

9.5a Factor Quadratic Equations When the Leading Coefficient "a=1"

VOCABULARY:

- Standard Form of a Quadratic Equation $Ax^2 + Bx + C = 0$;

Where a, b, c are real numbers ; and $A \neq 0$

- Factoring a quadratic trinomials when $a=1$ into a product of 2 binomial factors

Algebra: $x^2 + bx + c = (x + \#)(x + \#)$

When $A=1$, the question to ask yourself ... What 2 numbers add to "b" and their product is "c"?

Example: $x^2 + 5x + 6 = (x+2)(x+3)$ because $2+3=5$ and $2 \cdot 3=6$.

o Note: order of factors does not matter $(x+2)(x+3) \Leftrightarrow (x+3)(x+2)$ are both equal.

Example 1 Factor when b and c are positive

Steps to Factor : $x^2 + 10x + 16 = (x+2)(x+8)$ or $(x+8)(x+2)$

$\begin{array}{r} 1 \ 16 \\ 2 \ 8 \\ 4 \ 4 \end{array}$

1) Identify a, b, and c. $a=1$ $b=10$ and $c=16$

2) Write 2 sets of ()'s. One for each factor.

3) The first term in both factors is "x". Why? $x \cdot x = x^2 \leftarrow$ that is the 1st term

4) What must the **signs** have to be for each factor? **both signs are + b/c b+c are positive**

5) What are the factors of 16? Put them under the 16.

6) Find the 2 factors $2 + 8 = 10$ AND $2 \cdot 8 = 16$

Mentally

7) **CHECK** by Multiplying the factors $(x+2)(x+8) = x^2 + 8x + 2x + 16 = x^2 + 10x + 16 \checkmark$

CHECK POINT: Factor and Check by mentally multiplying

<p>2) $x^2 + 9x + 8$ $(x+1)(x+8)$</p> <p>$\begin{array}{r} 1 \ 8 \\ 2 \ 4 \end{array}$</p> <p>$1+8=9$ $1 \cdot 8=8$ check</p>	<p>3) $x^2 + 12x + 20$ $(x+2)(x+10)$</p> <p>$\begin{array}{r} 1 \ 20 \\ 2 \ 10 \\ 4 \ 5 \end{array}$</p>
<p>4) $x^2 + 9x + 18$ $(x+3)(x+6)$</p> <p>$\begin{array}{r} 1 \ 18 \\ 2 \ 9 \\ 3 \ 6 \end{array}$</p>	<p>5) $x^2 + 13x + 40$ $(x+5)(x+8)$</p> <p>$\begin{array}{r} 1 \ 40 \\ 2 \ 20 \\ 4 \ 10 \\ 5 \ 8 \end{array}$</p>

Example 6 Factor when b is negative and c is positive

Steps to Factor : $x^2 - 5x + 6 = \boxed{(x-2)(x-3)}$ or $\boxed{(x-3)(x-2)}$

$\begin{array}{r} 1 \ 6 \\ 2 \ 3 \end{array}$

- 1) Identify a , b , and c . $a = 1$ $b = -5$ and $c = 6$
- 2) What must the **signs** have to be for each factor? Both Negative b/c $-B$ and $+C$
- 3) What are the factors of 6? Put them under the 6.
- 4) Find the 2 factors $-2 + -3 = -5$ ✓ AND $-2 \cdot -3 = 6$ ✓
- 5) **CHECK** by Multiplying the factors

CHECK POINT: Factor and Check by mentally multiplying

<p>7) $x^2 - 10x + 21 = \boxed{(x-3)(x-7)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 21 \\ 3 \ 7 \end{array}$ <i>remember mentally mult. to check!</i> </p>	<p>8) $x^2 - 10x + 16 = \boxed{(x-2)(x-8)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 16 \\ 2 \ 8 \\ 4 \ 4 \end{array}$ </p>
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Example 9 Factor when c is negative

Steps to Factor : $x^2 + 3x - 10 = \boxed{(x+5)(x-2)}$ or $\boxed{(x-2)(x+5)}$

$\begin{array}{r} 1 \ 10 \\ 2 \ 5 \end{array}$

- 1) Identify a , b , and c . $a = 1$ $b = 3$ and $c = -10$
- 2) What must the **signs** have to be for each factor? OPPOSITE SIGNS(+,-) b/c C is Negative
- 3) What are the factors of 10? Put them under the 10.
- 4) Find the 2 factors $-2 + 5 = 3$ ✓ AND $-2 \cdot 5 = -10$ ✓
- 5) **CHECK** by Multiplying the factors

CHECK POINT: Factor and Check by mentally multiplying

<p>10) $x^2 - 5x - 50 = \boxed{(x+5)(x-10)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 50 \\ 2 \ 25 \\ 5 \ 10 \end{array}$ $-10+5 = -5$ $-10 \cdot 5 = -50$ </p>	<p>10) $x^2 + 2x - 24 = \boxed{(x+6)(x-4)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 24 \\ 2 \ 12 \\ 3 \ 8 \\ 4 \ 6 \end{array}$ </p>
<p>11) $x^2 + 4x - 21 = \boxed{(x+7)(x-3)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 21 \\ 3 \ 7 \end{array}$ </p>	<p>12) $x^2 - 4x - 32 = \boxed{(x+4)(x-8)}$</p> <p style="margin-left: 100px;"> $\begin{array}{r} 1 \ 32 \\ 2 \ 16 \\ 4 \ 8 \end{array}$ </p>

9.5b Solve Quadratic Equations by Factor**Example 1 Steps to Solve Quadratic Equations by Factor:**

$x^2 + 7x = 18$ $\begin{array}{r} -18 \quad -18 \\ \hline x^2 + 7x - 18 = 0 \end{array}$	1) Put in standard form $Ax^2 + Bx + C = 0$
$(x + 9)(x - 2) = 0$ $\begin{array}{r} x + 9 = 0 \\ -9 \quad -9 \\ \hline x = -9 \end{array}$ $\begin{array}{r} x - 2 = 0 \\ +2 \quad +2 \\ \hline x = 2 \end{array}$	2) Factor
$C: (-9)^2 + 7(-9) = 18$ $81 - 63 = 18$ $18 = 18 \checkmark$ $C: 2^2 + 7(2) = 18$ $4 + 14 = 18$ $18 = 18 \checkmark$	3) Set each factor to "0" and solve Notice: there are 2 solutions
	4) Check each solution in the original equation

CHECK POINT: Solve by Factoring and Check

$$2) \quad x^2 + x = 12$$

$$\begin{array}{r} -12 \quad -12 \\ \hline x^2 + x - 12 = 0 \\ \underline{12} \\ 12 \\ \underline{3} \end{array}$$

$$(x+4)(x-3) = 0$$

$$x+4 = 0$$

$$x = -4$$

$$x-3 = 0$$

$$x = 3$$

$$C: (-4)^2 + (-4) = 12$$

$$16 - 4 = 12$$

$$12 = 12$$

$$C: 3^2 + 3 = 12$$

$$12 = 12 \checkmark$$

$$3) \quad x^2 - 14x = -40$$

$$\begin{array}{r} +40 \quad +40 \\ \hline x^2 - 14x + 40 = 0 \\ \underline{1} \\ 2 \\ \underline{4} \\ 5 \end{array}$$

$$(x-4)(x-10) = 0$$

$$x-4 = 0$$

$$x = 4$$

$$C: 4^2 - 14(4) = -40$$

$$-40 = -40$$

$$x-10 = 0$$

$$x = 10$$

$$C: 10^2 - 14(10) = -40$$

$$100 - 140 = -40$$

$$-40 = -40 \checkmark$$

$$4) \quad x^2 + 12x = -36$$

$$\begin{array}{r} +36 \quad +36 \\ \hline x^2 + 12x + 36 = 0 \\ \underline{1} \\ 2 \\ 3 \\ 4 \\ 6 \end{array}$$

$$5) \quad x^2 + 3x + 10 = 10$$

$$\begin{array}{r} -10 \quad -10 \\ \hline x^2 + 3x = 0 \end{array}$$

$$x(x+3) = 0$$

$$x = 0$$

$$C: 10 = 10 \checkmark$$

$$x+3 = 0$$

$$x = -3$$

$$C: (-3)^2 + 3(-3) + 10 = 10$$

$$9 - 9 + 10 = 10$$

$$10 = 10 \checkmark$$

$$6) \quad x^2 - 80 = 20$$

$$\begin{array}{r} -20 \quad -20 \\ \hline x^2 - 100 = 0 \end{array}$$

$$x^2 - 100 = 0$$

$$\begin{array}{r} \downarrow \\ 1 \\ 2 \\ 4 \\ 5 \\ 10 \end{array}$$

$$\left(\begin{array}{c} 10 \\ 10 \end{array} \right) \left(\begin{array}{c} 10 \\ 10 \end{array} \right) \leftarrow \text{Think}$$

$$- + - = b = 0$$

$$- \cdot - = c = -100$$

(CHALLENGE PROBLEM ☺)

What are a, b, c?

$$a = 1 \quad b = 0 \quad c = -100$$

* So what are signs of factors?

$$(x + \quad)(x - \quad) = 0$$

$$(x+10)(x-10) = 0$$

$$x+10 = 0$$

$$x = -10$$

$$C: 100 - 80 = 20$$

$$20 = 20 \checkmark$$

$$x-10 = 0$$

$$x = 10$$

$$C: 100 - 80 = 20$$

$$20 = 20 \checkmark$$