Wednesday, April 13, 2011
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AP Calculus AB
Summer Packet

Summer Review Packet for Students Entering Calculus

Complex Fractions

When simplifying complex fractions, multiply by a fraction (equal to 1) which has the numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Simplifying Expressions

Make sure you are very comfortable manipulating exponents, positive/negative, fractional. Also, know the relationship between exponents and radicals; the appropriate radical will "undo" an exponent.

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \frac{1}{x^4}$

$\frac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$

$x^6 x^5 = x^{11}$

$\frac{x^3}{x^9} = x^{-6} = \frac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Simplify each expression. Write answers with positive exponents where applicable.

A. $\frac{1}{x+h} - \frac{1}{x}$

B. $\frac{\frac{2}{x^2}}{\frac{10}{x^3}}$

C. $\frac{12x^{-3}y^2}{18xy^{-1}}$

D. $\frac{15x^2}{5\sqrt{x}}$

E. $\left(5a^3\right)\left(4a^2\right)$

F. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

G. $\frac{\frac{1}{2} - \frac{5}{4}}{\frac{3}{8}}$

H. $\frac{5-x}{x^2-25}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________ 7. $g(-3) =$ ____________ 8. $f(t+1) =$ __________

9. $f[g(-2)] =$ __________ 10. $g[f(m+2)] =$ ___________ 11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ___________ 13. $f\left(\dfrac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h[f(-2)] =$ _______ 15. $f[g(x-1)] =$ _______ 16. $g\left[h(x^3)\right] =$ _______

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int}.\ (\textit{Let } y=0)$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1 \textit{ or } x=3$

$x-\text{intercepts } (-1,0) \textit{ and } (3,0)$

$y-\text{int}.\ (\textit{Let } x=0)$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$x^2 + y^2 - 16x + 39 = 0$

$x^2 - y^2 - 9 = 0$

<u>Elimination Method</u>

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

Find the point(s) of intersection of the graphs for the given equations in #23 and #24. For #25, simplify the expressions , writing answers with positive exponents where applicable.

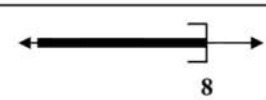
23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

<u>Interval Notation</u>

26. Complete the table with the appropriate notation or graph.

Solution	**Interval Notation**	**Graph**
$-2 < x \le 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9 = x-9+9 = x$$

$$g(f(x)) = \frac{(4x+9)-9}{4} = \frac{4x+9-9}{4} = \frac{4x}{4} = x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36. $f(x) = \dfrac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2,\ x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Using the Graphing Calculator

Spend some time getting comfortable with your graphing calculator, if you are not already. For calculus, you should be comfortable (1) graphing an arbitrary function, (2) finding zeros (or roots) of a function, (3) and finding the intersection between 2 functions. If you are not comfortable with these things, go to http://grovesite.com/vchs ->Mathematics->AP Calculus AB/BC->TI Calculator Review Elluminate Session and watch Mr. Nyberg's recorded session on some fundamental applications for your TI graphing calculator.

46. Given: $f(x) = x^4-3x^3+2x^2-7x-11$
    Find all roots to the nearest 0.001

    [You can solve for f(x)=0 by graphing f(x), then going to Calc menu and #2 zero function]

47. Given: $f(x) = 3\sin 2x - 4x + 1$ from $[-2\pi, 2\pi]$
    Find all roots to the nearest 0.001
    [NOTE: all trig functions are done in radian mode]

48. Given $f(x) = 0.7x^2 + 3.2x + 1.5$
    Find all roots to the nearest 0.001

49. Given: $f(x) = x^4 - 8x^2 + 5$
    Find all roots to the nearest 0.001

50. Given $f(x) = x^2 - 5x + 2$ and $g(x) = 3 - 2x$
    Find the coordinates of any points of intersection

Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

51. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

52. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

53. $\lim_{x \to 0} \cos x$

54. $\lim_{x \to 5} \frac{2}{x-5}$

55. $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

56. $\lim_{x \to 2} (4x^2+3)$

57. $\lim_{x \to 1} \frac{x^2+x+2}{x+1}$

58. $\lim_{x \to 0} \sqrt{x^2+4}$

59. $\lim_{x \to \pi} \cos x$

60. $\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)$ HINT: Factor and simplify.

61. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

62. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ HINT: Rationalize the numerator.

63. $\lim_{x\to 3}\frac{3-x}{x^2-9}$

64. $\lim_{h\to 0}\frac{2(x+h)-2x}{h}$

One-Sided Limits

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

65. $\lim_{x\to 5^+}\frac{x-5}{x^2-25}$

66. $\lim_{x\to -3^-}\frac{x}{\sqrt{x^2-9}}$

67. $\lim_{x\to 10^+}\frac{|x-10|}{x-10}$

68. $\lim_{x\to 5^-}\left(-\frac{3}{x+5}\right)$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

69. $f(x)=\frac{1}{x^2}$

70. $f(x)=\frac{x^2}{x^2-4}$

71. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree in the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

72. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

73. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

74. $f(x)=\frac{4x^5}{x^2-7}$

Determine each limit as x goes to infinity.

RECALL: This is the same process you used to find Horizontal Asymptotes for a rational function.

**** In a nutshell**

1. Find the highest power of x.
2. How many of that type of x do you have in the numerator?
3. How many of that type of x do you have in the denominator?
4. That ratio is your limit!

75. $\lim_{x\to\infty}\left(\frac{2x-5+4x^2}{3-5x+x^2}\right)$

76. $\lim_{x\to\infty}\left(\frac{2x-5}{3-5x+3x^2}\right)$

77. $\lim_{x\to\infty}\left(\frac{7x+6-2x^3}{3+14x+x^2}\right)$

Limits to Infinity

A rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

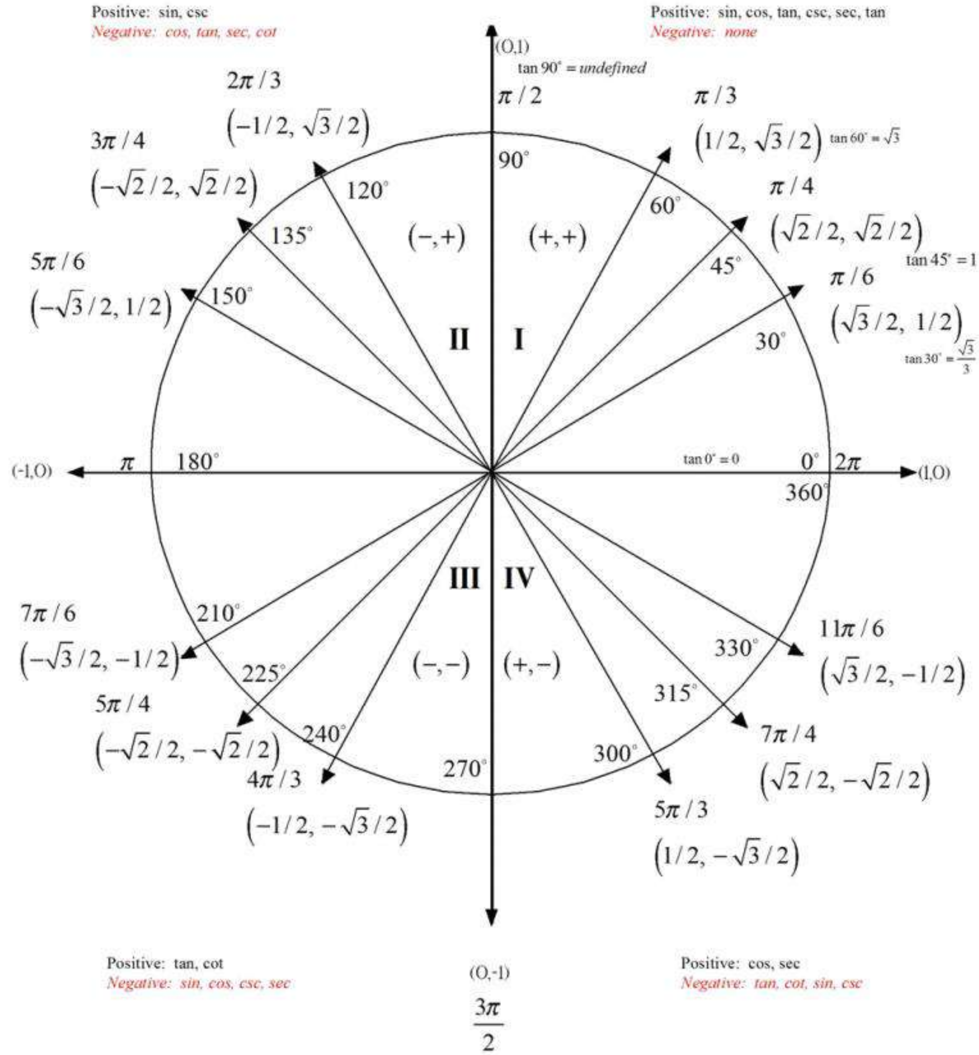
Determine each limit if it exists. If the limit approaches ∞ *or* $-\infty$, please state which one the limit approaches.

78. $\lim_{x \to -1^{+}} \frac{1}{x+1} =$

79. $\lim_{x \to 1^{+}} \frac{2+x}{1-x} =$

80. $\lim_{x \to 0} \frac{2}{\sin x} =$

Angles and Radians of a Unit Circle



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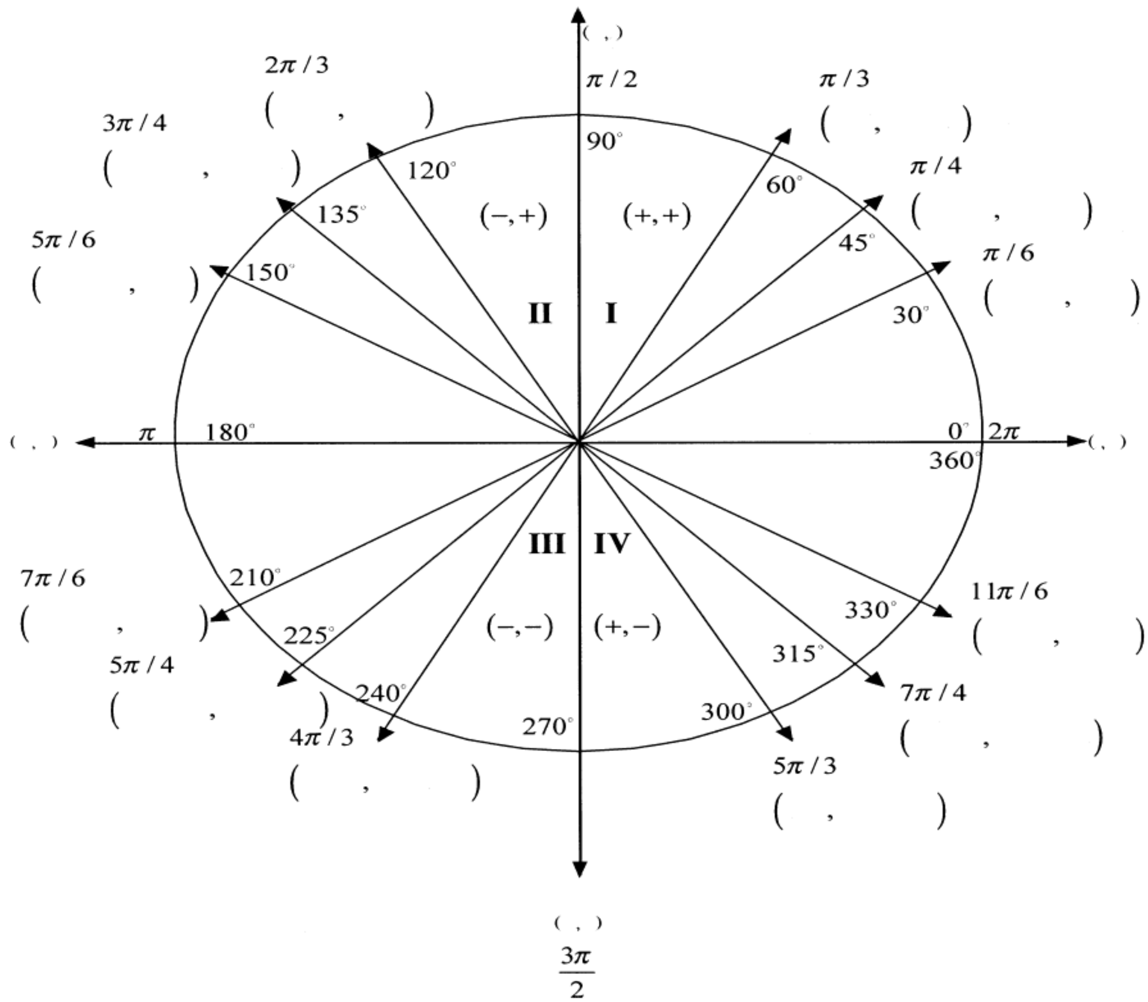
http://www.MathematicsHelpCentral.com

Unit Circle Worksheet



DIRECTIONS: Please fill in the (x, y) coordinates above for the Unit Circle. Then proceed by filling in the values of the appropriate sin, cos and tan functions below. You will also receive another copy of this unit circle with your summer letter, so you can practice repeatedly.

sin (0) = ____________	sin (π/6) = ______________
cos(0) = __________	cos (5π/6) = _____________
sin(3π/2) = ____________	tan (3π/2) = _____________
cos(π) = ____________	sin (π/4) = _____________
sin (5π/2) = ____________	cos (11π/6) = ____________
tan (0) = ____________	sin (π) = ________________

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$
$y = \ln x = \log_e x$ is equivalent to $x = e^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0

Thursday, April 14, 2011
7:45 AM

Any questions can be submitted to Mr. Zickert or Mrs. Letzel at: czickert@d211.org or kletzel@d211.org

AP Calculus AB Summer Homework – Answers

Pg. 2 Complex Fractions

1. $\frac{5-a}{a}$

2. $\frac{2x}{5(x+4)}$

3. $\frac{4x-12}{5x}$

4. $\frac{x^2-x-1}{x^2+x+1}$

5. $\frac{x-4}{3x^2-4x+32}$

Pg. 3 Simplifying Expressions

A. $\frac{-h}{x^2+xh}$ B. $\frac{x}{5}$ C. $\frac{2y^3}{3x^4}$ D. $3x^{3/2}$ E. $20a^5$

F. $8a^{5/2}$ G. -2 H. $\frac{-1}{x+5}$

Pg. 4 Functions

6. 5 7. 17 8. 2t + 3 9. 15 10. $8m^2+40m+49$

11. 2 12. 1 13. $\frac{\sqrt{3}}{2}$ 14. 15 15. $(2x+3)^2$

16. $2x^6+3$ 17. 9 18. -2

Pg. 5-6 Intercepts and Points of Intersection

19. (5/2, 0) and (0, -5)

20. (1,0) and (-2, 0) and (0,-2)

21. (0,0) and (-4, 0) and (4,0)

22. (0,0) and (-2, 0) and (2,0)

23. (3, 5)

24. (-1,5) and (2,2)

25. (6, -8) and (14, -8)

Pg. 6-7 Interval Notation

26. (-2, 4]	graph shows line segment from -2 to 4, with open circle on x=-2 and closed circle on x = 4
$-1 \le x < 7$	graph shows line segment from -1 to 7, with closed circle on -1 and open circle on 7
$-\infty < x \le 8$	$(-\infty, 8]$
27. [1/2, ∞)	graph shows closed circle on ½ with right arrow indicating to infinity
28. [-1/2, 7/2)	graph shows line segment from -1/2 to 7/2, open circle on 7/2
29. (30, ∞)	graph shows open circle on 30 with right arrow indicating to infinity

Pg. 7 Domain and Range

30. Domain: (-∞,∞) Range: [-5, ∞)
31. Domain: [-3, ∞) Range: (-∞, 0]
32. Domain: (-∞,∞) Range: [-3, 3]
33. Domain: (-∞, 1) and (1, ∞) Range: (-∞, 0) and (0, ∞)

Pg. 7-8 Inverses

34. $\frac{x-1}{2}$ 35. $\pm\sqrt{3x}$ 36 & 37. show that f(g(x)) = g(f(x)) = x

Pg. 9 Equation of a Line

38. $y = 3x + 5$

39. $x = 5$

40. $y = 2$

41. $(y-5) = (2/3)x$

42. $(y-8) = (5/6)(x-2)$

43. $y = 7$

44. $(y-2) = -(x-1)$ OR
$(y-6) = -(x+3)$

45. $y = (-3/2)x + 3$

Pg. 10 Using the Graphing Calculator

46. $x = -0.911, 3.329$

47. $x = 0.957$

48. $x = -4.041, -0.530$

49. $x = -2.705, -0.827, 0,827, 2.705$

50. $(-0.303, 3.606)$ and $(3.303, -3.606)$

51. Table of values
$F(x) = [\ 0.20408, 0.2004, 0.20004, 0.19996, 0.1996, 0.19608]$

Lim = 0.2

52. Table of values
$F(x) = [-.1662, -.1666, -.1667, -.1667, -1.667, -.1671]$

Lim = $-(1/6) \approx -.16666667$

53. lim = 1   54. DNE   55. 4

56. 19   57. 2   58. 2   59. -1

60. 2 61. -5 62. ½ 63. -1/6 64. 2

Pg. 12 One-Sided Limits

65. 1/10

66. -∞

67. 1

68. -0.3

Pg. 13 Vertical Asymptotes

69. x = 0

70. x = -2, 2

71. x = 0, 1

Pg. 13 Horizontal Asymptotes

72. y = 0

73. y = -5/3

74. no horizontal asymptotes

75. lim = 4

76. lim = 0

77. -∞

Pg. 14 Limits to Infinity

78. ∞

79. -∞

80. DNE