NAME:

SECTION 9.1 Exercises

Tip: Start sketching graph of Hypothesis H_0

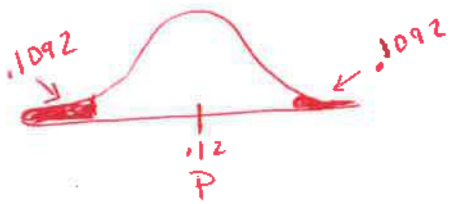
In Exercises 1 to 6, each situation calls for a significance test. State the appropriate null hypothesis H_0 and alternative hypothesis H_a in each case. Be sure to define your parameter each time.

1. **Lefties** Simon reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if 12% of the students at his large public high school are left-handed. Simon chooses an SRS of 100 students and records whether each student is right- or left-handed.

p = proportion of lefties in his school

$H_0: p = .12$

$H_A: p \neq .12$ (the proportion of lefties is NOT 12%)

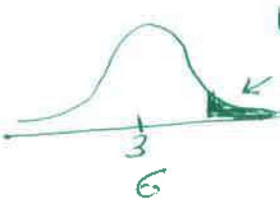


5. **Cold cabin?** During the winter months, the temperatures at the Colorado cabin owned by the Starnes family can stay well below freezing (32°F or 0°C) for weeks at a time. To prevent the pipes from freezing, Mrs. Starnes sets the thermostat at 50°F. The manufacturer claims that the thermostat allows variation in home temperature of $\sigma = 3$°F. Mrs. Starnes suspects that the manufacturer is overstating how well the thermostat works.

σ = standard deviation of the temperature in the cabin.

$H_0: \sigma = 3$

$H_A: \sigma > 3$ (manufacturer is overstating how well the thermostat works)



WHEN DEFINING H_0 + H_A ALWAYS:
① use population parameters (p, μ)
② define population parameter.

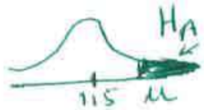
competition, the organizers estimate that the variation in distance flown by the athletes will be $\sigma = 10$ meters. An experienced jumper thinks that the organizers are underestimating the variation.

3. **Attitudes** The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures students' attitudes toward school and study habits. Scores range from 0 to 200. The mean score for U.S. college students is about 115. A teacher suspects that older students have better attitudes toward school. She gives the SSHA to an SRS of 45 of the over 1000 students at her college who are at least 30 years of age.

μ = mean attitude score on the SSHA for students at least 30 at the college

$H_0: \mu = 115$

$H_A: \mu > 115$ (older students have better attitudes)



11. **Lefties** Refer to Exercise 1. In Simon's SRS, 16 of the students were left-handed. A significance test yields a P-value of 0.2184. /2 = .1092

(a) Interpret this result in context.

(b) Do the data provide convincing evidence against the null hypothesis? Explain.

ⓐ IF THE PROPORTION OF LEFTIES AT SIMON'S SCHOOL IS REALLY .12 THERE IS A 21.84% CHANCE OF FINDING A SAMPLE OF 100 STUDENTS WITH A VALUE OF $\hat{p}$ THAT IS AS FAR FROM .12 AS THE SAMPLE VALUE IN EITHER DIRECTION.

ⓑ THE HIGH P-VALUE (21.84%) DOES NOT PROVIDE CONVINCING EVIDENCE. SOMETHING THAT HAPPENS OVER 20% OF THE TIME JUST BY CHANCE WHEN H_0 IS TRUE IS NOT STRONG EVIDENCE AGAINST H_0 (WE WOULD FAIL TO REJECT H_0).

In Exercises 7 to 10, explain what's wrong with the stated hypotheses. Then give correct hypotheses.

7. **Better parking** A change is made that should improve student satisfaction with the parking situation at a local high school. Right now, 37% of students approve of the parking that's provided. The null hypothesis $H_0: p > 0.37$ is tested against the alternative $H_a: p = 0.37$.

THE ALTERNATE HYPOTHESIS GIVES THE CURRENT SITUATION THAN WHAT WE ARE LOOKING FOR EVIDENCE FOR

CORRECTION:

$H_0: p = .37$

$H_A: p > .37$

9. **Birth weights** In planning a study of the birth weights of babies whose mothers did not see a doctor before delivery, a researcher states the hypotheses as

$$H_0: \bar{x} = 1000 \text{ grams}$$

$$H_a: \bar{x} < 1000 \text{ grams}$$

THE HYPOTHESES ARE ABOUT THE SAMPLE STATISTIC ($\bar{x}$) YOU ALWAYS USE population parameters.

CORRECTION:

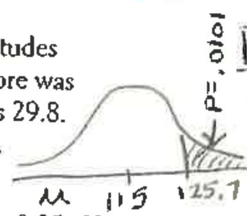
$H_0: \mu = 1000$ grams

$H_A: \mu < 1000$ grams

13. **Attitudes** In the study of older students' attitudes from Exercise 3, the sample mean SSHA score was 125.7 and the sample standard deviation was 29.8. A significance test yields a P-value of 0.0101.

(a) Interpret the P-value in context.

(b) What conclusion would you make if $\alpha = 0.05$? If $\alpha = 0.01$? Justify your answer.



(a) If the mean score on the SSHA for older students at this school is really 115, there is a 1.01% chance of finding a sample of 45 older students with a mean score of at least 125.7.

(b) If $\alpha = .05 >$ p-value $= .0101$ THEN REJECT THE NULL HYPOTHESIS H_0.

IF $\alpha = .01 <$ p-value $= .0101$ THEN FAIL TO REJECT THE NULL HYPOTHESIS H_0

15. **Is this what P means?** When asked to explain the meaning of the P-value in Exercise 13, a student says, "This means there is only probability 0.01 that the null hypothesis is true." Explain clearly why the student's explanation is wrong.

EITHER H_0 is TRUE (PROBABILITY THAT H_0 is TRUE is 1) OR H_0 is FALSE (PROBABILITY THAT H_0 is TRUE is 0)

A P-VALUE of 0.01 means that if H_0 is true, then the chance of observing a test statistic with the value we obtained or with a value that is more extreme is 1%.

	Ho TRUE	Ho FALSE
REJECT	TYPE I	CORRECT
FAIL TO REJECT	CORRECT	TYPE II

SECTION 9.1 (cont.) Exercises

Exercises 19 refer to the following setting. Slow response times by paramedics, firefighters, and policemen can have serious consequences for accident victims. In the case of life-threatening injuries, victims generally need medical attention within 8 minutes of the accident. Several cities have begun to monitor emergency response times. In one such city, the mean response time to all accidents involving life-threatening injuries last year was $\mu = 6.7$ minutes. Emergency personnel arrived within 8 minutes after 78% of all calls involving life-threatening injuries last year. The city manager shares this information and encourages these first responders to "do better." At the end of the year, the city manager selects an SRS of 400 calls involving life-threatening injuries and examines the response times.

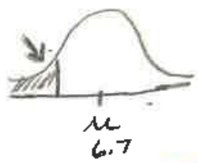
19. Awful accidents

(a) State hypotheses for a significance test to determine whether the average response time has decreased. Be sure to define the parameter of interest.

(b) Describe a Type I error and a Type II error in this setting, and explain the consequences of each.

(c) Which is more serious in this setting: a Type I error or a Type II error? Justify your answer.

19A μ = mean response time for all accidents involving life threatening injuries in the city



$H_0: \mu = 6.7$ min

$H_A: \mu < 6.7$ min want to do better

19B TYPE I ERROR: false positive α

The city council concludes the response time has improved when it has not.

TYPE II ERROR: false negative β

The city council concludes that the response time has not improved when it really has.

19C TYPE I ERROR would be worse. The city may stop trying to improve its response times because they think they have met the goal when in fact they have not. MORE PEOPLE COULD DIE.

21. Opening a restaurant You are thinking about opening a restaurant and are searching for a good location. From research you have done, you know that the mean income of those living near the restaurant must be over \$85,000 to support the type of upscale restaurant you wish to open. You decide to take a simple random sample of 50 people living near one potential location. Based on the mean income of this sample, you will decide whether to open a restaurant there.[8]

(a) State appropriate null and alternative hypotheses. Be sure to define your parameter.

(b) Describe a Type I and a Type II error, and explain the consequences of each.

(c) If you had to choose one of the "standard" significance levels for your significance test, would you choose $\alpha = 0.01$, 0.05, or 0.10? Justify your choice.

(a) μ = THE MEAN INCOME OF RESIDENTS NEAR THE RESTAURANT.

$H_0: \mu = \$85,000$

$H_A: \mu > \$85,000$

(b) TYPE I ERROR: OPEN THE RESTAURANT IN A LOCATION WHERE THE RESIDENTS WILL NOT BE ABLE TO SUPPORT IT.

TYPE II ERROR: DO NOT OPEN A RESTAURANT IN A LOCATION WHERE THE RESIDENTS COULD IN FACT SUPPORT IT FINANCIALLY.

(c) A TYPE I ERROR WOULD BE WORSE IN SELECTING A LOCATION TO OPEN THE RESTAURANT. SO IT WOULD BE BETTER TO CHOSE $\alpha = .01$ TO MINIMIZE THE RISK OF A TYPE I ERROR

23. Error probabilities You read that a statistical test at significance level $\alpha = 0.05$ has power 0.78. What are the probabilities of Type I and Type II errors for this test?

$\alpha = .05$

POWER $= .78$

POWER $= 1 - \beta$

$.78 = 1 - \beta$

$\rightarrow \beta = .22$

The P(TYPE I ERROR) $= \alpha = .05$

The P(TYPE II ERROR) $= \beta = .22$ $(1 - .78)$

9.1CQ

μ = the mean nicotine content of their cigarettes.

3. A certain cigarette brand advertises that the mean nicotine content of their cigarettes is 1.5 mg, but you are suspicious and plan to investigate the advertised claim by testing the hypotheses $H_0 : \mu = 1.5$ versus $H_a : \mu > 1.5$ at the $\alpha = 0.05$ significance level. You will do so by measuring the nicotine content of 30 randomly selected cigarettes of this brand.

$H_0 : \mu = 1.5$
$H_A : \mu > 1.5$

(a) Describe what a Type I error would be in this context. false positive (α)

Conclude that the mean nicotine content per cigarette is greater than 1.5 mg when it is EQUAL TO (OR LESS THAN) 1.5 mg.

(b) Describe what a Type II error would be in this context. false negative (β)

NOT Conclude that the mean nicotine level is greater than 1.5 mg per Cigarette when it is.

(c) From the perspective of public health, which error—Type I or Type II—is more serious? Explain.

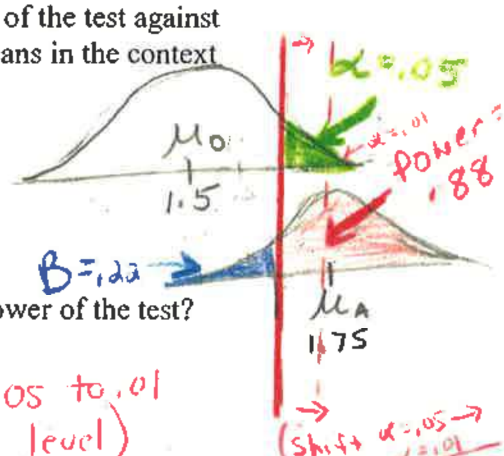
A TYPE II ERROR WOULD MEAN THAT YOU FAIL TO DISCOVER THAT THE CIGARETTES HAVE A HIGHER NICOTINE CONTENT THAT THE COMPANY CLAIMS, WHICH MEANS PEOPLE WILL BE EXPOSED TO MORE NICOTINE THAT THEY EXPECT AND THIS WOULD BE A PUBLIC HEALTH ISSUE! A TYPE I ERROR MIGHT BRING UNWARRANTED NEGATIVE PUBLICITY TO THE TOBACCO COMPANY BUT NOT A HEALTH ISSUE.

(d) Explain why it might be a good idea to increase the significance level to 0.10 for this test.

YOU WANT TO MINIMIZE THE CHANCE OF MAKING A TYPE II ERROR (NOT FINDING THAT THE NICOTINE LEVEL IS HIGHER THAN 1.5 WHEN IT IS), SO IT WOULD BE A GOOD IDEA TO USE A HIGHER SIGNIFICANCE LEVEL (α) WHICH WILL INCREASE THE POWER OF THE TEST.

(e) You have determined that at the $\alpha = 0.05$ significance level, the power of the test against the alternative $\mu = 1.75$ is 0.88. Explain what the power of the test means in the context of the problem.

Power = .88 measures the probability of rejecting the null hypothesis and concluding that the true mean nicotine level is above 1.5 when IT IS IN FACT 1.75 mg.



(f) What impact will reducing the significance level to 0.01 have on the power of the test?

Reducing α from .05 to .01 (the significance level) will increase the probability of a Type II error, so it reduces the power.

You can see this relationship by shifting the red line to the right

12. "Red tide" is a bloom of poison-producing algae—a few different species of a class of plankton called dinoflagellates. When weather and water condition cause these blooms, shellfish such as clams living in the area develop dangerous levels of a paralysis-inducing toxin. In Massachusetts, the Division of Marine Fisheries (DMF) monitors levels of the toxin in shellfish by regular sampling of shellfish along the coastline. If the mean level of toxin in clams exceeds 800μg (micrograms) of toxin per kg of clam meat in any area at a 5% level of significance, clam harvesting is banned there until the bloom is over and levels of toxin in clams subside. During a bloom, the distribution of toxin levels in clams on a single mudflat is distinctly non-Normal.

9.1C Quiz

(a) Define the parameter of interest and state appropriate hypotheses for the DMF to test.

μ = mean Concentration of Red Tide toxins in clams ($\mu g/kg$)

$H_0: \mu = 800\ \mu g/kg$
$H_A: \mu > 800\ \mu g/kg$

(b) Because of budget constraints and the large number of coastal areas that must be tested, the DMF would like to sample no more than 10 clams from any single area. Explain why this sample size may lead to problems in carrying out the significance test from (a).

The sample size of 10 clams is too small for a population that is known (given in the problem) to be "distinctly non-Normal."

(c) Describe a Type I and a Type II error in this situation and the consequences of each.

TYPE I ERROR: Concluding that the mean level of toxin is above 800 μg/kg when it is normal. CONSEQUENCE: THE DMF would close the area to clam harvesting which would have a negative economic impact on anyone who depends on the clam business, even though the clams are safe to eat.

TYPE II ERROR: NOT CONCLUDING THAT THE MEAN LEVEL OF TOXINS IS ABOVE SAFE LEVELS WHEN IT IS. CONSEQUENCE: THIS COULD CAUSE ANYONE WHO EATS CLAMS FROM THIS AREA TO BECOME SICK OR EVEN DIE.

(d) The DMF is considering changing the significance level of the test to 10%. Discuss the impact this might have on error probabilities and the power of the test, and describe the practical consequences of this change.

RAISING THE SIGNIFICANCE LEVEL TO 10% WOULD INCREASE THE PROBABILITY OF A TYPE I ERROR, BUT DECREASE THE PROBABILITY OF A TYPE II ERROR AND INCREASE THE POWER OF THE TEST. THIS WOULD DECREASE THE LIKELIHOOD OF PEOPLE EATING TOXIC CLAMS, SO IT MIGHT BE A GOOD IDEA. BETTER SAFE THAN SORRY.