8th Grade Mathematics

Functions: Define, evaluate, compare functions & understand connections between proportional relationships and linear equations

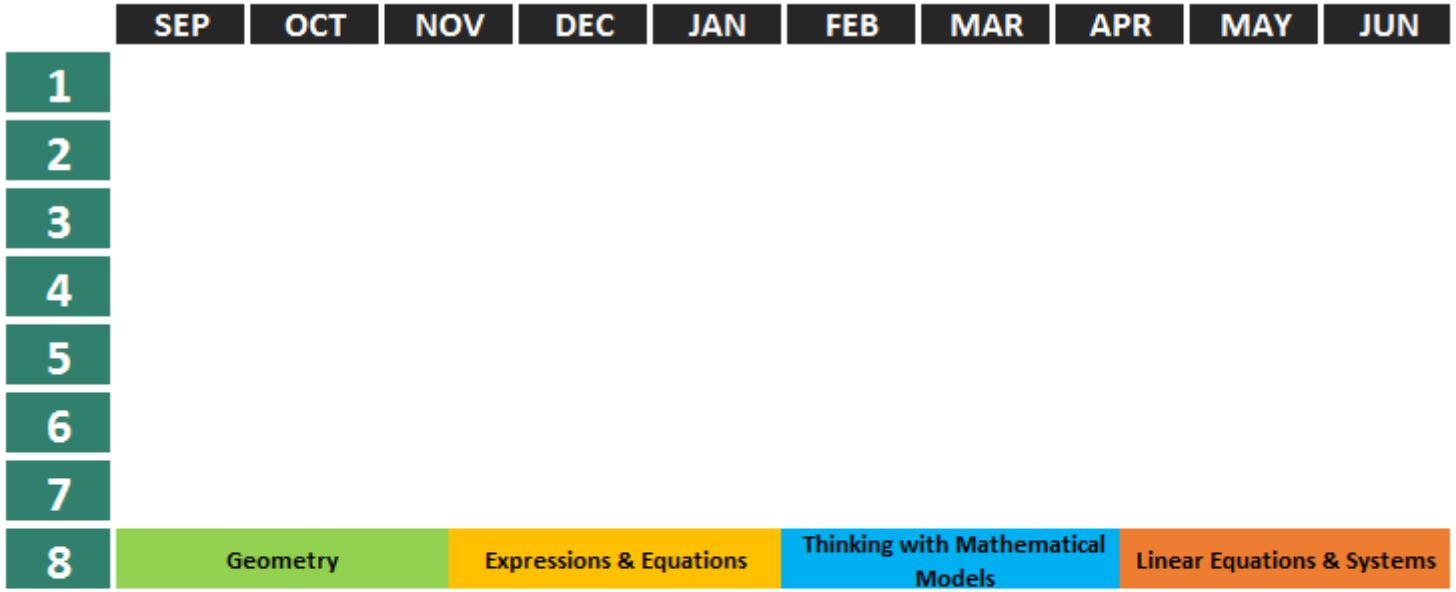
Unit 3 Curriculum Map: February 6th – March 31st

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

A STORY OF UNITS



Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software

Expressions and Equations: Expressions and Equations Work with radicals and interger exponents

Thinking with Mathematical Models: Define, evaluate, and compare functions, use similar triangles to explain slope, investigate patterns of association in bivariate data

Linear Equations and Systems: Use functions to model relationships between quantities, analyze and solve linear equations and pairs of simulataneous linear equations

Table of Contents

Unit 3 Overview

In this unit, students will ….

- Represent data patterns using graphs, tables, word descriptions, and algebraic expressions
- Investigate the nature of linear functions in contexts
- Use mathematical models to answer questions about linear relationships
- Write linear functions from verbal, numerical, or graphical information
- Analyze and solve linear equations
- Model situations with inequalities expressed as “at most” and “at least” situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships
- Compare inverse variation relationships with linear relationships
- Identify linear and nonlinear functions in graphs, equations, and tables
- Apply real world situations to the appropriate graphs
- Compare the slopes and y-intercepts of functions
- Create linear functions based on real world situations
- Use data to make predictions
- Fit a line to data that show a linear trend and measure closeness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear association between the two variables
- Use correlation coefficients informally to describe the strength of the linear association illustrated by scatter plots
- Use standard deviation to measure variability in univariate distributions
- Distinguish between categorical and numerical variables
- Use two-way tables and analysis of cell frequencies and relative frequencies to decide whether two variables are related

Enduring Understandings

- Functions provide a tool for describing how variables change together. Using a function in this way is called modeling, and the function is called a model.
- Functions can be represented in multiple ways—in algebraic symbols, graphs, verbal descriptions, tables, and so on—and these representations, and the links among them, are useful in analyzing patterns of change.
- Functions are single-valued mappings from one set—the domain of the function—to another—its range.
- A function can be represented in a table, graph, or equation

- Changes in varying quantities are often related by patterns
- A rate of change describes how one variable quantity changes with respect to another—in other words, a rate of change describes the co-variation between two variables.

Enduring Understandings (Continued)

- Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type for constants m and b.
- Formulating questions, designing studies, and collecting data about a characteristic shared by two populations or different characteristics within one population.
- Selecting, creating, and using appropriate graphical representations of data, including histograms, box plots, and scatter plots.
- Finding, using, and interpreting measure of center and spread, including mean and inter-quartile range.
- Discussing and understanding the correspondence between data sets and their graphical representations, especially histograms, and scatter plots.
- Using observations about differences between two or more samples to make conjectures about the populations from which samples were taken.
- Using proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
- Computing probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Pacing Guide

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Unit 3 Diagnostic	7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2d, , 7.EE.A.1, 7.EE.B.3, 7.EE.B.4a, 7.SP.A.2	½ Block
Thinking with Mathematical Models (CMP3) Investigation 1	8.F.A.3, 8.F.B.5, 8.SP.1	3½ Blocks
Assessment: Unit 3 Check Up 1 (CMP3)	8.F.A.3, 8.F.B.5, 8.SP.1	½ Block
Foundation of Algebra (Agile Mind) Topic 3.5	8.F.A.1	1½ Block
Working with Function and Equation (Agile Mind) Topic 6.1 and 6.2	8.F.A.2	2½ Block
Unit 3 Performance Task 1	8.F.A.1	1 Block
Unit 3 Assessment 1	8.F.A.1, 8.F.A.2	1 Block
Thinking with Mathematical Models (CMP3) Investigation 2	8.EE.C7.b, 8.EE.C.8a, 8.F.A.1, 8.F.A.2, 8.EE.B.5,	4½ Blocks
Assessment: Unit 3 Partner Quiz (CMP3)	8.EE.B.5, 8.EE.C7.b, 8.EE.C.8a, 8.F.A.1, 8.F.A.2	½ Block
Thinking with Mathematical Models (CMP3) Investigation 3	8.F.A.1, 8.F.A.3, 8.F.B.5, 8.SP.A.1	4 Blocks
Unit 3 Performance Task 2	8.F.A.2	1 Block
Unit 3 Assessment 2	8.EE.B.6	1 Block
Thinking with Mathematical Models (CMP3) Investigation 4	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.EE.B.5, 8.EE.C7, 8.F.A.1, 8.F.A.3, 8.F.B.4	4 Blocks
Assessment: Unit 3 Check Up 2 (CMP3)	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.EE.B.5, 8.EE.C7, 8.F.A.1, 8.F.A.3	½ Block
Thinking with Mathematical Models (CMP3) Investigation 5	8.SP.A.4, 8.EE.C.7, 8.F.A.3	3 Blocks
Unit 3 Performance Task 3	8.F.A.3	1 Block
Unit 3 Assessment 3	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4	1 Block
Total Time		**31 Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

FEBRUARY						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
			1	**2**	**3**	**4**
5	**6** **Unit 3: Thinking w/ Math Models** ***Unit 3 Diagnostic***	**7**	**8** **12:30 pm Student Dismissal**	**9**	**10** ***Assessment: Check Up 1***	**11**
12	**13**	**14**	**15**	**16** ***Performance Task 1 Due***	**17** ***Assessment: Unit 3 Assessment 1***	**18**
19	**20** **Winter Recess** **District Closed**	**21** **Winter Recess** **District Closed**	**22** **Winter Recess** **District Closed**	**23** **Winter Recess** **District Closed**	**24** **Winter Recess** **District Closed**	**25**
26	**27**	**28**				

MARCH

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3 ***Assessment: Partner Quiz***	4
5	6	7	8	9	10 ***Performance Task 2 Due***	11
12	13 ***Assessment: Unit 3 Assessment 2***	14	15	16	17	18
19	20 ***Assessment: Check Up 2***	21	22	23	24 ***Performance Task 3 Due***	25
26	27 ***Assessment: Unit 3 Assessment 3***	28 **Solidify Unit 3 Concepts**	29 **Solidify Unit 3 Concepts**	30 **Solidify Unit 3 Concepts**	31 **Unit 3 Complete**	

Math Background

Although the basic understandings of and skills for linear equations were addressed in the Grade 7 Unit Moving Straight Ahead, they need to be revisited and practiced to deepen student understanding. The Problems in Investigation 2 of Thinking with Mathematical Models are designed to promote this sort of review and extension.

In Moving Straight Ahead, students learned to recognize, represent symbolically, and analyze relationships in which a dependent variable changes at a constant rate relative to an independent variable. Students learned the connections between the equation y=mx+b, the rate of change, the slope of the line, and the y-intercept of the line.

Most students probably interpret the sentence “y varies directly with x” to mean “As x increases, y increases,” and “y varies inversely with x” means “As x increases, y decreases.” In mathematics, the meanings of direct variation and inverse variation are more specific.

A key idea of Thinking with Mathematical Models is to use mathematics to approximate real-world data. Mathematics provides precisely defined objects and operations that can be used to represent real-world data patterns that may not be as well behaved.

Effective use of mathematical modeling requires awareness of the overall modeling process and a set of mathematical concepts and skills for model building and analysis. This Unit is only an introduction to these ideas. It lays a foundation for a more sophisticated and thorough development of modeling strategies in high school and college mathematics and science courses.

In this Unit, students learn about situations in which linear or inverse variation models are particularly appropriate. For the linear examples, students “eyeball” a fitted line and then find the equation of the line. Students do not find the line of best fit for the data.

For the inverse variation examples, we suggest that students only experiment with data plots and test function rules to establish reasonable proportionality constants.

Using the plotting and function-graphing capabilities of a graphing calculator makes successive approximation an effective modeling technique. If you have a graphing calculator available, you may use it to extend student investigation for any of the Problems in this Unit.

When students work with data, they are often interested in the individual data items, particularly if the data are about themselves. Looking at the overall distribution of a data set rather than at individual items can reveal important information.

We use graphs to help provide a picture of a distribution of data. Distributions (unlike individual cases) have properties that include statistics such as measures of central tendency (i.e., mean, median, mode) or variability (e.g., outliers, range) and characteristics such as shape (e.g., clumps, gaps, skewed distributions).

Math Background (Continued)

When we look at distributions, we are often interested in the measures of center, which tell us that a value is “typical” of the distribution. Any measure of center alone can be misleading, however. It is important also to consider the variability of the distribution.

Generally, students’ earlier work with data analysis has emphasized describing what is typical about a distribution of data. During the middle grades, there is a shift toward consideration of variability; students are better prepared mathematically and developmentally to consider this concept.

Describing variability includes looking at measures of center, at the range of the data, at where data cluster or where there are gaps in a distribution, at the presence of outliers, and at the shape of the distribution.

Many problem-solving and decision-making situations require analyzing what statisticians call categorical variables. For example, as the student text points out, in comparing the popularity of wood-frame and steel-frame roller coasters, the type of roller coaster frame is a categorical variable that has values wood and steel.

Data may fall into more than two categories. For instance, a study comparing the popularity of cars might investigate the categorical variable type of vehicle with values sedan, convertible, SUV, pickup, and so on. A study comparing the popularity of dogs could investigate the categorical variable breed of dog with values poodle, terrier, Irish setter, German shepherd, Chihuahua, and so on.

Questions about categorical variables are often resolved by comparing frequencies of occurrence for each categorical value. The Common Core standards for Grade 8 call for an introduction to associations between pairs of categorical variables by analyzing two-way tables.

PARCC Assessment Evidence Statements

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator?
8.EE.5-1	Graph proportional relationships, interpreting the unit rate as the slope of the graph.	i) Pool should contain tasks with and without context. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	1, 5	Yes
8.EE.6-1	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.	i) Tasks do not have a context.	2, 7	Yes
8.F.1-1	Understand that a function is a rule that assigns to each input exactly one output.	i) Tasks do not involve the coordinate plane or the "Vertical line test." ii) Tasks do not require knowledge of the concepts or terms domain and range. iii) 20% of functions in tasks are non-numerical, e.g., the input could be a person and the output could be his or her month of birth	2	No
8.F.1-2	[Understand that] the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	i) Functions are limited to those with inputs and outputs in the real numbers. ii) Tasks do not require knowledge of the concepts or terms domain and range. iii) 80% of tasks require students to graph functions in the coordinate plane or read inputs and outputs from the graph of a function in the coordinate plane. iv) 20% of tasks require students to tell whether a set of points in the plane represents a function.	2, 5	No
8.C.3.1	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.F.3-2	i) Note especially the portion of 8.F.3 after the semicolon. Tasks require students to prove that a given function is linear or nonlinear.	3, 6	Yes

8.C.6	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 7.RP.A, 7.NS.A, 7.EE.A	Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 8.	3, 6	Yes
8.D.1	Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 8; requiring application of knowledge and skills articulated in the Evidence Statements on the PBA (excludes Reasoning Evidence Statements).	Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 8.	4, 1, 2, 5, 7	Yes
8.D.2	Solve multi-step contextual problems with degree of difficulty appropriate to Grade 8, requiring application of knowledge and skills articulated in 7.RP.A, 7.NS.3, 7.EE, 7.G, and 7.SP.B	Task may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 8.	4, 1, 2, 5, 7	Yes
8.D.3	Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in the Evidence Statements on the PBA (excludes Reasoning Evidence Statements).	Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 8.	4, 1, 2, 5, 7	Yes
8.D.4	Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills articulated in the Evidence Statements on the PBA (excludes Reasoning Evidence Statements).	Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 8.	4, 1, 2, 5, 7	Yes

8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.		3,5,7	No
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		2,5,7	No
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept*. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*		2,4,6,7	Yes
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	i) An equal number of tasks require students to : • Answer basic comprehension questions about a two-way table, or; • To compute marginal sums or marginal percentages, or; • To interpret patterns or association	2,4,5,7	Yes

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	- Students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends
2	Reason abstractly and quantitatively
	- Students consider the units involved; attending to the meaning of quantities
3	Construct viable arguments and critique the reasoning of others
	- Students learn to determine domains to which an argument applies
4	Model with mathematics
	- Students are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5	Use appropriate tools strategically
	- Students analyze graphs of functions and solutions generated using a graphing calculator, through tables, graphs, or equations
6	Attend to precision
	- Students are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem
7	Look for and make use of structure
	- Students apply properties to generate equivalent expressions and solve equations. - Students examine patterns in tables and graphs to generate equations and describe relationships.
8	Look for and express regularity in repeated reasoning
	- By paying attention to the calculation of slope as they repeatedly check whether points are on a line through a point given a slope, students might abstract the appropriate equation.

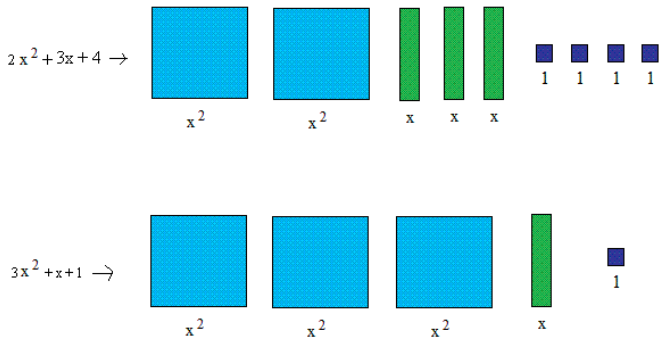
Vocabulary

Term	Definition
Function	A rule of matching elements of two sets of numbers in which an input value from the first set has only one output value in the second set.
Slope	The "steepness" of a line. The slope of a line can be found directly when a linear equation is in a slope-intercept form (y=mx+b). In this form, the slope is the coefficient of x and is represented by the letter m. The slope of a line can also be found by determining the ratio of the "rise" to the "run" between two points on the graph.
Unit Rate	A comparison of two measurements in which the second term has a value of one. Unit rates are used to compare the costs of items in a grocery store.
Initial Value	The y – intercept in a real world situation
Linear	A relationship or function that can be represented by a straight line
Nonlinear	A relationship or function that cannot be represented by a straight line
Categorical Variables	Variables that measure characteristics using "words" that represent possible responses within a given category. Frequency counts can be made of the values for a given category
Correlation Coefficient	A measure of the strength of a linear relationship between two variables using a decimal number between −1 and 1.
Mathematical Model	An equation or a graph that describes, at least approximately, the relationship between two variables. To make a mathematical model, acquire data, plot the data points, and, when the points show a pattern, find the equation of a line or curve that fits the trend in the data. A mathematical model allows you to make reasonable guesses for values between and beyond the data points.
Outlier	A value that lies far from the "center" of a distribution. Outlier is a relative term, but it indicates a data point that is much higher or much lower than the values that could be normally expected for the distribution.
Scatter Pot	A graph used to explore the relationship between two variables.
Standard Deviation	Standard deviation measures the spread of a data set. The greater the standard deviation, the greater the spread of the data. To calculate the standard deviation, find the differences between the actual values and the mean. These differences are squared and averaged by dividing by (n−1). This average is the variance. Take the square root of the variance to get standard deviation.
Variance	Variance is calculated from the differences between the actual value and the mean. These differences are squared and averaged by dividing by (n−1).

Potential Student Misconceptions

- Students may incorrectly calculate or identify the slope because they have memorized a formula and do not contextualize or conceptualize the idea of slope
- Students may incorrectly calculate or identify the y-intercept because they have not contextualized or conceptualized the idea of the y-intercept
- Students may incorrectly plot a line. To help these students, encourage that a table of values be used to aid in the process.
- Students may mistakenly believe that a slope of zero is the same as “no slope” and then confuse a horizontal line (slope of zero) with a vertical line (undefined slope).
- Students may interchange the meanings of *x* (independent variable) and *y* (dependent variable), particularly when graphing the line of an equation.
- Students may think that a ratio must be a pair of non-negative, non-zero numbers *A* and *B* in the form $\frac{A}{B}$, A:B, or A to B
- Confusing $\frac{rise}{run}$ with the ordered pair notation (x, y)
- Students may mix the meanings of *x* (independent variable) and *y* (dependent variable), particularly when graphing the line of an equation
- When plotting data on a scatter plot, students may confuse the independent (*y)* and the dependent *(x)* variable and on what axis each should be placed.
- The language used in questions to help students interpret two-way tables can be very confusing. Be sure to help students break down questions and understand how to determine what information each question is asking for.
- Students may attempt to create a line of best fit by “connecting the dots” on a scatter plot. Students need to understand that all lines are straight and they will need to pick a line to fit as many dots as possible.

Teaching Multiple Representations

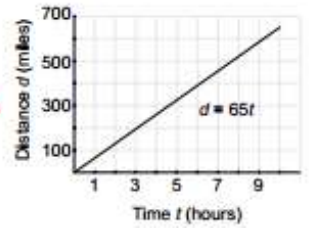
CONCRETE REPRESENTATIONS	
• Algebra Tiles	
• Counters	
• Grid/Graph Paper	
• Graphing Paper	

PICTORIAL REPRESENTATIONS

- Flow Charts
- Unit Rate in Tables

t (hours)	1	2	3	4	5	6
$60t$ (miles)	60	120	180	240	300	360

- Unit Rate in Graphs

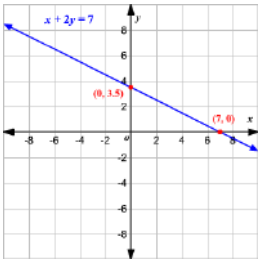


- Tabular Representation

CARPET CLEANER RENTAL

Number of Hours	Cost
1	$16
2	$22
3	$28
4	$34
5	?
6	?

- Graphing



- Two-Way Frequency Table

	Ritual A	Ritual B	Totals
Male	43	9	52
Female	44	4	48
TOTALS	87	13	100

ABSTRACT REPRESENTATIONS

- Equations, y = mx + b
- Application Problems

Assessment Framework

Unit 3 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 3 Diagnostic Assessment (Beginning of Unit)	7.SP.A.2, 7.EE.B.4a, 7.RP.A.2b, 7.RP.A.2a	1 Block	Individual	No
Unit 3 Check Up 1 (After Investigation 1) *Thinking with Mathematical Modeling CMP3*	8.F.A.3, 8.F.B.5, 8.SP.1	½ Block	Individual	Yes
Unit 3 Assessment 1 (After AgileMind Topic 6.2) *District Assessment*	8.F.A.1, 8.F.A.2	1 Block	Individual	Yes
Unit 3 Partner Quiz (After Investigation 2) *Thinking with Mathematical Modeling CMP3*	8.EE.B.5, 8.EE.C7.b, 8.EE.C.8a, 8.F.A.1, 8.F.A.2	½ Block	Group	Yes
Unit 3 Assessment 2 (After Investigation 3) *District Assessment*	8.EE.B.6	1 Block	Individual	Yes
Unit 3 Check Up 2 (After Investigation 4) *Thinking with Mathematical Modeling CMP3*	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.EE.B.5, 8.EE.C7, 8.F.A.1, 8.F.A.3	½ Block	Individual	Yes
Unit 3 Assessment 3 (Conclusion of Unit) *District Assessment*	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4	1 Block	Individual	Yes

Assessment Framework (continued)

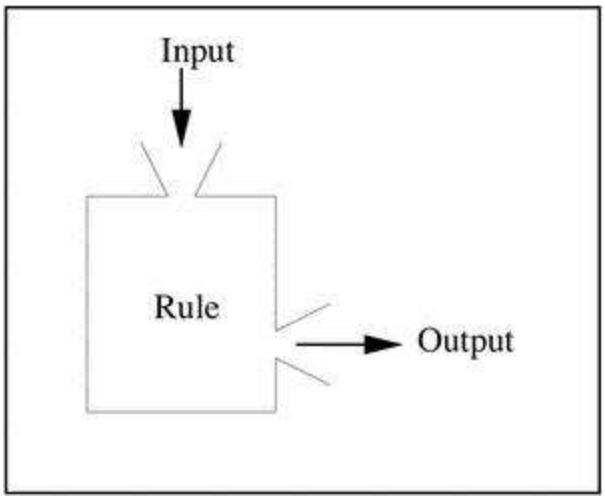
Unit 3 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 3 Performance Task 1 (Early February) *Function Rule*	8.F.A.1	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 3 Performance Task 2 (Early March) *Battery Charging*	8.F.A.2	½ Block	Individual w/ Interview Opportunity	Yes: rubric
Unit 3 Performance Task 3 (Late March) *Introduction to Linear Function*	8.F.A.3	½ Block	Group	Yes; Rubric
Unit 3 Performance Task Option 1 (optional)	8.F.B.4	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 3 Performance Task Option 2 (optional)	8.F.B.4	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

Unit 3 Performance Task 1

Function Rule (8.F.A.1)

A function machine takes an input, and based on some rule produces an output.



The tables on the next page show some input-output pairs for different functions. For each table, describe a function rule in words that would produce the given outputs from the corresponding inputs. Then fill in the rest of the table values as inputs and outputs which are consistent with that rule.

a. Input values can be any English word. Output values are letters from the English alphabet.

input	cat	house	you	stem			
output	t	e	u		z		

b. Input values can be any real number. Output values can be any real number.

input	2	5	−1.53	0	−4		
output	7	10	3.47	5		8	

c. Input values can be any whole number. Output values can be any whole number.

input	1	2	3	4	5	6	7
output	2	1	4				

d. Input values can be any whole number between 1 and 365. Output values can be any month of the year.

input	25	365	35	95	330	66	
output	January	December	February	April	November		October

For at least one of the tables, describe a second rule which fits the given pairs but ultimately produces different pairs than the first rule for the rest of the table.

Solution:

a. We can notice that the letters provided as output are the last letters in the words provided as input, so one possible rule is "take the last letter" of the input. Below is one possible way to complete the table consistent with this rule.

input	cat	house	you	stem	buzz	sky	picture
output	t	e	u	m	z	y	e

b. We can notice that the output and input pairs given all differ by five, so one possible rule is “add 5” to the input value. Below is one possible way to complete the table consistent with this rule.

input	2	5	-1.5	0	-4	3	0.1113
output	7	10	3.5	5	1	8	5.1113

c. The numbers seem to be "switching places" in a way. One possible precise rule could be that the output for an odd number is the even number which is one greater, and the output for an even number is the odd number which is one less. Below is how to complete the table consistent with this rule.

input	1	2	3	4	5	6	7
output	2	1	4	3	6	5	8

d. One possible rule here is “the month in which the input day of the year falls”, defining January 1st to be day one. To be precise, we specify that we are using a non-leap year to determine the output values. Below is one possible way to complete the table consistent with this rule.

input	25	365	35	95	330	66	280
output	January	December	February	April	November	March	October

Unit 3 Performance Task 1 PLD Rubric

SOLUTION

- **Student indicates that the rule for the first table is to "take the last letter of the input to get the output," and completes the table.**
- **Student indicates that rule for the second table is "input plus 5 to get the output."**
- **Student indicates that the rule for the third table is that "numbers are switching places" or "output for an odd number is an even number, which 1 more than the odd number, and output for an even number is an odd number, which 1 less than the odd number.**
- **Student indicates that the rule for the fourth table is "the month in which the input day of the year falls", defining January 1st to be day one. To be precise, we specify that we are using a non-leap year to determine the output values"**

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
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Unit 3 Performance Task 2

Battery Charging (8.F.A.2)

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

b. How much time would Sam need to charge the battery 100% on both devices?

Solution:

Solution: Using equations

a. The battery charge of both devices can be modeled with linear functions. The wording describing the MP3 player suggests a linear function since it uses a constant rate of change. The table of values for the video game player shows a constant rate of change for the first 30 minutes. It is a reasonable assumption that the battery will continue to charge at the same rate. However, it is an assumption on our part. (Another possibility would be that as the battery charge approaches 100%, the rate of change decreases, but that would be much harder to model.)

The MP3 player charges at a rate of 12 percentage points every 15 minutes, which is equal to 0.8 percentage points per minute. If we let y be battery charge of the device (in percentage points) we have:

$$y=0.8t+40,$$

where t is measured in minutes.

We know that the video game player is initially 20% charged and from the table we see that the charge increases by an additional 12 percentage points every 10 minutes, or 1.2 percentage points per minute. So for this function we get:

$$y=1.2t+20.$$

Sam's family is planning to leave the house 60 minutes after Sam started charging his devices. We are looking for the charge when $t=60$:

MP3 player: $y=0.8\cdot 60+40=88$ % charged

video game player: $y=1.2\cdot 60+20=92$ % charged

b. To answer this question, we need to find the values of t for which each function has output value 100.

MP3 player: Solving $100=0.8t+40$ for t we have, $t=75$ minutes.

Video game player: Solving $100=1.2t+20$ for t we have $t=67$minutes.

So if Sam's family could wait just 15 more minutes, Sam could have both devices fully charged for the car trip.

Solution: Using tables

a. Since the video game player's battery charge is given in a table, we can extend the table and see what value it will give after 60 minutes. Note that the rate of change of the data in the table is constant: For every 10 minutes the charge increases by 12 percentage points. Assuming that this pattern continues, we have:

time charging (minutes)	0	10	20	30	40	50	60
Video game player battery charge (%)	20	32	44	56	68	80	92

b. We can make a similar table for the MP3 player:

time charging (minutes)	0	15	30	45	60
MP3 player battery charge (%)	40	52	64	76	88

c. So after 60 minutes, the MP3 player's battery would be 88% charged and the video game player will be 92% charged.

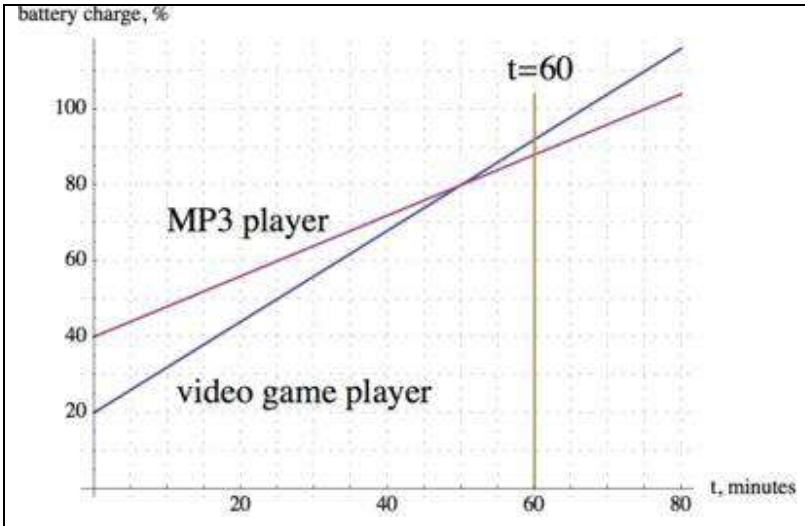
d. We can see from the table above that the MP3 player would be fully charged in another 15 minutes; we just have to add one more column to the table to find that answer.

The video game player will need less than 10 minutes to fully charge, since we are only missing 8 percentage points after 60 minutes. To be exact, using the rate of increase, we will need 2/3 of 10 minutes, which is just under 7 minutes.

Solution: Using graphs

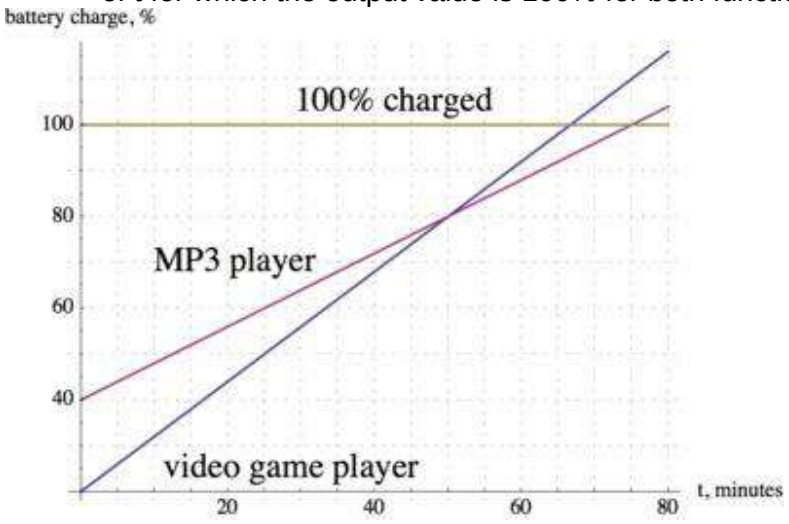
a. With the given information, it is quite straight-forward to graph the functions for both devices. For the MP3 player we have a starting value (i.e. vertical intercept) of 40% and a rate of change (i.e. slope) of 12/15 = 0.8 percentage points per minute.

For the video game player we have a starting value of 20% and the rate of change for the data in the table is constant at 12/10 = 1.2 percentage points per minute. Below are the two graphs.



We can estimate from the graph that after 60 minutes the MP3 player has a battery charge of just under 90% and the video game player has a battery charge of just over 90%. Zooming in on a graphing calculator or other graphing device would give us better estimates.

b. To find out how long it will take until both batteries are fully charged, we need to find values of t for which the output value is 100% for both functions.



From the graph we see that the MP3 player will take the longest to charge and it will take about 75 minutes total. So if Sam's family can wait an extra 15 minutes before they leave, Sam would have both devices fully charged.

Unit 3 Performance Task 2 PLD Rubric

SOLUTION

- **Student indicates that After 60 minutes Battery charge for the video game will be 92 percent and explains either with a table, equation or graph.**
- **Students indicates that if Sam's family needs to wait just 15 more minutes then he can charge both of the devices 100% and explains the reason**

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Unit 3 Performance Task 3

Introduction to Linear Function (8.F.A.3)

a. Decide which of the following points are on the graph of the function $y=2x+1$:
   - $(0,1),(2,5),(12,2),(2,-1),(-1,-1),(0.5,1)$.
   - Find 3 more points on the graph of the function.

b. Find several points that are on the graph of the function $y=2x^2+1$.
   - Plot the points in the coordinate plane. Is this a linear function?
   - Support your conclusion.

c. Graph both functions and list as many differences between the two functions as you can.

Solution:

a. We can find which of the given points are on the line $y=2x+1$ by seeing if the x and y coordinate satisfy the equation. For the first point, with $y=1$ and $x=0$, we see

$$1=2(0)+1=1$$

Which is true, and so the point $(0,1)$ is on the line. However, for $(2,-1)$, by substituting $x=2$ and $y=-1$, we have

$$-1\neq 2(2)+1=5$$

and so $(2,-1)$ is not on the line.

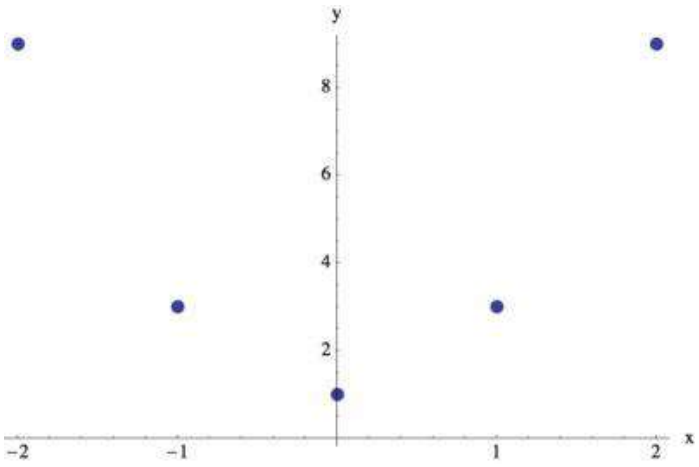
Continuing this way, we see that $(0,1)$, $(2,5)$, $(1/2,2)$, and $(-1,-1)$ are points on the line and $(2,-1)$, and $(.5,1)$ are not points on the line.

We can find three more points by arbitrarily choosing value for x and using the equation for the line to find the corresponding y values: $(-2,-3),(1,3),(-1/2,0)$.

b. By picking arbitrary x values, such as $x=0$, $x=1$, $x=2$, $x=-1$, and $x=-2$, we find the corresponding y values and have several points that lie on the graph of the function:

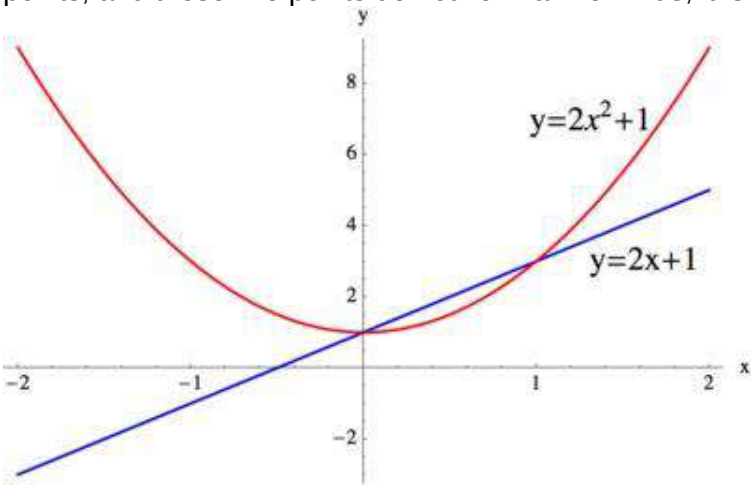
$$(0,1),(1,3),(2,9),(-1,3),(-2,9)$$

By plotting these points, we arrive at the following graph:



A linear function can be written in the form $y=mx+b$, and this is equation is not written in that form. We

might wonder whether it could be written in that form using some clever trick we haven't thought of yet. If it were a linear function, its graph would be a straight line. The graph of $y=2x^2+1$ contains the above five points, and these five points do not lie in a line. Thus, it is not a linear function.



c. Below is a list of differences between these two functions. Possible answers may include some, all, or more differences.

- The first graph is a straight line, the second graph is curved.
- The x term is squared in the second function, and not in the first.
- The first function has negative and positive y values, and the second function will never have negative y values.
- As x increases by equal amounts, the y values in the first function also increase by equal amounts (rate of change is constant) but the y values of the second function increase by bigger and bigger amounts (rate of change is increasing) as $|x|$ gets larger.
- The first function has a constant slope (steepness). The steepness of the second function changes.
- The second function is symmetrical about the y-axis, and the first function is not.
- The first function crosses the x-axis at $(-1/2,0)$, and the second function never crosses the x-axis.
- Each y value in the first function has exactly one x value. In the second function, most y values have 2 possible x values (example :$(-2,9)$ and $(2,9)$ both lie on the second graph).

Unit 3 Performance Task 3 PLD Rubric

SOLUTION

- **Student indicates that (0,1),(2,5),(1/2,2), and (−1,−1)are points on the line and (2,−1), and (.5,1) are not points on the line by substituting the ordered pairs into the equation**
- **Student indicates three more ordered pair by arbitrarily picking a value for the x variable**
- **Student arbitrarily picks some values for x and plots the second equation, ($y = 2x^2 + 1$)**
- **Student indicates that the second equation from part b is not a linear equation because it's not in the y = mx + b format and the line is not straight. Or the student indicates that the rate of change for the output is not constant and the graph is not a straight line.**
- **Student lists difference between two functions accurately from looking at their own graphs and given equations**

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Unit 3 Performance Task Option 1

Video Streaming (8.F.B.4)

You work for a video streaming company that has two monthly plans to choose from:

- Plan 1: A flat rate of $7 per month plus $2.50 per video viewed
- Plan 2: $4 per video viewed

A. What type of functions model this situation? Explain how you know.

B. Define variables that make sense in the context, and then write an equation with cost as a function of videos viewed, representing each monthly plan.

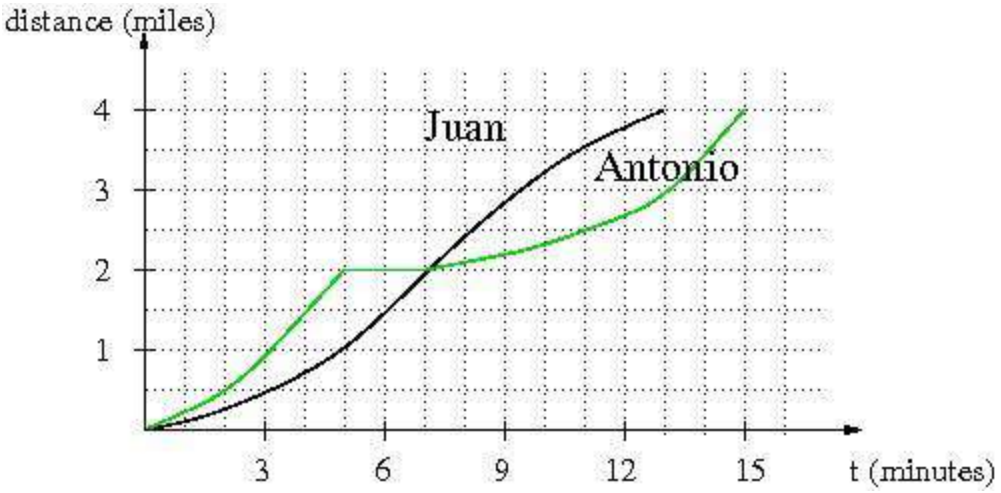
C. How much would 3 videos in a month cost for each plan? 5 videos?

D. Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

Unit 3 Performance Task Option 2

Bike Race (8.F.B.5)

Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).



A. Who wins the race? How do you know?

B. Imagine you were watching the race and had to announce it over the radio, write a little story describing the race.

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

Extensions and Sources

Assessment Resources
http://dashweb.pearsoncmg.com
- Online Connected Math 3 Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

Online Resources
http://www.agilemind.com/programs/mathematics/algebra-i/
-Working with function and equation

http://www.ixl.com/math/grade-8
Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Tracks student progress, video learning, interactive practice

http://www.doe.k12.de.us/assessment/files/Math_Grade_8.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Special Needs designed tasks, assessment resources

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Document Progressions