Absolute and Conditional Convergence **Name:** Answers

AP Calculus BC

Consider the series below. Think about what the first few terms will be. (Use your calculator to help you if needed.) What do you notice about this series? Do you think this series will converge or diverge or is it hard to tell?

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^3} = \cos 1 + \frac{\cos 2}{8} + \frac{\cos 3}{27} + \frac{\cos 4}{64} + \ldots$$

$= .540 + -.052 + -.037 + -.010 + .002$

You may have noticed that the series above has both positive and negative terms, but it's not an alternating series. One way to get information on a series such as this is to investigate the convergence of the absolute value of the series:

$$\sum_{n=1}^{\infty} \left| \frac{\cos n}{n^3} \right|$$

What do you think about the convergence or divergence of the absolute value of the series?

$|\cos n| \leq 1$

so $\left|\frac{\cos n}{n^3}\right| \leq \frac{1}{n^3}$

$\frac{1}{n^3}$ is a p-series with $p > 1$

so $\frac{1}{n^3}$ converges

$\therefore \sum_{n=1}^{\infty} \left|\frac{\cos n}{n^3}\right|$ also converges

Here's a useful theorem:

If the series $\sum |a_n|$ converges, then the series $\sum a_n$ also converges.

Think about this. Does this make sense? yes! What can you conclude, now, about the series at the top of the page? Can you determine whether it converges or diverges?

Since $\sum \left|\frac{\cos n}{n^3}\right|$ converges then $\sum \frac{\cos n}{n^3}$ converges

Definitions:

The series $\sum a_n$ is **absolutely** convergent when $\sum |a_n|$ converges.

The series $\sum a_n$ is **conditionally** convergent when $\sum a_n$ converges but $\sum |a_n|$ diverges.

What's an example? ans. alternating Harmonic Series

Determine whether the following series are convergent or divergent. Classify any convergent series as absolutely or conditionally convergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n(n+1)/2}}{3^n} = -\frac{1}{3} - \frac{1}{9} + \frac{1}{27} + \frac{1}{81} - \cdots$$

note: Not alternating

$$\sum \left| \frac{(-1)^{n(n+1)/2}}{3^n} \right| = \sum_{n=1}^{\infty} \frac{1}{3^n}$$ which is a geometric series with $|r|<1$

This series converges $\boxed{\text{absolutely.}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} = -\frac{1}{ln2} + \frac{1}{ln3} - \frac{1}{ln4} + \frac{1}{ln5} + \cdots$$

Alternating Series test

$a_{n+1} < a_n$ and $\lim\limits_{n\to\infty} a_n = 0$

says this converges

$$\sum \left| \frac{(-1)^n}{\ln(n+1)} \right| = \frac{1}{\ln 2} + \frac{1}{\ln 3} + \frac{1}{\ln 4}$$ which diverges by direct comparison with $\frac{1}{n}$ so

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$$ is conditionally convergent

$$\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n} = \frac{0!}{2^0} - \frac{1!}{2^1} + \frac{2!}{2^2} - \frac{3!}{2^3} + \cdots$$

n^{th} term test for Divergence tells us this $\boxed{\text{diverges}}$ since $\lim\limits_{n\to\infty} a_n \neq 0$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = -\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$$

Alternating Series test says this converges

$\boxed{\text{Conditionally convergent}}$

but $$\sum \left| \frac{(-1)^n}{\sqrt{n}} \right| = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} +$$ which diverges since its a p-Series with $p<1$. So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is conditionally conv.