Probability
A2RCC U10D6 Practice for Quest 10

Name Key

1) The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive

1) I, only
2) II, only
3) I and III
4) II and III

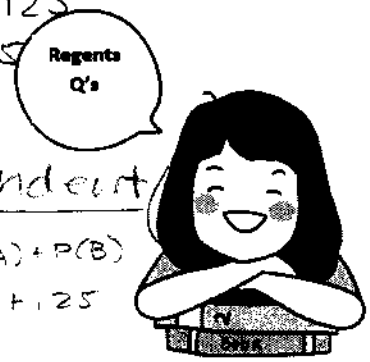
is $P(A) \cdot P(B) = P(A \cap B)$

$(.25)(.5) = .125$

$.125 = .125$

yes

independent



is $P(A \cup B) = P(A) + P(B)$

$.625 \neq .5 + .25$

$P(A \cup B) = .25 + .5 - .125$

$= .625$

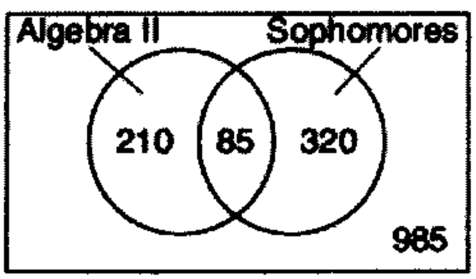
not mutually exclusive

2) On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

1) 73%
2) 36%
3) 23%
4) 12%

$(.48)(.25) = .12$

3) Data for the students enrolled in a local high school are shown in the Venn diagram below.



If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

1) $\frac{85}{210}$

2) $\frac{85}{295}$

3) $\frac{85}{405}$

4) $\frac{85}{1600}$

4) Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

1) independent
2) dependent
3) mutually exclusive
4) complements

$P(R \mid P) = .40$

$P(P) = .50$

$P(R) = .40$

independent

5) A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events D, that a customer uses the drive-thru, and F, that a customer orders French fries, are independent. The following data are given in a report:

$$P(F) = 0.8$$

$$P(F \cap D) = 0.456$$

Given this information, $P(F \mid D)$ is

1) 0.344
2) 0.3648
3) 0.57
4) 0.8

$P(F|D)=P(F)$

since independent:

$P(F) \cdot P(D) = P(F \cap D)$

$.8\, P(D) = .456$

$P(D) = .57$

$P(F \mid D) = \frac{.456}{.57} = .8$

6) Suppose events A and B are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

1) $P(A) = 0.4$, $P(B) = 0.3$, $P(A \text{ or } B) = 0.5$ $\quad (.4)(.3) \neq .2$
2) $P(A) = 0.8$, $P(B) = 0.25$
3) $P(A \mid B) = 0.2$, $P(B) = 0.2$ $\quad \frac{.02}{.2} \neq .2$
4) $P(A) = 0.15$, $P(B) = 0.05$

$(.15)(.05) \neq .2$

7) The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

[illegible]	[illegible]		
	0–10	11–50	Over 50
15–18	4	37	68
19–22	6	25	87
23–60	25	47	157

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If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1) $\frac{157}{229}$
2) $\frac{157}{312}$
3) $\frac{157}{384}$
4) $\frac{157}{456}$

8) The probability that a resident of a housing community opposes spending money for community improvement on plumbing issues is 0.8. The probability that a resident favors spending money on improving walkways given that the resident opposes spending money on plumbing issues is 0.85. Determine the probability that a randomly selected resident opposes spending money on plumbing issues and favors spending money on walkways.

P(opposed plumbing) = .8

P(favor walkways | opposed plumbing) = .85

$P(A \cap B) = ?$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$.85 = \frac{x}{.8}$

$P(A \cap B) = .68$

9) A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math? Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

and

$P(\text{math}) = \frac{1}{4}$

$P(\text{junior}) = \frac{116}{459}$

$P(\text{math} \cup \text{junior}) = \frac{47}{108}$

$\frac{47}{108} = \frac{1}{4} + \frac{116}{459} - x$

$\frac{47}{108} = \frac{923}{1836} - x$

$x = \frac{31}{459}$

not independent

$\frac{1}{4} \cdot \frac{116}{459} \overset{?}{=} \frac{31}{459}$

$\frac{29}{459} \neq \frac{31}{459}$

10) A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

Favorite Type of Program			
	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshmen	119	103	54

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A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$\frac{103}{213}$

11) Data collected about jogging from students with two older siblings are shown in the table below.

	Neither Sibling Jogs	One Sibling Jogs	Both Siblings Jog
Student Does Not Jog	1168	1823	1380
Student Jogs	188	416	400
		2239	1780

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

P(student | 1)

$\frac{416}{2239} \approx .19$

P(student | both)

$\frac{400}{1780} \approx .22$

12) The results of a poll of 200 students are shown in the table below:

	Preferred Music Style			
	Techno	Rap	Country	
Female	54	25	27	106
Male	36	40	18	94
		65		200

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

$P(F) \cdot P(R) \stackrel{?}{=} P(F \cap R)$

$\frac{106}{200} \cdot \frac{65}{200} \stackrel{?}{=} \frac{25}{200}$

$.17 \neq .125$

not independent

13) The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

$P(m) \cdot P(RS) \stackrel{?}{=} P(M \cap RS)$

$\frac{230}{490} \cdot \frac{180}{490} \stackrel{?}{=} \frac{70}{490}$

not independent

$.17 \neq .14$

✓ 14) A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

$P(D) = .5 \quad P(SP) = .5 \quad P(D \cap W) = .40$

$$P(W|D) = \frac{.40}{.5} = .8$$

✓ 15) The guidance department has reported that of the senior class, 2.3% are members of key club, *K*, 8.6% are enrolled in AP Physics, *P*, and 1.9% are in both. Determine the probability of *P* given *K*, to the *nearest tenth of a percent*. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

$P(K) = .023 \quad P(P) = .086 \quad P(P \cap K) = .019$

$$P(P|K) = \frac{.019}{.023} = .82608\ldots$$

82.6%

✓ 16) A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$P(S) = \frac{649}{1376} \quad P(M) = \frac{433}{1376} \quad P(S \cup M) = \frac{974}{1376}$

$P(S \cup M) = P(S) + P(M) - P(M \cap S)$

$$\frac{974}{1376} = \frac{649}{1376} + \frac{433}{1376} - x$$

$$x = \frac{108}{1376}$$

✓ 17) **Given events *A* and *B*, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether *A* and *B* are independent or dependent.**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.8 = .6 + .5 - x$

$x = .3$

$P(A) \cdot P(B) = P(A \cap B)$

$(.6)(.5) \stackrel{?}{=} .3$ independent

✓ 18) In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on *both* issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

$P(\text{salaries}) = .5$

$P(\text{ins}) = .7$

$P(\text{salaries} \cup \text{ins})^c = .2$

$P(\text{salaries} \cup \text{ins}) = .8$

$P(\text{salaries} \cup \text{ins}) = P(\text{salaries}) + P(\text{ins}) - P(\text{both})$

$.8 = .5 + .7 - x$

$x = .4$

$(.5)(.7) \neq .4$

independent