AP Statistics – 9.1C	Name: KEY
Goal: Understanding Power and review Type I and Type II Errors	Date:

I. **Review:**

a) **Fill in the table -** Type I error, Type II error, Power, α, β

Truth about the population

		H_0 true	H_0 false (aka → H_a *is* true)
Sample Results	Reject H_0	TYPE I ERROR (α)	CORRECT DECISION (Power $= 1-\beta$)
	Fail to reject H_0	CORRECT DECISION	TYPE II ERROR (β)

b) **Define (using your own words)**

- Type I error (α)

REJECT H_0, WHEN H_0 IS ACTUALLY TRUE

FALSE POSITIVE

NOTE: Researchers can control α AND MUST SET α BEFORE STUDY. Typically $\alpha = .05$.

- Type II error (β)

FAIL TO REJECT H_0, WHEN H_A IS ACTUALLY TRUE

FALSE NEGATIVE → OR (H_0 False)

NOTE: Researchers cannot set β or power.

- Power(1-β)

MAKING A CORRECT DECISION. REJECT H_0, WHEN H_A IS TRUE.

IMPORTANT: AT END OF ACTIVITY KNOW HOW TO IMPROVE β + POWER

II. **Example** "***Faster fast food?***" The manager of a fast-food restaurant want to reduce the proportion of drive-through customers who have to wait more than 2 minutes to receive their food once their order is placed. Based on store records, the proportion of customers who had to wait at least 2 minutes was 63%. To reduce this wait time proportion, the manager assigns an additional employee to assist with drive-through orders. During the next month the manager will collect a random sample of drive-through times.

Problem 1: State appropriate null and alternative hypotheses in symbols and words. Be sure to define your parameters.

P = the true proportion of customers who have to wait more than 2 minutes for their orders

$H_0: p = .63$ [wait time of 2^+ minutes is 63%]
$H_A: p < .63$ [wait time of 2^+ minutes is less than 63%]

Problem 2: Describe a Type I and a Type II error in this setting and explain the consequer es of each.

- **Type I error (α)** (REJECT H_0 WHEN H_0 IS TRUE ($p = .63$)
  CONTEXT - THE MANAGER BELIEVES THE WAIT TIME IS LESS THAN 63%, WHEN IN FACT IT HAS NOT CHANGED.

* **Consequence** The manager keeps the new employee because he thinks it is improving the wait time. But the new employee is NOT Needed and they are wasting money.

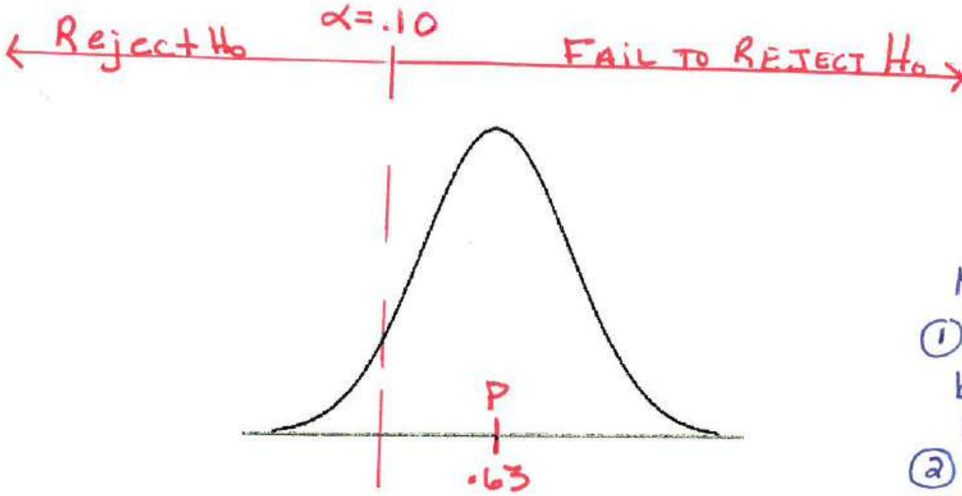
- **Type II error (β)** (FAIL TO REJECT H_0 WHEN H_A IS TRUE ($P < .63$)
  CONTEXT - THE MANAGER BELIEVES THE WAIT TIME HAS NOT CHANGED (63%), WHEN IN FACT, THE WAIT TIME WAS REDUCED (less than 63%)

* **Consequence** The manager does NOT think he needs the new employee. The drive through would be under staffed and customers would be unhappy with the wait time. The restaurant would lose customers business.

* CONSEQUENCES SHOULD FOCUS ON ECONOMICS ($'s)

Problem 3: Suppose that the manager decided to carry out this test using a random sample of 250 orders and a significance level of $\alpha = 0.10$.

- Make a graph labeling the population parameter, α, the rejection region and fail to reject region.
- What is the probability of a making a Type I error? P(TYPE I) = α = .10

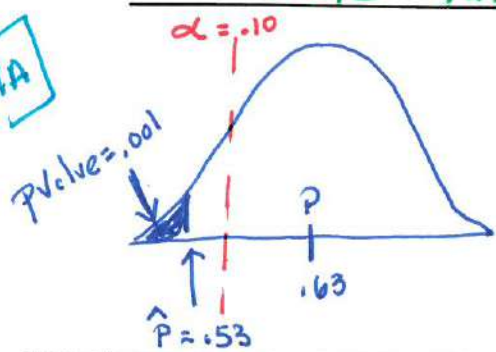


NOTES:
① We set α, before doing research
② α = SIGNIFICANCE LEVEL (willingness to make a Type I error)

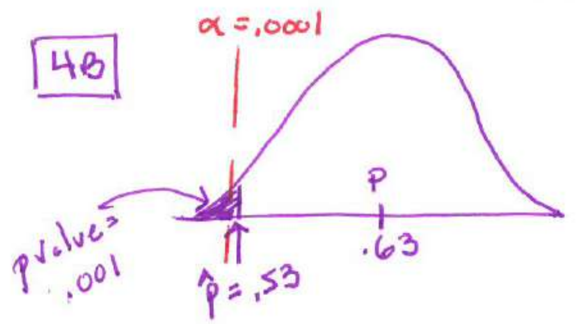
Problem 4: From the random sample of 250 orders, the manager found 134 customers (about 53%) waited more than 2 minutes to receive their food once their order was placed. Based on this sample, the resulting P-value was .001.

- Add the P-value and sample statistic to the above graph.
- 4A (Graph below) Based on this information what conclusion would you make? Since the pvalue (.001), is less than α = .10, we reject Ho. We have sufficient evidence to conclude the wait time is less than 63%
- 4B (Graph below) Would you make the same decision if α=.0001? NO Clearly explain. Since the pvalue (.001), is greater than α = .0001, we fail to reject Ho. We do NOT have sufficient evidence to conclude the wait time has been reduced.

4A



4B



General rule: * small pvalues → REJECT Ho
* large pvalue → FAIL TO REJECT Ho

Power Demonstration: *How would the following changes affect the power of the test?*

Example "***Faster fast food?***" $H_0 : p = 0.63$ versus $H_a : p < 0.63$

He found from the random sample of 250 orders, that 53% of customers waited more than 2 minutes to receive their food once their order is placed.

(a) **Reduces the significance level: α=0.10 → α=0.01.**

Launch applet
- ***Improved Batting Averages (Power)***
- **www.rossmanchance.com/applets**

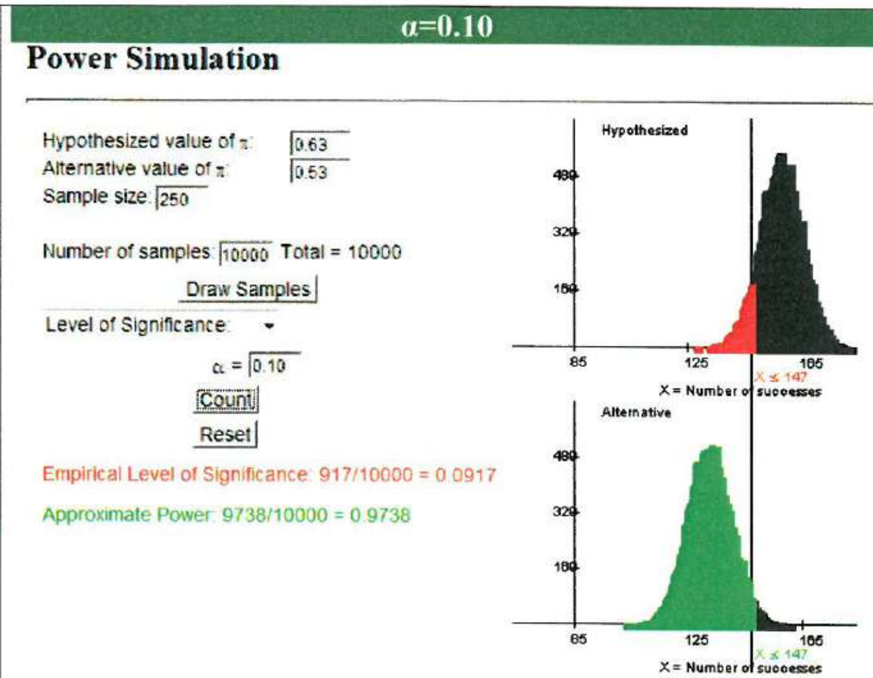
For our test of:
Ho: $p = 0.63$
Ha: $p < 0.63$

We assume:
phat = 0.53, n = 250, α = 0.10

Step 1 - Enter
- 0.63 for the hypothesized value of p or π
- 0.53 for the alternative hypothesis
- 250 for the sample size, and
- 10,000 for the number of samples.
- **Press Draw Samples.**

Step 2 - Enter
- In the drop down menu that says "Choose option," choose Level of Significance and enter 0.10 for α.
- Press "count"
- **Result: Power of the test is ~97% and β=.03**

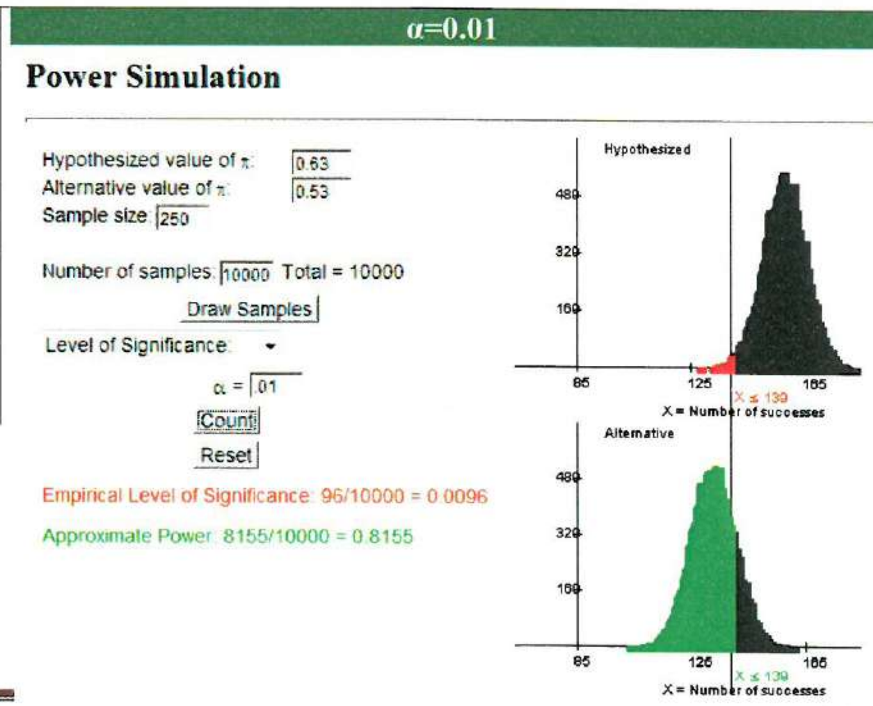
α=0.10



Step 3 -
- Change the value of $\alpha = 0.01$ and
- Press Count.

How does the power change?
- **Result: Power of the test is ~82% and β=.18**

α=0.01



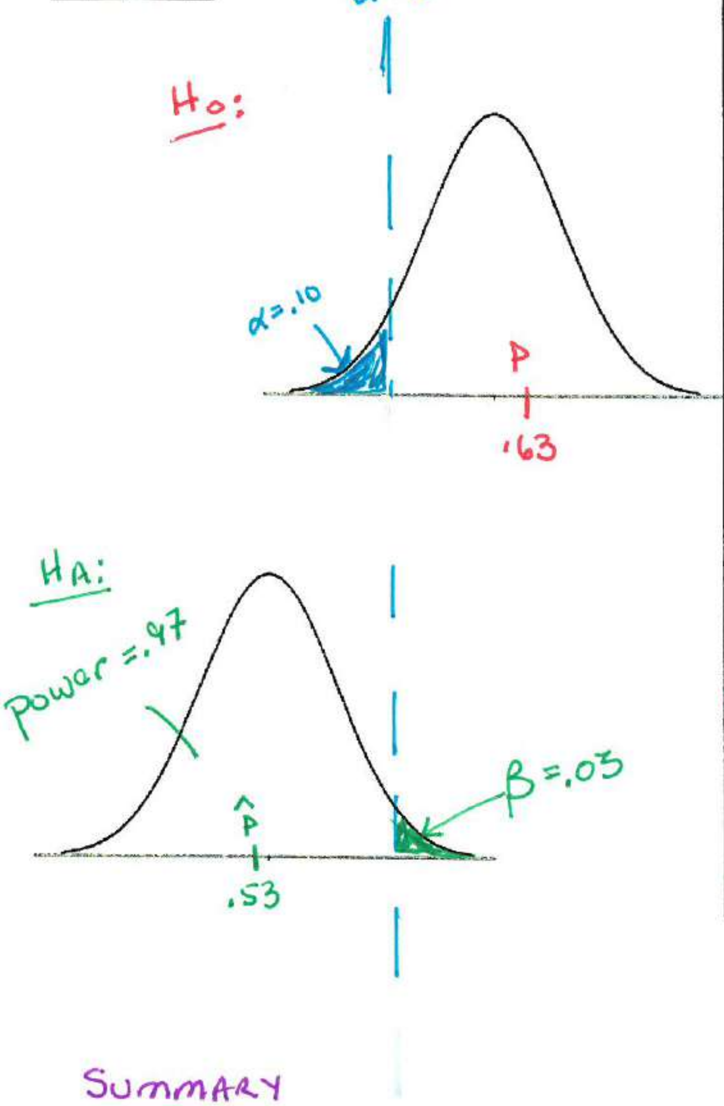
Example 5

Hypothesis	***Sample Results***		
$H_0: p = 0.63$	n=250	x=134	$\hat{p} = \frac{134}{250} \approx .53$
$H_a: p < 0.63$	P-value =.001		

"Power Demonstration" How would the following changes affect the power of the test?

(a) For **α=.10**, use a simulation to sketch the null and alternative hypothesis. Find β and power.

α=.10, n=250



SUMMARY

n = 250

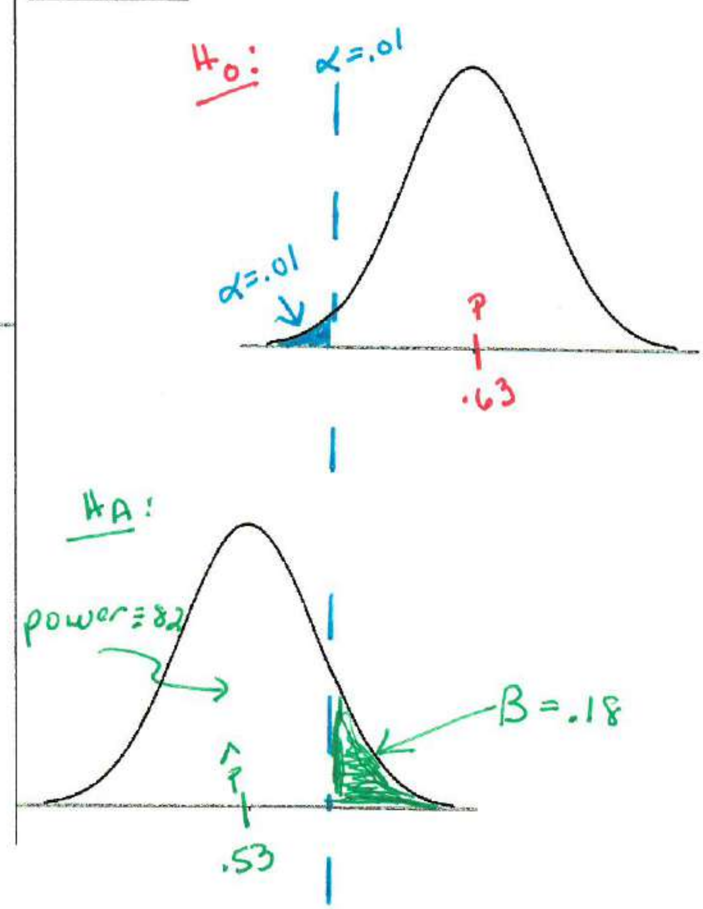
α = .10

β = .03

Power = .97

(b) To reduce the possibility of a Type I error and avoid the possibility of unnecessarily paying an extra employee, the manager **reduces the significance level from 0.10 to 0.01**.

α=.01, n=250



SUMMARY

n = 250

α = .01 ↓

β = .18 ↑

Power = .82 ↓

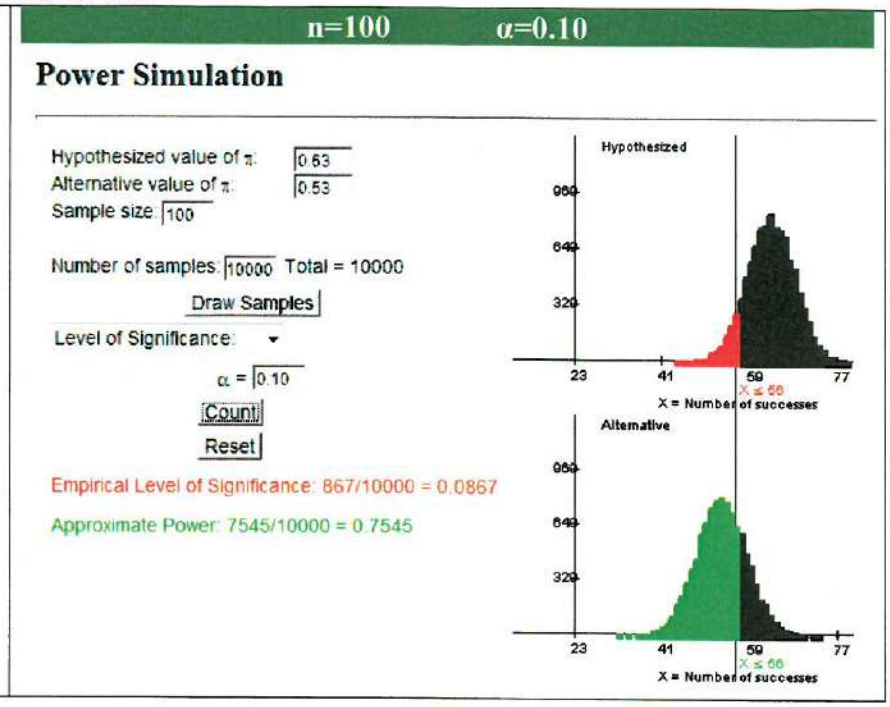
(b) **Reduces the sample size:** n=250 → n=100. **Use α=0.10**

Step 4 -

- Reset
- Change the sample size to 100
- Draw Samples
- Choose α=.10
- Press "count"

How does the power change?

- **Result: Power of the test is ~75% and β=.25**



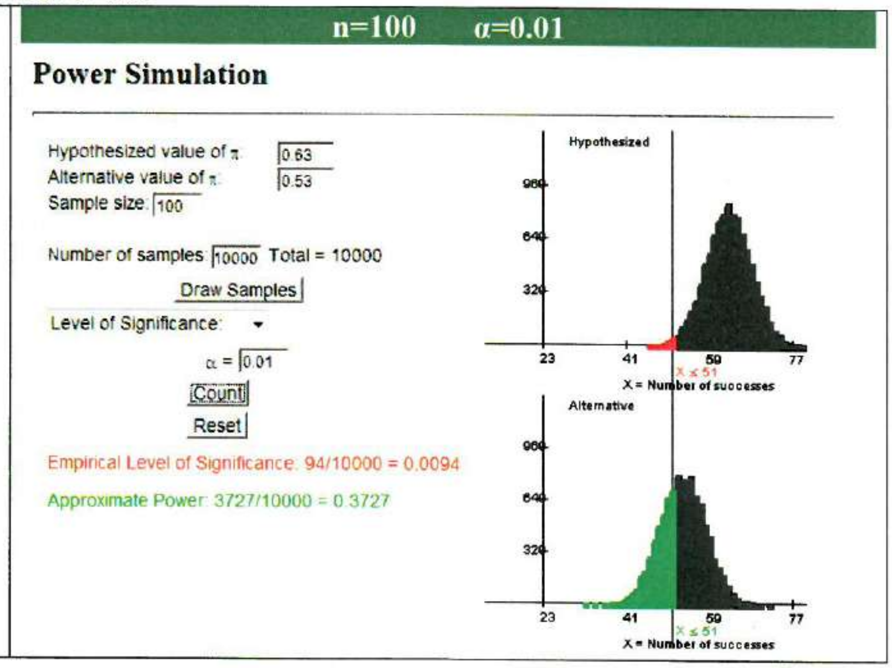
(c) **Reduces the sample size:** n=250 → n=100. **Use α=0.01**

Step 5 -

- Change the value of α = 0.01
- Press "count"

How does the power change?

- **Result: Power of the test is ~37% and β=.63**



"Power Demonstration (continued)" How would the following changes affect the power of the test?

(c) To get faster results, the manager reduces the sample size from 250 to 100. **Use α=.10**

$\boxed{\alpha=.10,\ n=100 \downarrow}$

$\alpha = .10$

$\beta = .25$

Power = .75

(d) To get faster results, the manager reduces the sample size from 250 to 100. **Use α=.01**

$\boxed{\alpha=.01,\ n=100 \downarrow}$

$\alpha = .01$

$\beta = .37$

power = .63

Time permitted

$n = .10 \quad n = 500$

Then → $\beta = .01$

Power = .99

Larger samples increase power

(e) Summarize what you have learned about the factors that affect the power of a test.

What happens when you reduce **α**? $\alpha\downarrow$ $\beta\uparrow$ Power↓ α + β have inverse relationship	What happens when you increase **α**? $\alpha\uparrow$ $\beta\downarrow$ Power↑ α + Power are directly related
What happens when you reduce **sample size (n)**? ↓n α No change β ↑ Power ↓	What happens when you increase **sample size (n)**? * Sample size and power are directly related
Which error(s) can the researcher control? Researcher can set "α" only (but should consider impact of β and Power)	

What do you do to maximize the power of a test?

Researchers want to maximize power!!

(1) Choose as high an α (TYPE I ERROR) as you are willing to risk

(2) Choose as large a sample size as you can afford.

Power↑ then → α↑ and n↑

2003 Problem #2

When a law firm represents a group of people in a class action lawsuit and wins that lawsuit, the firm receives a percentage of the group's monetary settlement. That settlement amount is based on the total number of people in the group - the larger the group and the larger the settlement, the more money the firm will receive.

A law firm is trying to decide whether to represent car owners in a class action lawsuit against the manufacturer of a certain make and model for a particular defect. If 5 percent or less of the cars of this make and model have the defect, the firm will not recover its expenses. Therefore, the firm will handle the lawsuit only if it is convinced that more than 5 percent of cars of this make and model have the defect. The firm plans to take a random sample of 1,000 people who bought this car and ask them if they experienced this defect in their cars.

Scoring:

(a) Define the parameter of interest and state the null and alternative hypotheses that the law firm should test. [Partial IF ONLY STATE 1]

① p=true proportion of DEFECTS FOR CARS OF A CERTAIN MAKE + MODEL

② $H_0: p = .05$
$H_A: p > .05$

ELEMENT #1

E P I

(b) In the context of this situation, describe Type I and Type II errors and describe the consequences of each of these for the law firm.

TYPE I ERROR Reject H_0 WHEN H_0 IS TRUE

CONTEXT → THE LAW FIRM BELIEVES THAT THE PROPORTION OF CARS THAT HAS DEFECTS GREATER THAN .05, WHEN IN FACT IT HAS NOT CHANGED

CONSEQUENCE THE LAW FIRM TAKES THE CASE BUT WILL NOT RECOVER EXPENSES (LOSING $'S)

TYPE II ERROR FAIL TO REJECT H_0 WHEN H_A TRUE.

CONTEXT → THE LAW FIRM BELIEVES DEFECTS IS 5%, WHEN IN FACT DEFECTS ARE GREATER THAN 5%

CONSEQUENCE THE LAWFIRM REFUSES THE CASE, AND MISSES AN OPPORTUNITY TO MAKE MONEY.

ELEMENT #2

E P I

P - IF NOT IN CONTEXT
P - IF DEFINE ONLY 1 ERROR
P - IF REVERSE ERRORS

ELEMENT #3

E P I

CONSEQUENCES
P - DOES NOT GIVE ECONOMIC CONSEQUENSE

Total:__/4