Welcome to A-Level Maths! This course will continue on from your studies in AS Maths and we will be preparing all year to take Papers 3 & 6 (Pure Mathematics 2 & 3 and Probability & Statistics 2) in May of 2020. This is a fast-paced, rigorous course, and I can say with certainty that you will be well prepared for advanced mathematics courses in college with successful completion of this course. With that being said, this summer assignment is meant to keep your math brain fresh and it contains problems from some of the major concepts of AS Maths with a focus on the skills and content you will need for A Maths. Completion of the summer assignment is mandatory and will be due on the first day of class. If you complete this packet early in the summer, take time to review your work in August so that the math is fresh in your mind. My suggestion is that your spread these problems out across the summer. You want to be able to remember and use these concepts and skills when school starts! If you are stuck, consult Khan Academy- there are many great tutorials out there. I am looking forward to a great year! Enjoy your summer and welcome again to A-Level Maths!

Instructor: Mrs. Julie Krohn   Email: julie1.rotthoff@cms.k12.nc.us

Statistics Review: Binomial and Normal Distributions

1. Expected value of a binomial random variable X is 10 and the probability of failure is 0.9. Calculate the standard deviation of X.

2. Sam has not prepared for the upcoming exam. Exam is completely based on multiple choice questions with 5 options for each question and only one correct answer. There are 30 questions in the exam and Sam attempted all questions betting on his luck.
    a. What is the probability that exactly 17 questions are correct?
    b. What is the probability that at least 2 questions are wrong?

3. It is known that 13% of all items produced by a machine are defective. If 7 items are selected at random,
    a. What is the probability that exactly 2 items are defective?
    b. What is the probability that at least one the items selected is defective?

4. Let X be a binomial random variable with probability of success as 0.3. If 60 trials are made,
    a. What is the expected number of successes?
    b. Calculate the standard deviation of number of successes.

5. It is known that 15% of all the items produced from a factory are defective.
    a. What is the expected number of items to be drawn to get a non-defective item?
    b. Calculate the expected number of items to be drawn to get a defective item?

6. An urn contains several marbles. It is known that 35% of all the marbles in the urn are blue. If a sample of 200 marbles are selected at random, what is the probability of getting at least 80 blue marbles?

7. A fair dice is rolled 5 times. What is the probability that a number greater than 4 occurs at least 3 times?

8. It is a known that 70% of the electric bulbs procured from a particular manufacturer lasts for at least 240 days before they fail. If a sample of 6 bulbs are drawn at random, find the probability that at least 5 bulbs fail before 240 days.

9. A fair dice is rolled 6 times, what is the probability that a number between 3 and 6 (including both) occurs at least 3 times and no more than 5 times?

10. Replacement times for CD players are normally distributed with a mean of 7.1 years and a standard deviation of 1.4 years (data from Consumer Reports). What is the probability that a randomly-selected CD player will have to be replaced in 8 years or less?

11. Replacement times for CD players are normally distributed with a mean of 7.1 years and a standard deviation of 1.4 years (data from Consumer Reports). If you are the manufacturer and want to provide a warranty such that 98% of the players need replacement after the warranty expires, how long should the warranty period be?

12. A mile-runner's times for the mile are normally distributed with a mean of 4 min. 3 sec. (This would have to be expressed in decimal minutes -- 4.05 minutes), and a standard deviation of 2 seconds (0.0333333··· minutes (the three dots indicate a repeating decimal)). What is the probability that on a given run, the time will be 4 minutes or less?

13. A mile-runner's times for the mile are normally distributed with a mean of 4 min. 3 sec. (This would have to be expressed in decimal minutes -- 4.05 minutes), and a standard deviation of 2 seconds (0.0333333··· minutes (the underline indicates a repeating decimal)). What does the mean have to be for a 0.20 probability of the time being 4 minutes or less?

14. A machine fills 24-ounce (according to the label) boxes with cereal. The amount deposited into the box is normally distributed with a standard deviation of 0.25 ounce. What does the mean have to be in order for 99.5% of the boxes to contain 24 ounces or more of cereal?

15. A machine fills 24-ounce (according to the label) boxes with cereal. The amount deposited into the box is normally distributed with a standard deviation of 0.20 ounce. What does the mean have to be in order for only 0.2% of the boxes to contain more than 24.5 ounces or more of cereal?

16. A student gets a 70 on a test where the mean score was 64. What does the standard deviation have to be in order for the student to be in the 95th percentile?

17. A machine fills 24-ounce (according to the label) boxes with cereal. The amount deposited into the box is normally distributed with a mean of 24.8 ounces. What does the standard deviation have to be in order for 96% of the boxes to contain 24 ounces or more?

18. On a standardized test, scores are normally distributed with a mean of 400 and a standard deviation of 80. What score must one have to be in the 80th percentile?

Pure Math Review: Solving Algebraic Equations (with a focus on exponential and logarithmic equations)

1. $3^{x-1} = 81$

2. $8^x = 4$

3. $e^x = 5$

4. $-14 + 3e^x = 11$

5. $-6 + \ln 3x = 0$

6. $\log(3x + 1) = 2$

7. $\ln x - \ln 3 = 4$

8. $2 \ln 3x = 4$

9. $5^{x+2} = 4$

10. $\ln(x + 2)^2 = 6$

11. $4^{-3x} = 0.25$

12. $2e^{2x} - 5e^x - 3 = 0$

13. $\log_7 3 + \log_7 x = \log_7 32$

14. $2 \log_6 4x = 0$

15. $\log_2 x + \log_2(x - 3) = 2$

16. $\log_2(x + 5) - \log_2(x - 2) = 3$

17. $4 \ln(2x + 3) = 11$

18. $\log x - \log 6 = 2 \log 4$

19. $2^x = 64$

20. $5^x = 25$

21. $4^{x-3} = \frac{1}{16}$

Pure Math Review: Derivatives & Antiderivatives

1. Find y': $y = (3x + 5)^4$

2. Find y': $y = (4x - 1)^3$

3. Find y': $y = (2x + 5)^4$

4. Find y': $y = \frac{4}{x^2+1}$

5. Find y': $y = \sqrt{x^2 + 6x - 5}$

6. Find the value of the derivative at the given point. $y = \sqrt{2x + 1}$ at (4, 3)

7. Find the equation of a line tangent to the given curve at the given point. $y = (x^2 + 1)^3$ at (-1, 0)

8. Find the antiderivative: $f(x) = 1 - 3x^2 - 6x$

9. Find the antiderivative: $f(x) = x - x^{2/3}$

10. Find the antiderivative: $f(x) = \sqrt[5]{x} + 1$

11. Find the indefinite integral: $\int (x^2 + \sqrt[3]{x})dx$

12. Evaluate: $\int_0^5 (3x^2 - x + 1)dx$

13. Evaluate: $\int_1^3 \frac{1}{x^2} dx$

14. Evaluate: $\int_2^6 \sqrt{x}dx$