (0,0) that passes through (3,4) in the standard (x,y) coordinate plane?

A. $x - y = 1$
B. $x + y = 25$
C. $x^2 + y = 25$
D. $x^2 + y^2 = 5$
*E. $x^2 + y^2 = 25$



Circle Formula

$(x-h)^2 + (y-k)^2 = r^2$

(h,k) = Center point.

Find r

$3^2 + 4^2 = r^2$

$r^2 = 25$

$r = 5$

$x^2 + y^2 = 5^2$

5) Which of the following is equivalent to $\frac{5}{k} + \frac{k+3}{k+5}$?

A. $\frac{k+8}{2k+5}$
B. $\frac{k+8}{k(k+5)}$
C. $\frac{5(k+3)}{k(k+5)}$
D. $\frac{k^2+3k}{5k+25}$
*E. $\frac{k^2+8k+25}{k(k+5)}$

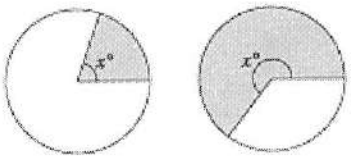
$\frac{k+5}{k+5} \cdot \frac{5}{k} + \frac{k+3}{k+5} \cdot \frac{k}{k}$

$\frac{5k+25 + k^2+3k}{k(k+5)}$

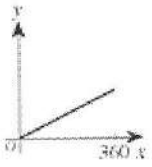
$\frac{k^2+8k+25}{k(k+5)}$

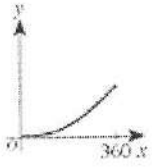
Functions

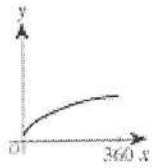
1) The 2 diagrams below show a circle of radius 1 inch with shaded sectors of angle $x°$, for 2 different values of x.

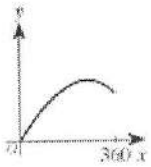


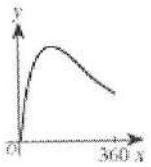
One of the following is the graph in the standard (x,y) coordinate plane of the area, y, of a shaded sector with angle $x°$, for all values of x between 0 and 360. Which is that graph?

* A. 

B. 

C. 

D. 

E. 

$AREA = \pi r^2$

$A = \frac{x}{360} \pi (1)^2$

$A = \frac{x}{360} \cdot \pi$ OR $A = \frac{\pi}{360} \cdot x$

$y = mx + b$

Slope is $\frac{\pi}{360}$

LINEAR

2) If $h(x) = x^3 + x$ and $g(x) = 2x + 3$, then $g(h(2)) = ?$

A. 7
B. 10
C. 17
D. 19
* E. 23

Composition of functions

$h(2) = 2^3 + 2$
$= 8 + 2$
$= 10$

$g(10) = 2(10) + 3$
$= 23$

Solving Systems of Equations

1) If $xy = 144$, $x + y = 30$, and $x > y$, what is the value of $x - y$?

A. 4
B. 6
* C. 18
D. 22
E. 24

$y = 30 - x$ (solve for y)

$x(30 - x) = 144$ (substitute)

$30x - x^2 = 144$

$x^2 - 30x + 144 = 0$

$x^2 - 30x + 144 = 0$

$(x - 6)(x - 24) = 0$

$x = 6$ OR $x = 24$

$x = 24$ and $y = 6$

$24 - 6 = 18$

2) A neighborhood recreation program serves a total of 280 children who are either 11 years old or 12 years old. The sum of the children's ages is 3,238 years. How many 11-year-old children does the recreation program serve?

x = 11 yr olds y = 12 yr olds

A. 55
* B. 122
C. 132
D. 158
E. 208

$x + y = 280$

$11x + 12y = 3238$

$y = 280 - x$

$11x + 12(280 - x) = 3238$

$11x + 3360 - 12x = 3238$

$-x = -122$

$x = 122$

122 11 yr olds

Probability, Statistics & The Fundamental Counting Principle

1. 1) Ding's Diner advertised this daily lunch special: "Choose 1 item from each column—only $4.95!" Thus, each daily lunch special consists of a salad, a soup, a sandwich, and a drink.

Salads	Soups	Sandwiches	Drinks
cole slaw	onion	meat loaf	milk
lettuce	tomato	chicken	cola
potato		hamburger	coffee
		ham	tea
		tenderloin	

How many different daily lunch specials are possible?

A. 4
B. 14
C. 30
* D. 120
E. 180

Salad choices: 3 × Soup choices: 2 × Sandwich choices: 5 × Drink choices: 4 = 120 TOTAL

2) What is the probability that a number selected at random from the set {2, 3, 7, 12, 15, 22, 72, 108} will be divisible by both 2 and 3 ?

12, 72, & 108 are divisible by 2 & 3

3 out of 8 or $\frac{3}{8}$

A. $\frac{1}{4}$

* B. $\frac{3}{8}$

C. $\frac{3}{5}$

D. $\frac{5}{8}$

E. $\frac{7}{8}$

3) Ms. Hernandez began her math class by saying:

I'm thinking of 5 numbers such that their mean is equal to their median. If 4 of the numbers are 14, 8, 16, and 14, what is the 5th number?

What is the 5th number Ms. Hernandez is thinking of?

$\frac{8+14+14+16+x}{5}$ = MEAN

$\frac{x+52}{5} = 14$

$x+52 = 70$

$x = 18$

8, 14, 14, 16 & x

on either side the median is 14

A. 13

B. 14

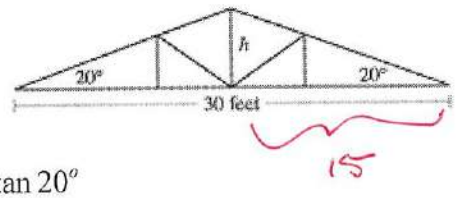
C. 15

D. 16

* E. 18

Basic Trigonometry Concepts

1) Which of the following expressions is the closest approximation to the height *h*, in feet, of the roof truss shown below?



$\frac{\tan 20^\circ}{1} = \frac{h}{15}$

$15 \tan 20^\circ = h$

* A. $15 \tan 20^\circ$

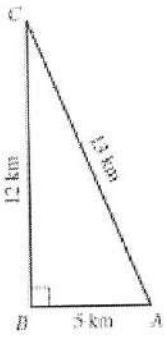
B. $15 \sin 20^\circ$

C. $30 \tan 20^\circ$

D. $30 \sin 20^\circ$

E. $\frac{15}{\sin 20^\circ}$

2) Which of the following is the sine of $\angle A$ in the right triangle below?



$\sin = \frac{\text{side opp}}{\text{hyp}}$

$\sin A = \frac{12}{13}$

A. $\frac{5}{13}$

B. $\frac{5}{12}$

* C. $\frac{12}{13}$

D. $\frac{12}{5}$

E. $\frac{13}{5}$

3) Which of the following is equivalent to $\sin\theta \cdot \csc(-\theta)$ wherever $\sin\theta \cdot \csc(-\theta)$ is defined?

* A. -1

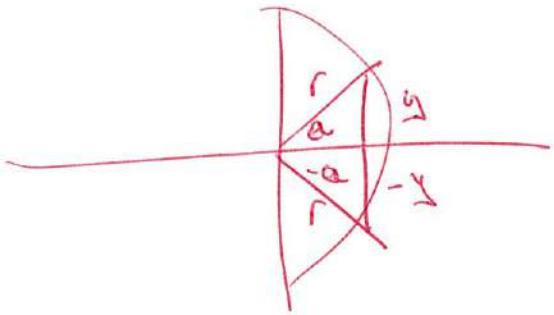
B. 1

C. $-\tan\theta$

D. $\tan\theta$

E. $-\sin^2\theta$

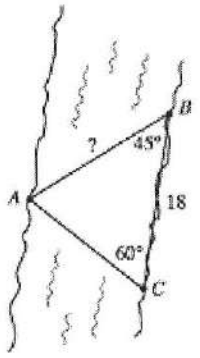
$\sin = \frac{y}{r}$ $\quad \csc = \frac{r}{y}$



$\sin\theta \cdot \csc(-\theta)$

$\frac{y}{r} \cdot \frac{r}{-y} \rightarrow -1$

4) In the figure below, points A and B are on opposite banks of a small stream. Point C is on the same bank of the stream as point B and approximately 18 meters from B. The measure of $\angle CBA$ is 45°, and the measure of $\angle BCA$ is 60°.



Which of the following expressions gives the approximate distance, in meters, between point A and point B ?

(Note: For ΔABC, $\frac{\sin\angle A}{A} = \frac{\sin\angle B}{B} = \frac{\sin\angle C}{C}$)

$\measuredangle A = 180° - (60° + 45°)$

$\measuredangle A = 75°$

$\frac{\sin 60°}{AB} = \frac{\sin 75°}{18}$

$AB \sin 75° = 18 \sin 60°$

$AB = \frac{18 \sin 60°}{\sin 75°}$

A. $\frac{\sin 60^o}{18 \sin 45^o}$

B. $\frac{\sin 60^o}{18 \sin 75^o}$

C. $\frac{18 \sin 45^o}{\sin 60^o}$

D. $\frac{18 \sin 60^o}{\sin 45^o}$

* E. $\frac{18 \sin 60^o}{\sin 75^o}$

The Pythagorean Theorem

1) A boat departs Port Isabelle, Texas, traveling to an oil rig. The oil rig is located 9 miles east and 12 miles north of the boat's departure point. About how many miles is the oil rig from the departure point?

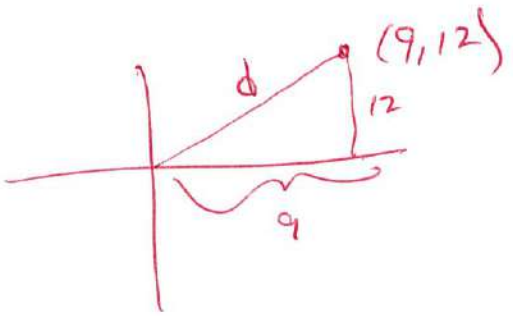
A. 3

B. $\sqrt{63}$

* C. 15

D. 21

E. 225



$9^2 + 12^2 = d^2$

$81 + 144 = d^2$

$d^2 = 225$

$d = 15$

Factoring

1) Which of the following is a factor of the polynomial $2x^2 - 3x - 5$?

$2x^2-3x-5$

$(2x-5)(x+1)$

A. $x-1$
B. $2x-3$
* C. $2x-5$
D. $2x+5$
E. $3x+5$

2) What is the x-intercept of the graph of $y = x^2 - 4x + 4$?

$y=(x-2)^2$ $y=0$ at the x-int.

$(x-2)^2=0$

$x=2$ DZ.

A. 4
B. -1
C. 0
* D. 2
E. 1

Basic Linear & Quadratic Concepts

1) What is the slope of any line parallel to the line $9x + 4y = 7$?

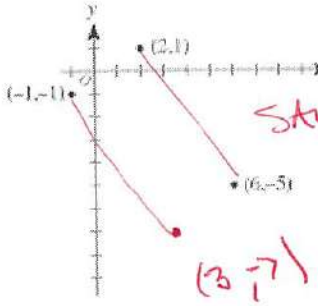
$9x+4y=7$

$4y=-9x+7$

$y=-\frac{9}{4}x+\frac{7}{4}$

A. -9
* B. $-\frac{9}{4}$
C. $\frac{9}{7}$
D. 7
E. 9

2) In the standard (x, y) coordinate plane below, 3 of the vertices of a rectangle are shown. Which of the following is the 4th vertex of the rectangle?



Same slope $\frac{-6}{3}=-2$

(3,-7)

* A. (3,-7)
B. (4,-8)
C. (5,-1)
D. (8,-3)
E. (9,-3)

3) Which of the following equations represents the linear relationship between time, t, and velocity, v, shown in the table below?

t	0	1	2
v	120	152	184

A. $v = 32t$
* B. $v = 32t + 120$
C. $v = 120t$
D. $v = 120t = 32$
E. $v = 120t + 120$

Slope $= \frac{152 - 120}{1}$

$m = \frac{32}{1}$

$(0, 120)$ is the intercept.

$v = 32t + 120$

Basic Geometry Concepts & Transformations

1) A circle has a circumference of 16π feet. What is the radius of the circle, in feet?

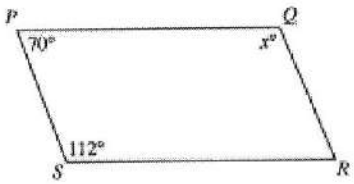
A. $\sqrt{8}$
B. 4
* C. 8
D. 16 * d = 16 false answer
E. 32

$C = \pi d$

$\frac{16\pi}{\pi} = \frac{\pi d}{\pi}$

$d = 16$ so the radius = 8

2) In quadrilateral $PQRS$ below, sides PS and QR are parallel for what value of x ?



A. 158
B. 132
C. 120
* D. 110
E. 70

$70° + x° = 180°$

$x = 110°$

3) In the standard (x,y) coordinate plane, what are the coordinates of the midpoint of a line segment whose endpoints are (–3,0) and (7,4) ?

Midpoint formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{-3+7}{2}, \frac{0+4}{2}\right)$

$(2,2)$

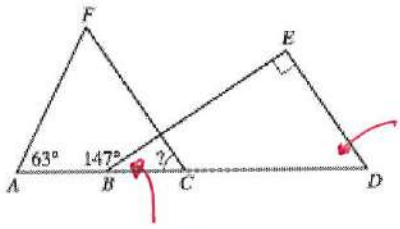
* A. (2,2)
B. (2,4)
C. (5,2)
D. (5,4)
E. (5,5)

4) What is the degree measure of the acute angle formed by the hands of a 12-hour clock that reads exactly 1 o'clock?

$360^\circ \div 12 \rightarrow 30^\circ$

A. 15^o
* B. 30^o
C. 45^o
D. 60^o
E. 72^o

5) In the figure below, *A*, *B*, *C*, and *D* are collinear, *FC* is parallel to *ED*, *BE* is perpendicular to *ED*, and the measures of $\angle FAB$ and $\angle EBA$ are as marked. What is the measure of $\angle FCB$?

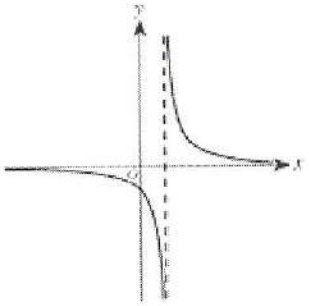


$90^\circ - 33^\circ = 57^\circ$

33°

A. 33^o
* B. 57^o
C. 63^o
D. 84^o
E. Cannot be determined from the given information.

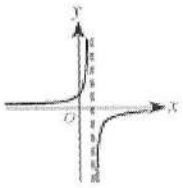
6) The graph of a certain hyperbola, $y = h(x)$, is shown in the standard (x,y) coordinate plane below.



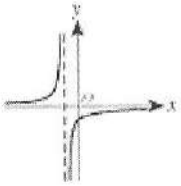
Among the following graphs, which best represents $y = -h(x)$?

REFLECTION ACROSS x-AXIS

* A.



B.



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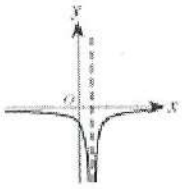

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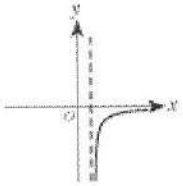
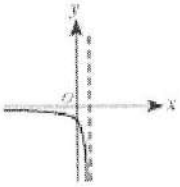
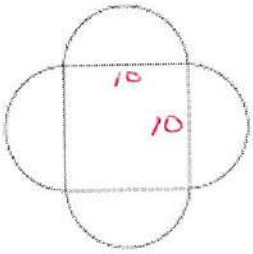


E.



7) The geometric figure shown below consists of a square and 4 semicircles. The diameters of the semicircles are the sides of the square, and each diameter is 10 centimeters long. Which of the following is the closest approximation of the total area, in square centimeters, of this geometric figure?



SQUARE
$10 \times 10 = 100$

CIRCLES
πr^2
$\pi (5)^2$
25π

$\times$ 2 CIRCLES $= 50\pi \approx 157.1$

A. 100
* B. 160
C. 260
D. 400
E. 730

Exponent Laws & Logarithms

1) If $n = 8$ and $16 \cdot 2^m = 4^{n-8}$, then $m = ?$

$16 \cdot 2^m = 4^{8-8}$
$16 \cdot 2^m = 4^0$
$16 \cdot 2^m = 1$
$2^m = \frac{1}{16}$
$m = -4$

* A. -4
B. -2
C. 0
D. 1
E. 8

Exponent Laws:

$b^m \cdot b^n = b^{m+n}$

$\frac{b^m}{b^n} = b^{m-n}$

$(b^m)^n = b^{mn}$

$b^0 = 1$

Logarithm Laws:

$\log_b mn = \log_b m + \log_b n$

$\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log_b m^n = n \cdot \log_b m$

$\log_b b^m = m$