Course Title: Advanced Placement Calculus (AB) – Period 5

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**Textbook**: CALCULUS: Graphical, Numerical, Algebraic; Third Edition

Authored by: Finney, Demana, Waits, and Kennedy

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## **Course Objectives**

To expose students to a first semester college calculus course;

- To prepare students for advanced coursework in mathematics, science, technology, engineering, etc;
- To prepare students for the Advanced Placement Exam in May;
- To provide opportunities for students to integrate the use of modern technologies along with traditional methods of analysis in problem solving and discovery;
- To provide opportunities for students to work cooperatively in problem solving and discovery.

## Course Goals

- Students should be able to work with functions represented in various ways: graphically, numerically, analytically, and/or verbally. They should also understand the, and be able to make, connections among these representations;
- Students should understand the meaning of the <u>derivative</u> both as a <u>limit of a difference quotient</u>, and as a <u>rate of change</u>; and should be able to use derivatives to solve problems;
- Students should understand the meaning of the <u>definite integral</u> both as a <u>limit of Reimann sums</u>, and as a <u>net accumulation of change</u>; and should be able to use definite integrals to solve problems;
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the <u>Fundamental</u> <u>Theorem of Calculus</u>;

- Students should be able to communicate mathematics both orally and in well-written sentences, and should be able to explain solutions to problems;
- Students should be able to model a written description of a physical situation with a function, a <u>differential equation</u>, or a <u>definite integral</u>;
- Students should be able to use technology to help: solve problems, explore situations, experiment with ideas, investigate phenomena, interpret results, and verify conclusions;
- Students should be able to use approximation methods to estimate solutions, and understand how approximation techniques are used to develop exact methods;
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement;
- Students should develop an appreciation of calculus as a meaningful and coherent body of knowledge and as a human accomplishment.

## **Topical Outline**

## I. Functions, Graphs, and Limits

**Analysis of graphs**. With the aid of technology, graphs of functions are often easy to produce. Hence the emphasis is on the interplay between geometric and analytic information and on the use of calculus both to predict and explain the observed local and global behavior of a function.

**Limits of a function**. An intuitive understanding of the limit process is sufficient for this course.

- \* Calculating limits using algebra.
- \* Estimating limits from graphs or tables of data.

  (If time permits we will investigate the formal definition of limits)

### Asymptotic and unbounded behavior.

- \* Understanding asymptotes in terms of graphical behavior.
- \* Describing asymptotic behavior in terms of limits involving infinity.
- \* Comparing relative magnitudes of functions and their rates of change.

<u>Continuity</u> as a property of functions. The central idea of <u>continuity</u> is that close values of the domain lead to close values of the range.

- \* Understanding continuity in terms of limits.
- \* Geometric understanding of graphs of <u>continuous functions</u> (IVT and EVT).

## II. <u>Derivatives</u>

**Concept of a <u>derivative</u>**. The concept of the <u>derivative</u> is presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change.

- \* Derivative defined as a limit of the difference quotient.
- \* Relationship between differentiability and continuity.

## Derivative at a point.

- \* Slope of a curve at a point. Examples are emphasized, including points at which there are <u>vertical tangents</u> and points at which there are no <u>tangents</u>.
- \* Tangent line to a curve at a point and local linear approximation.
- \* Instantaneous rate of change as the limit of average rate of change.
- \* Approximate <u>rate of change</u> from graphs and tables of values.

#### Derivative of a function.

- \* Corresponding characteristics of graphs of **f** and **f**'.
- \* Relationship between the <u>increasing /decreasing behavior</u> of <u>f</u> and the sign of **f**'.
- \* The Mean Value Theorem (MVT) and its geometric consequences.
- \* Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

#### Second derivatives.

- \* Corresponding characteristics of the graphs of f, f', and f".
- \* Relationship between the concavity of f and the sign of f".
- \* Points of inflection as places where the concavity changes.

#### Applications of derivatives.

- \* Analysis of curves, including the notions of monotonicity and concavity.
- \* Optimization, both absolute (global) and relative (local) extrema.
- \* Modeling rates of change, including related rates problems.
- \* Use of <u>implicit differentiation</u> to determine the derivative of an inverse function.
- \* Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- \* Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

#### Computation of derivatives.

- \* Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- \* Basic rules for the derivative of sums, products, and quotients of

functions.

\* Chain rule and implicit differentiation.

## III. Integrals

## Reimann sums and properties of definite integrals.

- \* Concept of a Reimann sum over equal subdivisions.
- \* Computation of <u>Reimann sums</u> using left, right, and midpoint evaluations.
- \* Definite integral as a limit of Reimann sums.
- \* <u>Definite integral</u> of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: f'(x)dx = f(b) f(a).
- \* Basic properties of definite integrals.

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, social, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to yield accumulated change or using the method of setting up an approximating Reimann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross section, the average value of a function, and the distance traveled by a particle along a line.

#### **Fundamental Theorem of Calculus.**

- \* Use of the Fundamental Theorem to evaluate definite integrals.
- \* Use of the <u>Fundamental Theorem</u> to represent a particular <u>antiderivative</u>, and the analytical and graphical analysis of functions so defined.

## Techniques of antidifferentiation.

- \* Antiderivatives following directly from derivatives of basic functions.
- \* Antiderivatives by substitution of variables.

## Applications of antidifferentiation.

- \* Determining specific <u>antiderivatives</u> using <u>initial conditions</u>.
- \* Solving separable differential equations and using them in modeling.

**Numerical approximation to definite integrals.** Use of <u>Reimann sums</u> and the <u>Trapezoidal rule</u> to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

# **Homework and Grading Policy**

Homework is an integral part of any learning experience and it is necessary for students to complete the assignments on a regular basis in order to be successful. In addition to appropriate attention to homework, proper effort and attitude are essential for student success. AP Calculus is a college level course, and in an attempt to prepare students for a college level experience, quarter and final grades are based solely on the grades achieved on tests and quizzes. However, in order to acknowledge these aspects of a successful approach to calculus, the scale used reflects (and incorporates) credit for effort, attitude, and homework.

<u>AVERAGE</u>	<u>GRADE</u>
88 - 100 82 - 87 76 - 81 68 - 75 62 - 67 56 - 61 48 - 55 42 - 47 36 - 41 32 - 35	A A- B+ B C+ C C- D+
0 – 31	F