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Resonant Acoustic Characterization of Coins: An Inquiry-Based Learning Activity for Everyone with a Smartphone

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November 10, 2021

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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(Dated: October 22, 2021)

Abstract

This study describes a new instructional laboratory activity where the acoustic frequencies produced by the resonant vibrational modes of coins are measured using smartphones. The resulting acoustic spectra allow for unique characterization of coins with different physical properties. This is an extremely accessible inquiry-based learning opportunity for students fostered by the powerful capabilities of modern smartphones and the widespread distribution of precision test objects: coins produced by national mints around the globe.

I. Introduction

Inquiry-based learning allows students to engage deeply with phenomena, both natural and designed, through exploration, reflection, and discussion that involves science and engineering practices, disciplinary core ideas, and crosscutting concepts. This approach underpins the Next Generation Science Standards framework[1] and the recommendations from the National Academies of Sciences, Engineering, and Medicine to put investigation and design at the center of the classroom.[2] The most impactful inquiry-based learning activities are easily accessible to a diverse student population and spark students' intrinsic curiosity to explore the world around them. Measuring and interpreting the resonant acoustic frequencies of coins provides an extremely rich inquiry-based learning opportunity for students fostered by the powerful capabilities of modern smartphones and the widespread distribution of precision sample objects: coins produced by national mints around the globe. Nearly uniform access of students to smartphones makes this activity equitable because it can be done for free in almost any classroom.

This inquiry-based learning activity includes opportunities for students to: 1) explore disciplinary core ideas of force, waves, and energy, 2) investigate crosscutting concepts of characterizing resonance frequency patterns that depend on variations in scale and material properties, and 3) engage in science and engineering practices which include defining a problem and the requirements for a solution, planning and validating an optimized experimental design, analyzing data, comparing experimental results to models, constructing explanations, and effectively communicating observations.

Waves are a foundational phenomenon in physics and the principles of waves are routinely applied by scientists and engineers to explain the origins of the universe and design much of the technology we use daily. The concepts of superposition and standing waves are often introduced to students through vibrations of strings and of columns of air.[3,4] Investigation of these resonant systems include the introduction of boundary conditions and models for the normal modes of oscillation. These concepts can be extended to more complex three-dimensional solid structures which also have resonant modes of vibration. In both simple and complex systems, the vibrations of the system can be characterized by measuring the sound waves that result from the vibrations. Engineers regularly make use of resonant vibrations to evaluate the quality of manufactured parts or the integrity of mechanical systems. Using a method often referred to as resonant acoustic testing, engineers will measure the resonant acoustic frequencies produced by an object following a rapid impulse that excites resonant vibrational modes.[5] The resonant frequencies are used to determine if the object has the desired physical properties or if the part has unexpected physical and structural differences from the design specifications.[6]

The resonant acoustic method is also well-suited to investigate the physical properties of coins. The resonant frequencies can provide a "fingerprint" (i.e., a unique signature of a coin) which allows coin recognition, counterfeit coin detection, and a potentially more detailed investigation of individual coins. Despite the potential, there have been limited published investigations,[7,8,9,10] and there is still much to learn about the benefits and limitations of using this approach to effectively characterize coins.

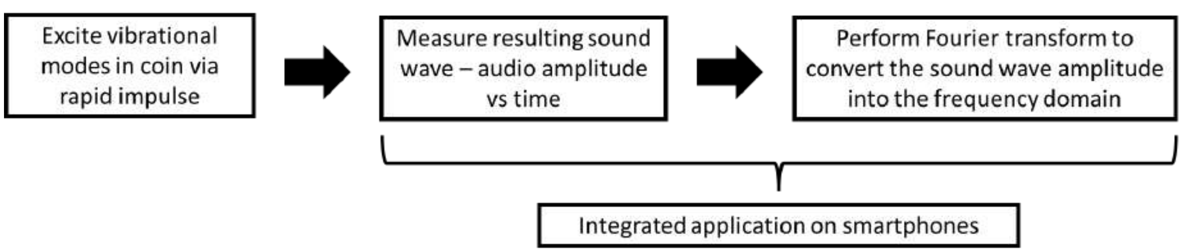
Several physics education-oriented papers have described the use of smartphones to characterize the frequencies of sound waves produced in model systems including air columns[11] and rods.[12] The incorporation of high-quality microphones in smartphones combined with powerful software applications allows students to investigate sound[13] with unprecedented detail, including the many beautiful details of music and the foundations of voice recognition. These standard, yet powerful, smartphone capabilities also provide students with the ability to measure the sound waves produced by resonant vibration of coins.

This paper will introduce the application of the resonant acoustic method for characterizing coins in the context of providing an extremely rich area for student inquiry. While the basic concept of differentiating between the denominations of recently minted coins is an easy place to start, this paper will illustrate the potential for scaffolded levels of investigation where students can probe many unknown questions about the structure and composition of coins. In addition to the STEM benefits of this particular activity, there is broad general interest in coins[14] that are stamped with a date and designed with symbolic images. Investigating a variety of coins is a pedagogical tool that can excite students by combining physics with their interest in art, society, and history.[15]

II. Experimental Method

The principal objective of this activity is to evaluate the effectiveness of using resonant acoustic frequencies (resulting from the quantized vibrational modes) to uniquely characterize coins with different physical and/or structural properties. An additional requirement of the activity is to assure the characterization can be conducted reproducibly by different individuals in almost any location without any specialized equipment beyond a common smartphone.

A schematic of the experimental approach is provided in the block diagram below.



Exciting the Vibrational Modes in the Coin

The first step in the experimental method is to impart energy into the coin to excite a number of well-defined vibrational modes. The vibrating coin will generate sound waves that can be accurately measured using the microphone of a smartphone. Students can easily verify different approaches to put energy into a coin in order to generate audible sound waves. Physics students will also be familiar with the influence of boundary conditions on the standing waves patterns in other resonant systems such as the strings or the air columns in open and closed pipes. Since any contact with the coin by another object will influence its boundary conditions and subsequent vibrational modes, the ideal experimental design should allow measure of the resonant acoustic frequencies while the coin is vibrating in free space. A simple experimental design meeting these conditions is shown in Fig. 1 where the coin is initially held between two fingers a few centimeters above a hard surface. The coin is then released and allowed to bounce repeatedly on the hard surface (e.g., wooden table, wooden floor, granite countertop, kitchen sink, or lab bench). The coin will lose energy after each collision, resulting in a series of bounces of decreasing height (and shorter times between collisions) until the coin comes to rest on the surface.[16] During the periods between impulses, the coin will be free to vibrate in space and produce sound waves associated with its resonant vibrational modes.

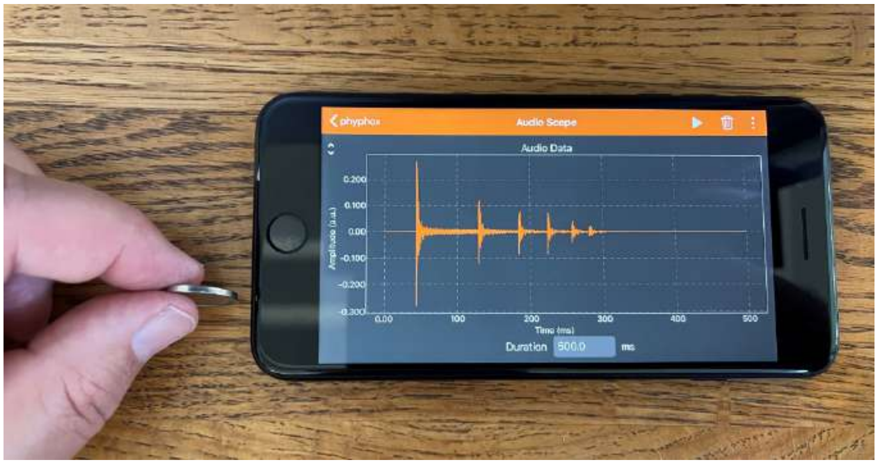


Figure 1: Experimental design used to measure the resonant acoustic frequencies of coins. The coin is dropped from a few centimeters onto a hard surface near the microphone of a smartphone. The coin is allowed to bounce repeatedly on the surface until it comes to rest.

The sound waves produced by the repeated collisions and the vibrating coin can be measured using the internal microphone of a smartphone. Most smartphones are designed to collect data from their microphones at rates up to 48,000 Hz. The audio amplitude vs time for a sound wave produced by bouncing/vibrating nickel (2017 D) on a hardwood table (3 cm thick) is shown in Fig. 2. The data was collected using the free application phyphox[17] ("Audio Scope" submodule) and exported for further analysis. The first bounce for the data presented in Fig. 2 occurs at 28 ms. The amplitude of the sound wave demonstrates an initial rapidly decay followed by a lower amplitude soundwave which persists until the next bounce occurs at 108 ms. Sequential collisions producing sound waves of decreasing amplitude occurring with decreasing time intervals between collisions can be observed. It is easy to identify more than 10 individual impulses in Fig. 2.

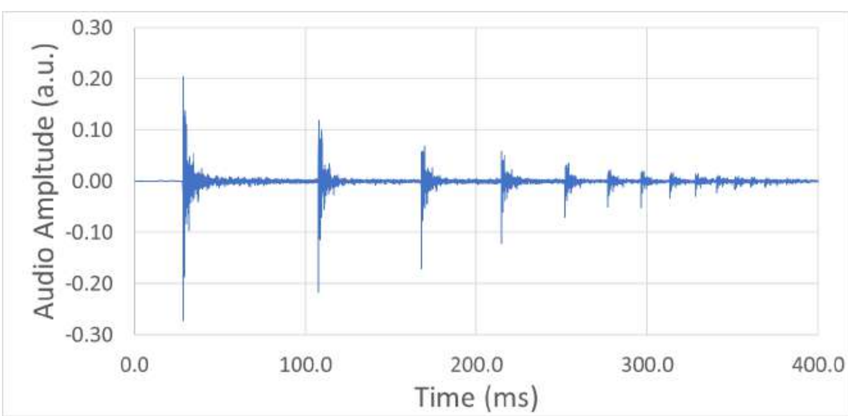


Figure 2: Audio amplitude vs time produced by dropping a nickel onto a hard wooden table as shown in Fig. 1. The sound was recorded at 48 kHz using the microphone on a smartphone.

Transforming Data from Amplitude to Frequency Domains

The audio amplitude vs time data can be transformed to the frequency domain using a mathematical algorithm known as a Fast Fourier transform (FFT). While the sound wave of a vibrating coin is very hard to interpret from an amplitude vs time graph, transformation to the frequency domain will allow identification of a small number of resonant frequencies that result from well-defined vibrational modes of the coin. Applications for conducting this transformation and displaying real-time frequency data are available for smartphones. Two different free applications (found to be very high quality, simple to use, and provide significant flexibility in choosing the analysis parameters) were used to collect the data presented in this study. The two applications used were Advanced Spectrum Analyzer Pro[18] on Android phones and SpectrumView[19] on IOS phones (the authors are not aware of a free application with the desired functions available on both operating systems). An example acoustic resonance spectrum produced by a nickel (2017 D) and analyzed by Advance Spectrum Analyzer Pro is shown in Fig. 3. Clear resonant frequencies are observable at 12.26, 14.60, 21.83 kHz. This example provides evidence that a vibrating solid object can produce a series of well-defined acoustic frequencies. Understanding the variation of these acoustic frequencies, resulting from changes in the physical properties of different coins, will be the focus of this investigation.

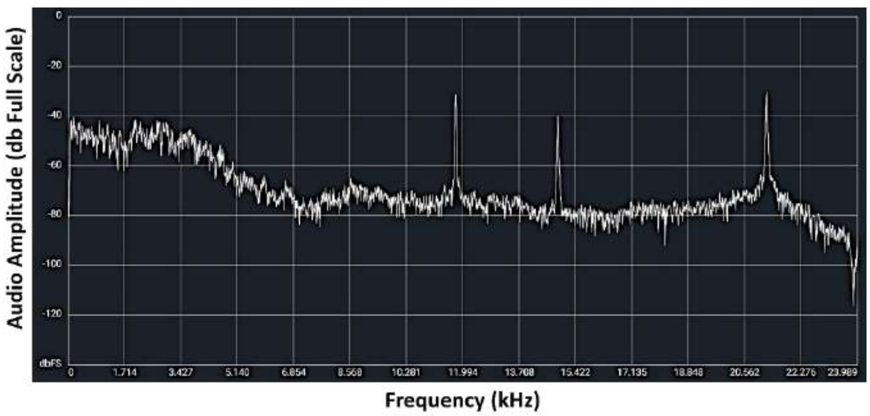


Figure 3: Resonant acoustic frequency spectrum for a nickel. The graph is a screen capture of amplitude vs frequency using Advanced Spectrum Analyzer Pro application on a Samsung Galaxy Note9 smartphone. Frequency is displayed on a linear scale.

The observable frequency range is limited by a combination of the microphone response and the sampling frequency. For current generation smartphones, it is possible to analyze frequencies that range from approximately 20 Hz (low end of frequencies audible to humans) to the maximum observable frequency for a data collection rate of 48 kHz, which is 24 kHz (defined by Nyquist Criterion). The microphones used in smartphones have reduced response characteristics for frequencies approaching 24 kHz but multiple different brands of smartphones used in this study demonstrated the ability to measure frequencies up to the 24kHz limit. There are several parameters that are important in conducting the transformation from the time domain to the frequency domain that will determine the bandwidth (i.e., precision) of the calculated frequencies and the time resolution of each measurement. The first analysis parameter is the number of data points to be included in each FFT calculation (i.e., 256, 512, 1024, 2048, 4096, 8192 or 16385). The larger the number of data points included in the calculation, the smaller the bandwidth in the frequency domain. Using a smaller number of data points will enable the measurement of frequencies at shorter time intervals but will result in increasing the bandwidth. The second parameter is the number of FFT calculations, averaged to improve the signal-to-noise ratio of the measurement. Both the number of data points included in the FFT calculation and the number of FFT calculations to average can be selected in the settings menu for both applications. The parameters used to collect the data in Fig. 3 were 2048 data points for the FFT calculation and four calculations averaged. This resulted in a new frequency vs time spectrum determined (and displayed) every 0.17 seconds ((2048 samples per FFT calculation x 4 FFT calculations per displayed spectrum)/48,000 samples per second). Exploration of the influence of these settings provides a useful exercise for students to learn about tradeoffs between time resolution and frequency bandwidth in electronic signal processing.

A more detailed understanding of the physical processes producing the sound wave can be achieved by examining a single impulse and the subsequent "bounce" in greater detail as shown in Fig. 4. The resulting sound waves will be a combination of the sound waves that are produced by the surface involved in the collision as well as the sound wave produced by the vibrating coin. The coin will only be in contact with the surface for a short period of time during each impulse and will spend most of the time vibrating in free space between sequential impulses. Students can use the slow-motion video on their phone cameras to verify that the time of the impulse is much less than 4.2 ms which is the frame-to-frame resolution for cameras operating at 240 fps. From Fig. 4a we can see the large amplitude of the sound wave following the initial impulse. The sound wave decays relatively quickly, reducing amplitude by more than 10x in the first 10 ms after the collision. However, Fig. 4b shows that the sound wave persists for greater than 60 ms following the collision.

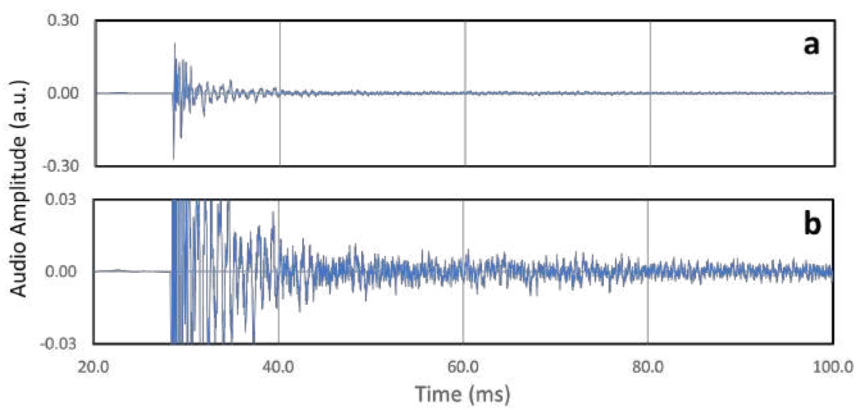


Figure 4: The audio amplitude from the experiment shown in Fig. 1, is shown on an expanded time scale for the time following the initial impact which occurs at ~28 seconds: a) audio amplitude at full scale, b) audio amplitude x10.

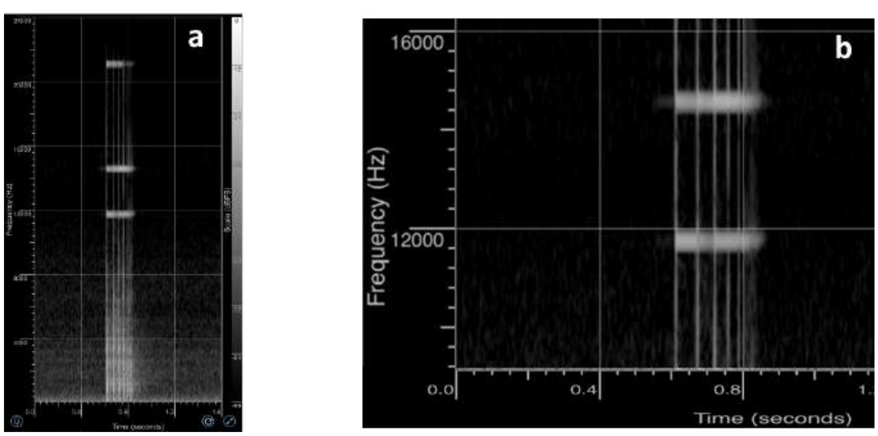


Figure 5: a) Spectrogram resulting from dropping a nickel on a hard wooden surface as described in Fig. 1. The frequencies were calculated in real-time using the SpectrumView application. b) Expanded view for same spectrogram showing two of the resonant frequencies.

The nature of this persisting sound wave can be further understood by looking at the frequency spectrum of the soundwave as a function of time. A "spectrogram" from a bouncing nickel is shown in Fig. 5a, generated by the SpectrumView application. The spectrogram consists of a waterfall plot of frequency vs time where the amplitude of the sound wave at a particular frequency is indicated using a color table (shown on the side of the graph). The analysis parameters were set to conduct a FFT every

512 data points and chart the results from each calculation on the waterfall plot (i.e., a new frequency spectrum every ~10 ms). Three persistent frequencies are easily identified by the three horizontal "bands" near 12, 15 and 21 kHz from Fig. 5a. In addition, there is an increase in the amplitude for lower frequencies over a broad spectral range (0–6kHz) while the coin is bouncing. In Fig. 5b, the scale is expanded to look more closely at the time the coin is bouncing for a narrower frequency range. In Fig. 5b, a series of vertical lines are observed that correlate with each impulse. This suggests that each impulse produces sound waves composed of a broad range of frequencies (non-resonant) that damp out quickly. The features of the spectrogram of greatest interest for this experiment are the discrete frequency bands that persist for tens of milliseconds. These bands represent the resonant acoustic frequencies generated by the nickel vibrating in free space. These well-defined frequencies are associated with the persistent sound wave shown in Fig. 4b.

One of the design requirements is that students in different locations will be able to produce identical results when using different surfaces to create the impulse. While the observations discussed above support the physical model of measuring the resonant frequencies of a coin freely vibrating in air, students may still question if the surface or the drop height (i.e., changing the energy imparted to the coin) might affect the observed frequencies. In fact, several of the published research reports made significant effort to use fixed experimental designs to assure reproducible impulses despite that fact they acknowledged the measured frequencies should be independent of the way the energy was placed in the coin.[7,9] Students should be encouraged to design and conduct tests to validate their ability to measure identical frequencies as they change surfaces and/or the magnitude of the impulse. Figure 6 shows the acoustic resonances frequencies for a Kennedy half-dollar coin (1972) where four different household surfaces were used in the experiment. The measured frequencies are independent of the surface within resolution of the experimental measurement. In contrast, the relative amplitudes of the different resonant frequencies demonstrate considerable variation. The amplitudes of the resonant acoustic frequencies vary based on how the coin is dropped and as a function of time following each impulse. It would be expected that different vibration modes could have different degrees of excitation depending on the nature of the impulse and the decay constant may vary for different vibrational modes. While these additional variations could contribute to the unique characterization of coins, the experimental design required for reproducible measurement of amplitudes and time dependent decays presents a significant challenge and are not addressed further in this study.

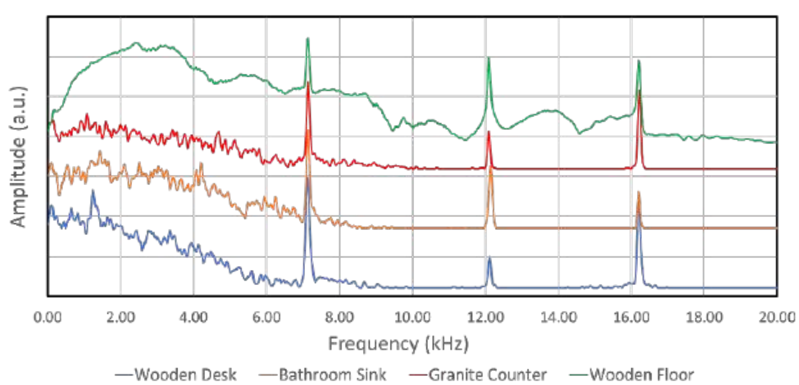


Figure 6: The acoustic resonance frequencies measured for a Kennedy half dollar coin (1972) using four different surfaces.

Students may want to spend additional time investigating alternative experimental methods to introduce energy into the coins. Several alternative approaches have been discussed in the literature[8] including resting the coin on the pad of one's finger or dangling a coin by a thread, followed by inducing an impulse by striking the coin with a hard object such as a second coin. Students may also consider observing the resonant acoustic spectra as the coin spins on the table and "rattles" to a stop. Students can discuss the merits and drawbacks of the various approaches. The authors found the method from Fig. 1 provided the most reproducible results for a well-defined physical state of the coin (i.e., free edge vibration).

III. Results and Analysis

Modeling of the Vibrational Modes of Coins[20]

Analysis of the natural vibrations of a coin can be guided by constructing a simple model, the coin as a flat disk of diameter *d*, thickness *h*, and composed of a uniform material. The motion is assumed to be small so that the material can be treated as simply elastic (meaning that the material deforms in proportion to the force applied and a temporary shape change is reversable after the force is removed). The response of an elastic material to applied forces depends on the material properties of ρ density, E Young's modulus, and ν Poisson's ratio. Young's modulus (named for Thomas Young, a British polymath) is a measure of stiffness as the force/area needed to get some fractional change in length (values range from E $\approx$ 3 GPa [PET, the plastic used in a soda bottle] to E $\approx$ 200 GPa [common steel]). Poisson's ratio (named for Siméon Poisson, a French polymath) is a measure of how much a material expands or contracts in directions perpendicular to the direction of squeezing or stretching (for an isotropic material the value is less than one-half, for materials used in coins the values range from about 0.25 for Zn to 0.42 for Au). The vibrations are displacements from the flat shape as a function of position from the center to edge (radius, $0 \le r \le d/2$), angle around the edge (azimuth $0 \le \theta \le 2\pi$) and time, *t*. The solution for displacement can be written,

$$w_{sn}(r,\theta,t) = R_{sn}(r/h)\cos(n\theta)\sin(2\pi f_{sn}t) \quad (1)$$

where the function *R* has a weak dependence on Poisson's ratio but otherwise is the same for every disk. The indices *s* and *n* refer to different radial and azimuthal modes. The lowest order modes for a coin (a disk treated as having edges that are free to move, versus a drumhead as a disk treated as having edges that are rigidly fixed) are shown in Fig 7. The modes (s = 0, n = 2) and (0,3) have a single radial inflection but have two or three azimuthal cycles, respectively. The modes (1, 0) and (1,1) have two radial inflections but have zero or one azimuthal cycle, respectively. Depending on how the coin is excited many modes can exist at the same time and produce a sound wave composed of several frequencies. The individual frequencies can be written,

$$f_{sn} = (\lambda_{sn})\left(\frac{h}{\pi d^2}\right)\sqrt{\frac{E}{3\rho(1-\upsilon^2)}} \quad (2)$$

where the first term is a numerical factor that comes from the analysis (again with just a weak dependence on Poisson's ratio), the second term carries the geometry, the third term carries that material properties. The ratios between frequencies for a given coin type (same size and material) are then purely ratios of the factors λ_{sn}. The ratios between frequencies of particular modes for a specific denomination of coin (e.g., a copper penny and a zinc penny) are the ratios of the material terms. The factors λ_{sn} have been previously tabulated for many different types of plates (e.g., disks, rectangles, annular disks) and for many different types of boundary conditions (e.g., free edge, clamped, simply supported).[21]

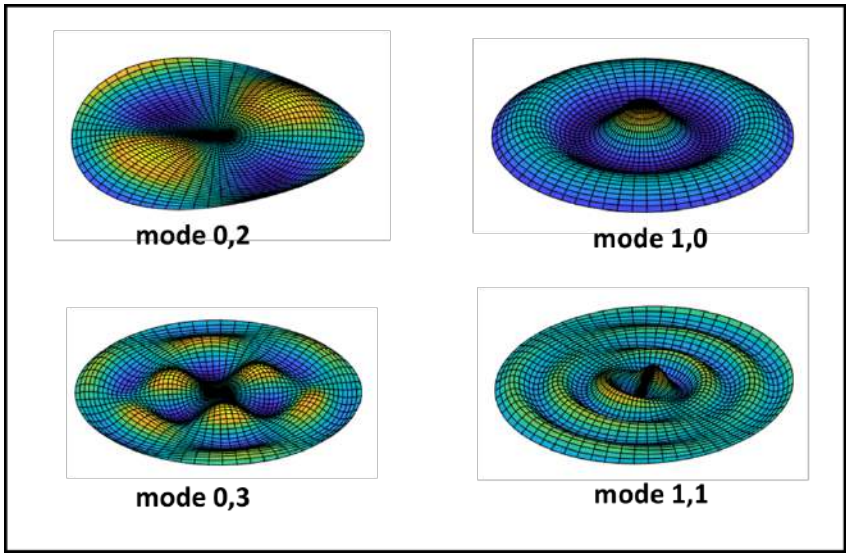


Figure 7: The first four vibrational modes of a free edge disk.

The diameter, thickness, mass *m*, and material for every coin used in this study is specified and highly controlled by the United States (US) Mint. However, coins are not simple flat plates, and the specifications appear to be for a peak-to-peak thickness. For the analysis presented in this study, the effective thickness $h' = m/(\rho\pi\, d^2/4)$ is used in Eq. 2 to represent the height of the coin, h.[22]

Using values from the US Mint, material properties from the literature,[23] and the equations presented above, the resonant vibrational frequencies for selected coins were calculated and are presented in Table I. Values for the five lowest frequency modes are included in the table with the frequencies within our detection range presented in bold type. The model predicts that the modern US penny, nickel, dime, quarter, half dollar, and dollar coin will all produce at least one acoustic resonance frequency within the measurement range (i.e., <24 kHz). The model also predicts significant differences in resonant acoustic frequencies for each denomination except for the penny and the dime which are predicted to have similar resonant frequencies. The model also predicts significant change in the resonant frequencies for coins of the same denomination but composed of different materials.

	Resonant Frequency (kHz)								
Mode	Penny (Steel)	Penny (Cu)	Penny (Zn)	Nickel (CuNi)	Dime (CuNi)	Quarter (CuNi)	Half Dollar (Ag)	Half Dollar (CuNi)	Dollar (CuNi)
0,2	**17.4**	**13.0**	**13.0**	**14.3**	**12.5**	**9.4**	**4.28**	**7.4**	**9.1**
1,0	28.1	**22.8**	**21.0**	**24.6**	**21.9**	**16.4**	**7.64**	**12.9**	**15.9**
0,3	40.2	30.3	30.2	33.3	29.1	**21.8**	**10.0**	**17.2**	**21.2**
1,1	64.4	51.3	51.3	55.7	49.3	37.0	**17.2**	29.2	35.9
0,4	70.4	53.2	53.2	58.5	51.2	38.4	**17.6**	30.3	37.2

Table I: Calculated resonant frequencies for selected US coins.

Comparing Resonant Acoustic Frequencies: Penny, Nickel, Dime, Quarter and Half Dollar

Figure 8 shows the acoustic frequency spectrum for five different denominations of coins, acquired using the spectrum analyzer feature in SpectrumView. The experiments were conducted using 2048 data points for the FFT and the results of four FFT calculations were averaged, resulting in a new spectrum displayed every 0.17 s. Each spectrum in Fig. 8 was captured by pausing the data collection during the experiment to "capture" an individual spectrum for further analysis using SpectrumView. Several resonant frequencies are easy to identify for each coin and all the observed frequencies were greater than 6 kHz. Table II lists the observed frequencies from the resonant acoustic spectra in Fig. 8. The calculated values from Table I are included in parentheses for comparison. The experimental measurements for the penny, dime, quarter, and half dollar are found to be in good qualitative agreement with the model. The measured acoustic resonance frequencies are typically within 1 kHz of the frequencies predicted by the model. In contrast, the nickel shows significant deviation from the model predictions. It should be noted that the acoustic resonant spectrum in Fig. 8 were selected to demonstrate the "simple case" for each of the different denominations. As will be discussed in the next section, the (0,2) vibrational mode frequently "splits" into two observable resonances.

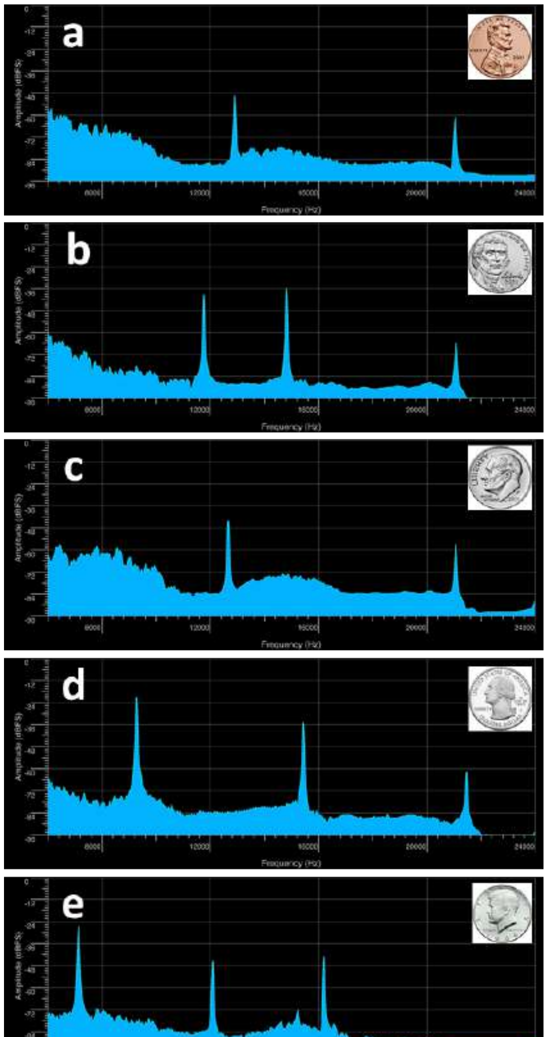


Figure 8: Representative acoustic resonance frequency spectra for 5 different US coins are shown: a) Lincoln penny, b) Jefferson nickel, c) Eisenhower dime, d) Washington quarter, and e) Kennedy half dollar.

Denomination	F_1 (kHz)	F_2 (kHz)	F_3 (kHz)
Penny	12.90 (13.0)	21.04 (21.0)	
Nickel	11.76	14.81 (14.3)	21.07 (24.6)
Dime	12.69 (12.5)	20.07 (21.9)	
Quarter	9.28 (9.4)	15.59 (16.4)	21.55 (21.8)
Half-Dollar	7.12 (7.4)	12.08 (12.9)	16.19 (17.2)

Table II: The observed frequencies for the data in Fig. 8. Calculated values from Table I are included in parentheses.

The ability of students to collect high-quality resonant acoustic spectra for coins similar to that presented in this study has been successfully demonstrated in multiple classrooms over the past year using a diverse set of smartphones.[24] This success can be compared to the only previously reported analysis of US coins where a "standard computer" microphone was used in the experiments. In the previous study[7], Steed reported measurements for multiple US coins but was not able to detect resonances for a penny, which is easily measured using a smartphone. In one validation of the smartphone-based approach, students were given graphs of resonant acoustic spectra collected by the teacher for six coins. The students were then asked to determine what denomination of coin was responsible for each of the resonant acoustic spectra. Students conducted experiments on their own set of coins (penny, nickel, dime, quarter, and half dollar) and attempted to match their observations to the resonant acoustic spectra of the reference graphs. (Since students were not provided with a dollar coin, its corresponding spectrum had to be determined by the process of elimination.) Very high agreement was achieved by the students except for differentiating a dime from a penny (often reversed due to their similar resonant acoustic frequencies).

Ideally, these initial experiments will inspire students to extend their investigation to include a wide range of other types of coins produced around the world, counterfeit coins, and further investigate the potential impact of other experimental parameters on their measurements (e.g., temperature, visible defects in the coins, …).

Evaluating the Precision of the Experimental Method

One of the fundamental practices implemented by scientists and engineers in experimental studies is to determine the precision of their measurement tools. Before exploring the differences between coins in greater detail, it is necessary to understand the precision of the experimental method. First, it is important to quantify the precision of measuring the resonant frequencies for a single coin with an individual phone. Secondly, it is important to characterize the precision of measurements made on a single coin using different devices. Table III provides data collected for five different devices on a single US quarter (2020 D – US Virgin Islands) intended to evaluate both aspects of the experimental method. The analysis parameters were set to 4096 samples for the FFT and four FFT calculations were averaged for each analysis. The data in Table III demonstrates the high precision of repeated measurements with a single phone as well as the precision achieved using different devices (standard deviation of ~10 Hz). The high precision enabled by the smartphone technology as well as the reproducible experimental method provides the opportunity to interrogate the detailed properties of coins that might extend beyond differentiation of denominations.

Device	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)
Samsung Galaxy Note9	8840	9470	14930	21280
	8850	9470	14930	21270
	8850	9470	14930	21270
iPhone 12	8830	9460	14920	21260
	8830	9460	14920	21260
	8840	9460	14920	21270
iPhone 8	8830	9450	14910	21260
	8810	9450	14910	21260
	8840	9450	14920	21260
iPad Pro (2nd Gen)	8840	9450	14930	21260
	8830	9460	14910	21260
	8830	9450	14910	21270
iPhone 7	8830	9450	14910	21250
	8840	9460	14920	21250
	8820	9460	14910	21250
Average (Hz)=	**8834**	**9458**	**14919**	**21262**
Standard Deviation (Hz) =	**10**	**7**	**8**	**8**

Table III: The resonant frequencies for an individual quarter (2020 D) are shown for 5 different devices. Three trials for each device are included. The average value and the standard deviation were determined for each of the four frequencies determined from the 15 experiments.

Students can design a variety of approaches to collect similar data to quantify the precision of the experimental approach while carefully considering which variables should be controlled. Partnering with classmates will enable the comparison of different devices as well as potentially identify any differences that might be introduced by different experimenters. Comparisons could also be extended to different classrooms or schools via shared databases. This exercise in collaboration allows students to experience some of the challenges that are associated with shared data bases where understanding the reliability of data supplied by diverse collaborators is critical and is one of the great challenges in data science. One example is GenBank,[25] which host large amounts of genomic data, where differences in the published genetic sequences are impacted by the sequencing methods and the computational tools used to assemble the reported sequences.

Comparing the Resonant Acoustic Frequencies of Coins of the Same Denomination

The variation in the resonant acoustic frequencies between different denominations of coins was illustrated in Fig. 8. However, using patterns of resonant acoustic frequencies to identify specific denominations with high confidence will also depend on the variations within coins of the same denomination. An example of the typical variation in the resonant acoustic spectra between coins of a similar demonization is shown in Fig. 9 for three different Washington quarters. The resonant acoustic frequencies associated with different vibrational modes of the quarters from Fig. 9 are displayed in Table IV. The two resonant acoustic frequencies at ~15.4 kHz and ~21.2 kHz, associated with the (1,0) and (0,3) vibrational modes respectively, appear to be single frequencies and each of the resonant frequencies have variances of a few 100 Hz between the three coins. The resonant acoustic resonance at ~9.2 kHz, associated with the (0,2) mode, appears as a single resonance in the 2016 quarter but is clearly split into two resolvable frequencies in the 2005 and 1986 quarters. The splitting of the (0,2) mode is also frequently observed for pennies, quarters, dimes, half dollars, and dollar coins. Figure 10 illustrates the splitting of the (0,2) mode for an Eisenhower dime, a Kennedy half dollar and a van Buren dollar. In an experimental survey of multiple coins of each denomination (penny, dime, quarter, half dollar, and dollar) the (0,2) modes can be observed as a single frequency, within the resolution to the measurement, or can be observed as two resonances with spacings of 100's of Hz. Manas[8] notes that the coins are not simple flat disks but have an asymmetry due to the minting process which may be responsible for the splitting. However, the significant variation in the degree of splitting is observed for coins with the same date and mint mark which are visibly indistinguishable, leaving the explanation of the splitting as a subject for further investigation. The frequencies associated with the (1,0) and (0,3) have only been observed as single resonances for the hundreds of coins explored in this study.

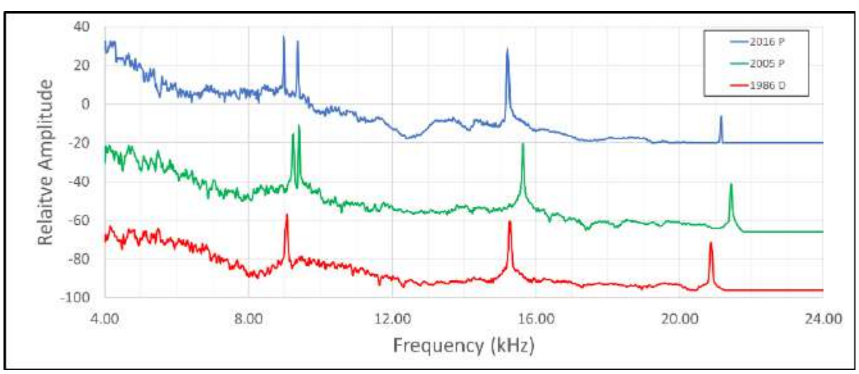


Figure 9: Resonant acoustic spectra for 3 different Washington quarters which are nominally the same composition and size. The three coins demonstrate a progressive change in the splitting of the (0,2) mode that is observed with no obvious correlation to date or mint.

Quarter Date Mint	mode 2, 0	mode 0, 1	mode 3, 0
1986 D	8.98, 9.38	15.21	21.15
2005 P	9.23, 9.41	15.63	21.43
2016 P	9.07	15.27	20.88

Table IV: The resonant frequencies determined for the three resonant acoustic spectra of quarters shown in Fig. 9.

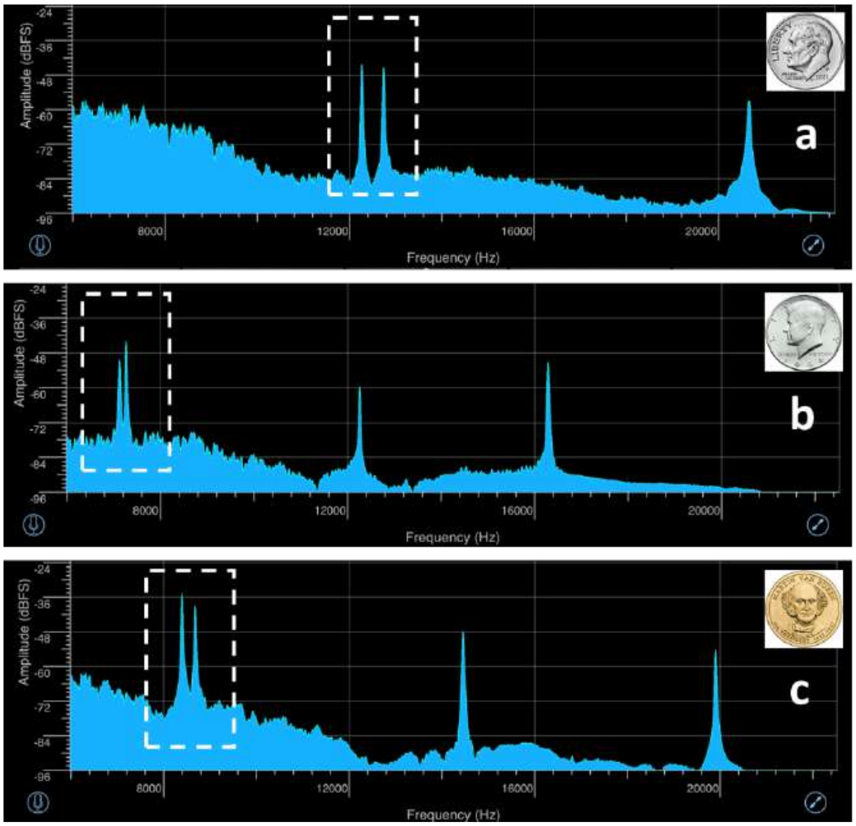


Figure 10: Resonant acoustic spectra for: a) Eisenhower dime (2016 D), b) Kennedy half dollar (1978 D), c) Martin van Buren dollar (2008 D). The splitting of the resonant acoustic frequency associated with the (0,2) vibrational mode is indicated by the dotted white box

The largest variations in resonances acoustic frequencies observed in modern US coins are seen in the Jefferson nickel. Three resonances are observed in most nickels, as shown in Fig. 11a labeled as F1, F2, and F3. The resonant acoustic frequencies were measured in 43 Jefferson nickels minted in Denver. The coins were minted between 1960–2020 with one coin for each year included in the analysis when available. The resonant frequencies for each of the coins in plotted in Fig. 11b. If one looks carefully at the relative variation of F1, F2, and F3, it becomes visibly obvious that there is a strong correlation between the frequency variations of F3 and F1. When the ratio of F3/F1 is calculated, the standard deviation of the ratio is 3x smaller than the standard deviation observed for F3 alone. It is also apparent from the data that F2 and F3 appear to be anticorrelated. While it was not obvious from the resonant acoustic spectrum from a single nickel in Fig. 11a, it seems likely that the F1 and F2 frequencies are the result of the splitting of the (0,2) mode predicted to be at approximately 14 kHz. It is not completely evident to the authors how to explain the significantly larger splitting for the nickel relative to the other US coins. Student investigation of coins from other nations with a range of physical and structural characteristics may provide additional insight into the physical origins of the phenomena.

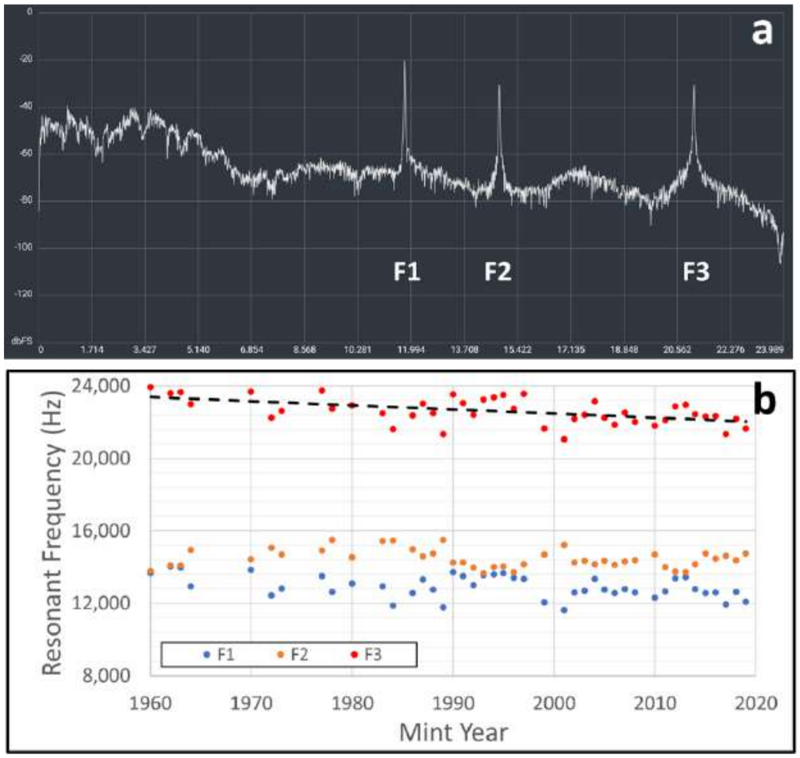


Figure 11: The resonant acoustic frequencies for Jefferson nickels, minted in Denver, for the time period of 1960-2020. a) Representative resonant acoustic frequency spectra; b) Resonant frequencies (F1, F2, and F3) observed for 43 different Jefferson nickels minted in different years.

The highest resonant acoustic frequency, F3, is observed at 22–24 kHz, a slightly lower frequency than the frequency for the (1,0) mode predicted to occur at 24.6 kHz. A gradual decrease in F3 is observed from 1960–2020 as illustrated by the linear fit (dotted red line). Further investigation will be necessary to determine the physical property changes over the years that might result in this trend.

Nickels originating from the Philadelphia mint demonstrated similar trends for all three resonant acoustic frequencies.[26]

Comparing the Resonant Acoustic Frequencies of Coins of the Same Denomination Composed of Different Materials

Throughout the history of the minting of coins used in commerce, the materials available for fabrication changed due to availability and cost. When changing materials used in the minting process, it is desirable to maintain the same dimensions and mass since various mechanism of exchange, such as vending machines, depend on identical denominations with similar properties. The changes in the coin composition will result in changes in Young's modulus, Poisson's ratio, and potentially the effective height (resulting from different strike depth during minting process). These changes can lead to differences in the normal modes of vibration for coins with nominally identical physical dimensions. The impact on the normal vibrational modes predicted by the model can be seen in Table I for three different compositions of the Lincoln penny and two different compositions of the Kennedy half dollar.

There have been several changes in the composition of the Lincoln penny during its production history, including the use of zinc plated steel in 1943 during World War II when the US conserved copper for use in munitions. The composition of the penny was also changed in 1982 from copper to zinc as the cost of copper used to make a penny began to exceed one cent. Figure 12 shows the resonant acoustic spectra for three pennies constructed out of steel, copper (95% copper / 5% zinc) and zinc (97.6% zinc / 2.4% copper). As predicted by the model, there is a significant increase in the resonant frequencies for the steel coin. Only the (0,2) vibrational mode is observed near 17 kHz which is very close to the value of 17.4 kHz predicted by the model. This is significantly higher than the (0,2) mode in either the copper or zinc penny, predicted and observed around 13 kHz. The higher frequency of the (0,2) vibrational mode of the steel coin is primarily the result of the high Young's modulus for steel (200 GPa), significantly larger than copper (130 GPa) or zinc (97 GPa). The (0,2) vibrational mode frequency for the copper and zinc penny are fortuitously predicted at the same frequency even though the Young's modulus differs between copper and zinc. The key counterbalancing factor is the significant difference in density of zinc and copper, 7.13 and 8.95 g/cm^3 respectively. As can be seen in Fig. 12, the acoustic frequency associated with the (0,2) vibrational mode is observed to be very close to the predicted value of 13 kHz for both the copper and zinc pennies. The (1,0) mode is predicted to be slightly higher for the copper penny, which is also qualitatively observed but not quantified in this study.

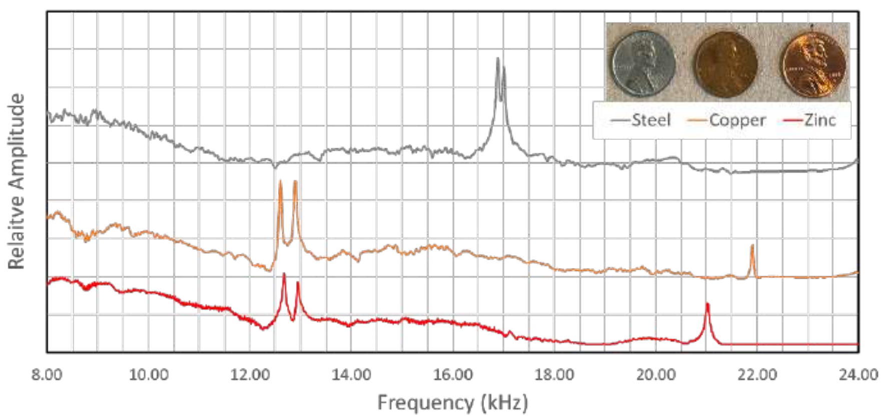


Figure 12: The resonant acoustic spectra for three pennies with different composition: top trace – 1943 (zinc coated steel), middle trace – 1975 (95 % copper / 5% zinc), and bottom trace – 2014 D (97.6 % zinc / 2.4% copper).

The Kennedy half dollar was first minted in 1964 as a memorial to John F. Kennedy following his assassination.[27] The coins during that first year of production were made of 90% silver and 10% copper. A very large demand for the coins contributed to a rapid depleting of the Treasury's stock of silver and led to congress passing the Coinage Act of 1965[28] which changed the composition of the half dollar to a 40% silver composition (an outer layer of 80% silver and 20% copper surrounding inner core of 21% silver and 79% copper) from 1965–1970. In 1971 the composition was changed[29] again to a composition of 75% copper and 25% nickel (alloy often referred to as a cupronickel) which has remained the composition to date. Figure 13 shows the resonant acoustic spectra for three half dollars that include a coin from each of the three periods representing the three different compositions. It is easy to see that the changes in material properties results in a substantial change in the observed frequencies subsequently predicted by the model for both the silver and the cupronickel half dollars. The observation of three resonances in the 17–20 kHz range where only two resonances were predicted might indicate a splitting of the (0,4) vibrational mode. The material properties for half dollars minted between 1965–1970 were more complex and were not included in the model calculations. The observed resonant acoustic spectra for the 1965 half dollar demonstrate an intermediate trend as might be expected based on the approximate 40% silver composition and the resonant frequencies of the other two half-dollar coins.

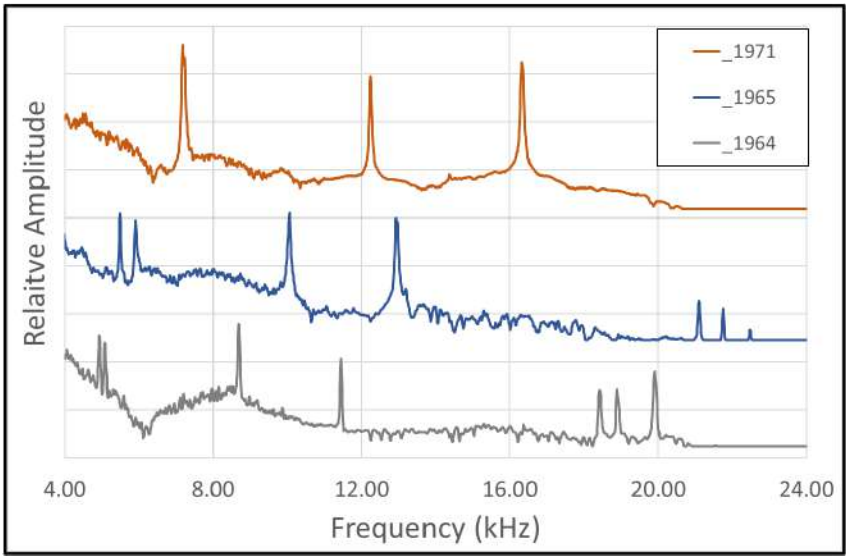


Figure 13: The resonant acoustic spectra for three half dollar coins which represent three different compositions: top trace (1971) has outer layer of 75% copper and 25% nickel surrounding a copper core, middle trace (1965) has an outer layer of 80% silver and 20% copper surrounding inner core of 21% silver and 79% copper, and bottom trace (1964) has a composition of 90% silver and 10% copper.

IV. Conclusions

Investigating coins through their unique resonant acoustic frequencies provides an extraordinary opportunity for inquiry-based learning. The investigation includes the use and reinforcement of multiple disciplinary core ideas as well as the opportunity to practice many of the foundational skills used by scientist and engineers.   In addition, the investigation of coins allows students to connect science to their interests in history, art, and society. The overall experimental investigation is enabled by the remarkable advances in smartphone technology and the wide accessibility of smartphones and coins (high precision test objects) to everyone.

This paper introduces an experimental approach and an application with extensive opportunity for further investigation. Rarely does an opportunity exist for students to conduct investigations where they can make novel discoveries and see patterns for the first time. The extraordinary capabilities of smartphones provide students with opportunities for novel discovery, in line with the practices of the "real" science community where the advent of novel scientific tools frequently propel new discoveries.

Finally, the resonant acoustic method outlined in this paper can be extended to characterize a wide range of objects. Students can use their imaginations to design investigations using the resonant acoustic method to characterize other more complicated structures. Inspired students could examine the resonant acoustic spectra of silverware, glasses, cans, or bottles, and will be amazed at the detail of the signatures they observe.

Acknowledgements

This work is performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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[13] A series of inquiry-based learning activities for students using smartphones to investigate sound can be found at: Mechanical Waves and Sound | Science and Technology (llnl.gov).
[14] See for example: The American Numismatic Association: https://www.money.org/.
[15] Post by Edward Van Orden, *Common Cents: Coins as Pedagogical Tools*, National Council for History Education (2021). Common Cents: Coins as Pedagogical Tools (ncheteach.org).
[16] Students will find that when coins are dropped on some surfaces too much energy is lost in the collision process. Such an impulses will not result in effective excitation of vibrational modes. Students might draw on a comparison of sound of a basketball bouncing on a gym floor verse the sound of a ball dropped onto sand at the beach.
[17] The phyphox application is available for free for both IOS and Android phones. Supporting documentation can be found at: phyphox – Physical Phone Experiments.
[18] Advanced Spectrum Analyzer Pro free application for frequency analysis of sound waves for Android phones: Advanced Spectrum Analyzer PRO - Apps on Google Play.
[19] SpectrumView free application for frequency analysis of sound waves for IOS phones: SpectrumView | Oxford Wave Research.
[20] Individual instructors will need to determine to what extent they should include the details of the mathematical model into their implementation of this activity. Some instructors may choose to include this model in a more qualitative manner.
[21] A. W. Leissa, "Vibration of Plates", NASA Tech. Brief SP-160, US Gov. Printing Office, (1969).
[22] Students could be asked to determine/validate the "effective" thickness of coins by conducting an experiment to determine the volumetric displacement and using the specified diameter.
[23] A list of the material properties used in the calculation is available from the authors on request.
[24] One of the authors (David Rakestraw) remotely led two different high school physics classes, one in California and one in New Mexico, where the resonant acoustic method was introduced to students and the students carried out investigations to successfully characterize coins.
[25] GenBank ® is the NIH genetic sequence database, an annotated collection of all publicly available DNA sequences GenBank Overview (nih.gov).
[26] The raw data is available upon request including data for 38 nickels from the Philadelphia which was not presented in this paper.

[27] John F. Kennedy Half Dollar Coin Legislation, Pub. L. No 88-256, (1963).
[28] Coinage Act of 1965, Pub. L. No. 89-81, (1965).
[29] Bank Holding Company Act Amendments of 1970, Pub. L. No. 91-607, Sec. 201, (1970).