8th Grade Mathematics

Congruence and Similarity

Unit 1 Curriculum Map: September 8th – October 23rd

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

In this unit students will ….

- Recognize properties of reflection, rotation, and translation transformations
- Explore techniques for using rigid motion transformations to create symmetric design
- Use coordinate rules for basic rigid motion transformations
- Recognize that two figures are congruent if one is derived from the other by a sequence of reflection, rotation, and/or translation transformations
- Recognize that two figures are similar if one can be obtained from the other by a sequence of reflections, rotations, translations, and/or dilations
- Use transformations to describe a sequence that exhibits the congruence between figures
- Use transformations to explore minimum measurement conditions for establishing congruence of triangles
- Use transformations to explore minimum measurement conditions for establishing similarity of triangles
- Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations
- Use properties of congruent and similar triangles to solve problems about shapes and measurements

Enduring Understandings

- Reflections, translations, and rotations are actions that produce congruent geometric objects.
- The notation used to describe a dilation includes a scale factor and a center of dilation. A dilation of scale factor k with the center of dilation at the origin may be described by the notation (kx, ky).
- Two shapes are similar if the lengths of all the corresponding sides are proportional and all the corresponding angles are congruent.
- When parallel lines are cut by a transversal, corresponding, alternate interior and alternate exterior angles are congruent.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.
- When two angles in one triangle are congruent to two angles in another triangle, the third angles are also congruent and the triangles are similar.

Pacing Guide

Activity	Common Core Standards	Estimated Time
Unit 1 Diagnostic	7.G.A.1, 7.G.A.2, 7.G.B.5, 8.G.A.1, 8.G.A.2	½ Block
Butterflies, Pinwheels, and Wallpaper (CMP3) Investigation 1	8.G.A.1, 8.G.A.1a, 8.G.A.1b, 8.G.A.1c	5½ Blocks
Assessment: Check Up 1 (CMP3)	8.G.A.1, 8.G.A.1a, 8.G.A.1b, 8.G.A.1c	½ Block
Butterflies, Pinwheels, and Wallpaper (CMP3) Investigation 2	8.G.A.2, 8.G.A.1a, 8.G.A.1b	4 Blocks
Unit 1 Assessment 1	8.G.A.1, 8.G.A.2	1 Block
Performance Task 1	8.G.A.1	½ Block
Butterflies, Pinwheels, and Wallpaper (CMP3) Investigation 3	8.G.A.3, 8.G.A.1c, 8.G.A.5	5½ Blocks
Unit 1 Assessment 2	8.G.A.3, 8.G.A.4	1 Block
Performance Task 2	8.G.A.2	1 Block
Butterflies, Pinwheels, and Wallpaper (CMP3) Investigation 4	8.G.A.3, 8.G.A.4, 8.G.A.5, 8.EE.B.6	4½ Blocks
Unit 1 Assessment 3	8.G.A.5	½ Block
Performance Task 3	8.G.A.3	½ Block
Total Time		**26 Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

SEPTEMBER						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
		1 **OPENING DAY** **SUP. FORUM PD DAY**	**2** **PD DAY**	**3** **PD DAY**	**4** **PD DAY**	**5**
6	**7** **LABOR DAY District Closed**	**8** **1st Day for students**	**9**	**10** **Unit 1: Congruence & Similarity** ***Unit 1 Diagnostic***	**11**	**12**
13	**14**	**15**	**16**	**17**	**18** ***Assessment: Check Up 1***	**19**
20	**21**	**22**	**23** ***Assessment: Unit 1 Assessment 1***	**24** **12:30pm Student Dismissal**	**25**	**26**
27	**28** ***Performance Task 1 Due***	**29**	**30**			

OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6 *Assessment: Unit 1 Assessment 2*	7 *Performance Task 2 Due*	8	9	10
11	12 COLUMBUS DAY District Closed	13	14	15 *Assessment: Unit 1 Assessment 3*	16 *Performance Task 3 Due*	17
18	19 Solidify Unit 1 Concepts	20 Solidify Unit 1 Concepts	21 Solidify Unit 1 Concepts	22 12:30 pm Student Dismissal	23 Unit 1 Complete	24
25	26	27	28	29 12:30 pm Student Dismissal	30	31

Math Background

In this Unit, students study symmetry and transformations. They connect these concepts to congruence and similarity. Symmetry and transformations have actually been studied in the Grade 7 Unit Stretching and Shrinking. In this Unit, students learn to recognize and make designs with symmetry, and to describe mathematically the transformations that lead to symmetric designs. They explore the concept and consequences of congruence of two figures by looking for symmetry transformations that will map one figure exactly onto the other.

In the first Investigation, students learn to recognize designs with symmetry and to identify lines of symmetry, centers and angles of rotation, and directions and lengths of translations.

Once students learn to recognize symmetry in given designs, they can make their own symmetric designs. Students may use reflecting devices, tracing paper, angle rulers or protractors, and geometry software to help them construct designs.

The concepts of symmetry are used as the starting point for the study of symmetry transformations, also called distance-preserving transformations, rigid motions, or isometries. The most familiar distance-preserving transformations—reflections, rotations, and translations—“move” points to image points so that the distance between any two original points is equal to the distance between their images. The informal language used to specify these transformations is slides, flips, and turns. Some children will have used this language and will have had informal experiences with these transformations in the elementary grades.

The question of proving whether two figures are congruent is explored informally. An important question is what minimum set of equal measures of corresponding sides and/or angles will guarantee that two triangles are congruent. It is likely that students will discover the following triangle congruence theorems that are usually taught and proved in high school geometry. This engagement with the ideas in an informal way will help make their experience with proof in high school geometry more understandable. Symmetry and congruence give us ways of reasoning about figures that allow us to draw conclusions about relationships of line segments and angles within the figures.

In Investigation 3, we return to transformations and look at transformations of figures on a coordinate plane.

In very informal ways, students explore combinations of transformations. In a few instances in the ACE Extensions, students are asked to describe a single transformation that will give the same result as a given combination. For example, reflecting a figure in a line and then reflecting the image in a parallel line has the same result as translating the figure in a direction perpendicular to the reflection lines for a distance equal to twice the distance between the lines.

In everyday language the word similar is used to suggest that objects or ideas are alike in some way. In mathematical geometry, the word similar is used to describe figures that have the same shape but different size. You can formally define the term with the concepts and language of transformations.

In everyday language, the word dilation usually suggests enlargement. However, in standard mathematical usage, the word dilation is used to describe either an enlargement or stretching action (scale factor greater than 1) or a reduction or shrinking action (scale factor between 0 and 1).

Math Background (Continued)

You can use the relationships between corresponding parts of similar triangles to deduce unknown side lengths of one of the triangles. This application of similarity is especially useful in situations where you cannot measure a length or height directly.

PARCC Assessment Evidence Statements

CCSS	Evidence Statement	Clarification	Math Practices	Calculator?
8.G.1a	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length	i) Tasks do not have a context.	3, 5, 8	No
8.G.1b	Verify experimentally the properties of rotations, reflections, and translations: b. Angles are taken to angles of the same measure.	i) Tasks do not have a context.	3, 5, 8	No
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	i) Tasks do not have a context. ii) Tasks do not reference similarity (this relationship will be assessed in 8.C.3.2) iii) Tasks should not focus on coordinate Geometry Tasks should elicit student understanding of the connection between congruence and transformations i.e., tasks may provide two congruent figures and require the description of a sequence of transformations that exhibits the congruence or tasks may require students to identify whether two figures are congruent using a sequence of transformations.	2, 7	No
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates	i) Tasks have “thin context” or no context ii) Tasks require the use of coordinates in the coordinate plane iii) For items involving dilations, tasks must state the center of dilation.	2, 3, 5	No
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	i) Tasks do not have a context. ii) Tasks do not reference congruence (this relationship will be assessed in 8.C.3.2) iii) Tasks should not focus on coordinate Geometry	2, 7	No

		iv) Tasks should elicit student understanding of the connection between similarity and transformations i.e., tasks may provide two similar figures and require the description of a sequence of transformations that exhibits the similarity or tasks may require students to identify whether two figures are similar using a sequence of transformations. v) Tasks do not require students to indicate a specified scale factor. vi) Similarity should not be obtained through the proportionality of corresponding sides		
8.C.3.2	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	None	3, 5, 6	Yes
8.C.3.3	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.5	None	3, 5, 6	Yes
8.C.5.2	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	None	2, 3, 5	Yes
8.C.5.3	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.B	None	2, 3, 5	Yes

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them - Students interpret geometric happenings with multiple transformations - Students reason about angles given information in a problem or visual
2	Reason abstractly and quantitatively - Students interpret similarity and congruence with or without a coordinate grid - Students interpret transformations
3	Construct viable arguments and critique the reasoning of others - Students learn to determine accuracy in transformations and understanding angle and congruency relationships
4	Model with mathematics - Students are able to recognize a specific set of transformations to map one figure onto its congruent image. - Students prove that all triangles have interior angles with a sum of 180 degrees
5	Use appropriate tools strategically - Students use coordinate grids, given angle measures, and given directions to prove congruence and similarity
6	Attend to precision - Students are careful about using transformations to prove congruence and similarity - Students use knowledge of parallel lines to reason appropriately
7	Look for and make use of structure - Students discover how rotations, translations, reflections, and dilations can be done without a coordinate grid from understanding the changes that happen in each of the quadrants after a rotation
8	Look for and express regularity in repeated reasoning - Students are able to prove congruence using transformations

Vocabulary

Term	Definition
Angle of Rotation	Then number of degrees that a figure rotates
Center of Rotation	A fixed point about which a figure rotates.
Congruent Figures	Figures that have the same size and shape (same angles and sides)
Corresponding Sides	Sides that have the same relative positions in geometric figures
Corresponding Angles	Angles that have the same relative positions in geometric figures
Dilation	A transformation that grows a geometric figure by a scale factor. A scale factor less than 1 will shrink the figure, and scale factor greater than 1 will enlarge the figure
Reflection	A transformation that “flips” a figure over a line of reflection
Reflection Line	A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.
Rotation	A transformation that turns a figure about a fixed point through a given angle and a given direction.
Scale Factor	The ratio of any two corresponding lengths of the sides of two similar figures.
Similar Figures	Two figures are similar if one is an image of the other under a sequence of transformations that includes a dilation. If the scale factor is greater than 1, the side lengths of the image are greater than the corresponding side lengths of the original figure. If the scale factor is less than 1, the side lengths of the image are less than the corresponding side lengths of the original figure. If the scale factor is equal to 1, then the two figures are congruent.
Transformation	The mapping or movement of all the points of a figure in a plane according to a common operation.
Translation	A transformation that “slides” each point of a figure the same distance in the same direction
Transversal	A line that crosses two or more lines

Potential Student Misconceptions

- Students confuse the rules for transforming two-dimensional figures because they rely too heavily on rules as opposed to understanding what happens to figures as they translate, rotate, reflect, and dilate. It is important to have students describe the effects of each of the transformations on two-dimensional figures through the coordinates but also the visual transformations that result.

- By definition, congruent figures are also similar. It is incorrect to say that similar figures are the same shape, just a different size. This thinking leads students to misconceptions such as that all triangles are similar. It is important to add to that definition, the property of proportionality among similar figures.

- Students do not realize that congruent shapes can be "checked" by placing one atop the other.

- Students may think the terms translation, reflection and rotation are interchangeable.

- Students do not conceptualize that clockwise and counterclockwise can be the same depending on the angle specified.

- Students are unfamiliar with the symbolic notation used to identify angles and their measures.

- Students interchange the terms supplementary and complementary.

- Students believe that all adjacent angles are either complementary or supplementary.

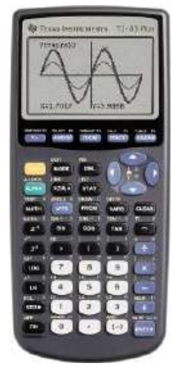
- Students may think vertical angles, alternate exterior, alternate exterior angles sum is 180° like adjacent angles on transversals.

- Students may think that angle properties are the same for transversals through non parallel lines.

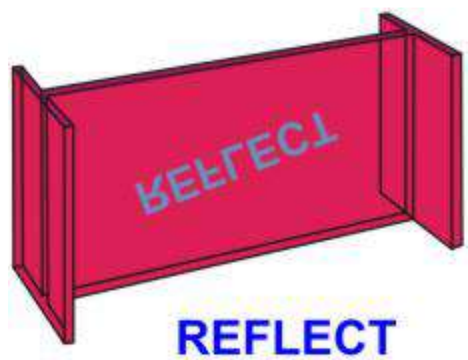
Teaching Multiple Representations

CONCRETE REPRESENTATIONS

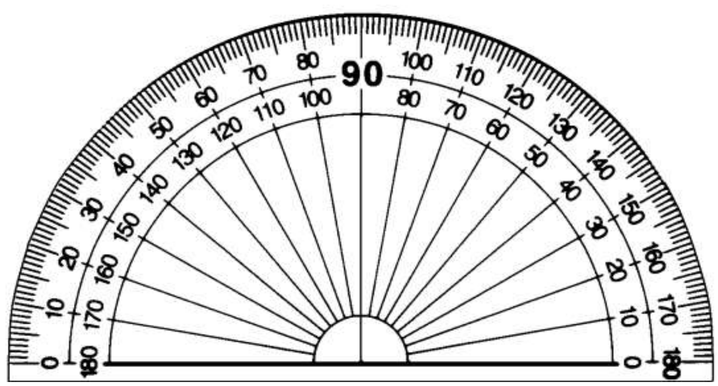
- Grid/Graph Paper
- Graphing Calculator



- Mirrors
- Transparent Reflection Tools

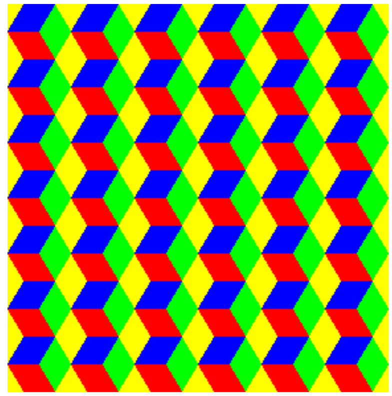


- Protractors

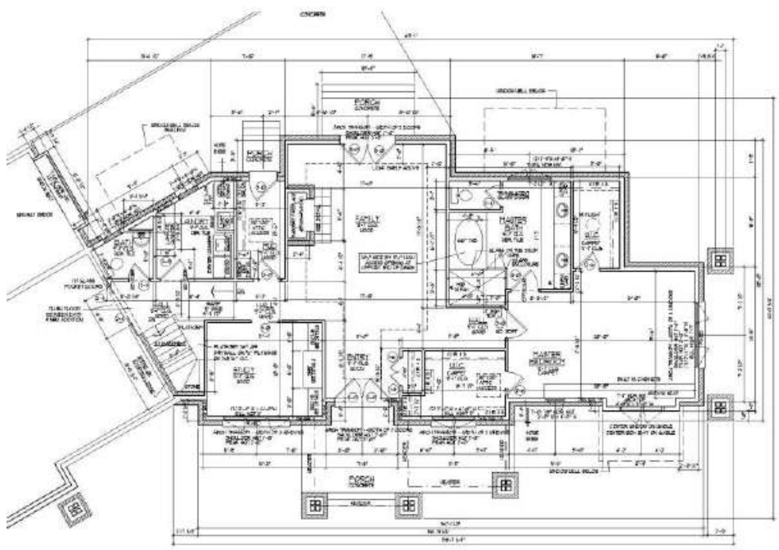


PICTORIAL REPRESENTATIONS

- Tessellations



- Architecture



ABSTRACT REPRESENTATIONS

- Describing similarity transformations in words and with coordinate rules
- Performing and analyzing transformations in designs

Assessment Framework

Unit 1 Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 1 Diagnostic (Beginning of Unit)	7.G.A.1, 7.G.A.2, 7.G.B.5, 8.G.A.1, 8.G.A.2	½ Block	Individual	No
Unit 1 Check Up 1 (After Investigation 1) *CMP3*	8.G.A.1, 8.G.A.1a, 8.G.A.1b, 8.G.A.1c	½ Block	Individual	Yes
Unit 1 Assessment 1 (After Investigation 2) *Model Curriculum*	8.G.A.1, 8.G.A.2	1 Block	Individual	Yes
Unit 1 Assessment 2 (After Investigation 3) *Model Curriculum*	8.G.A.3, 8.G.4	1 Block	Individual	Yes
Unit 1 Assessment 3 (Conclusion of Unit) *Model Curriculum*	8.G.A.5	½ Block	Individual	Yes
Unit 1 Partner Quiz (Optional) *CMP3*	8.G.A.2, 8.G.A.1a, 8.G.A.1b	½ Block	Group	Yes
Unit 1 Check Up 2 (Optional) *CMP3*	8.G.A.3, 8.G.A.1c, 8.G.A.5	½ Block	Individual	Yes

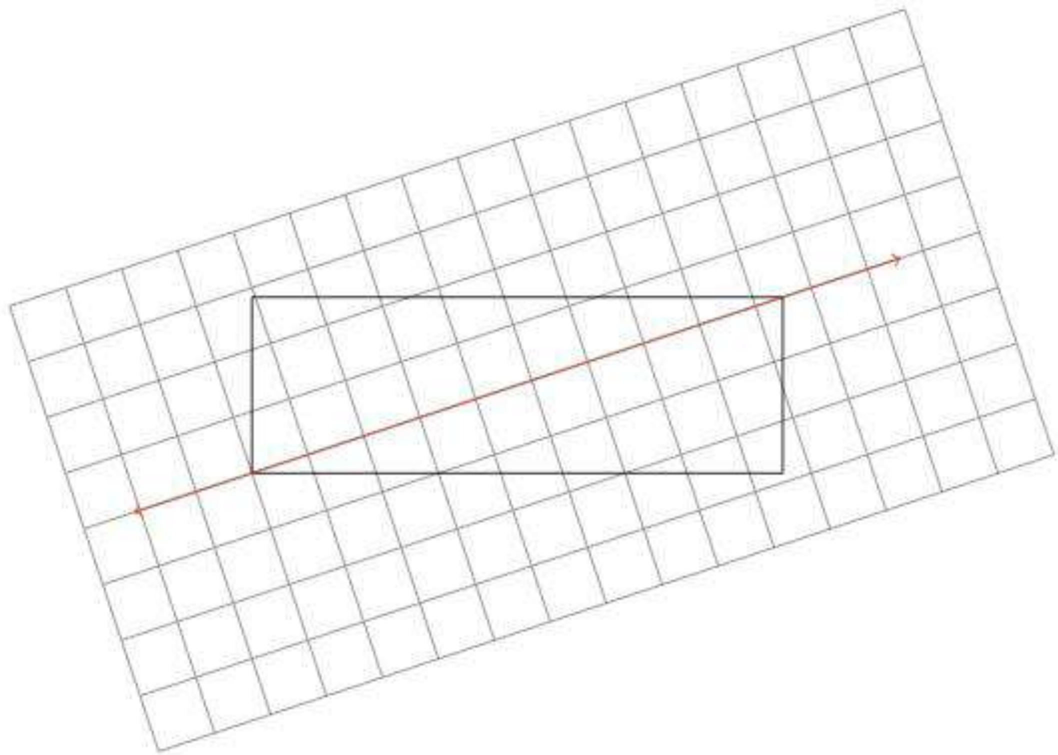
Unit 1 Performance Assessment Framework				
Task	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 1 Performance Task 1 (After Investigation 2) *Reflecting a Rectangle Over a Diagonal*	8.G.A.1	½ Block	Group	Yes; Rubric
Unit 1 Performance Task 2 (After Investigation 3) *Triangle Congruence with Coordinates*	8.G.A.2, 8.G.A.3, 8.F.A.1	1 Block	Group	Yes; Rubric
Unit 1 Performance Task 3 (After Investigation 4) *Effects of Dilations on Length, Area, and Angles*	8.G.A.3, 8.F.A.1	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 1 Performance Task Option 1 (Optional)	8.G.A.4	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 1 Performance Task Option 2 (Optional)	8.G.A.5	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

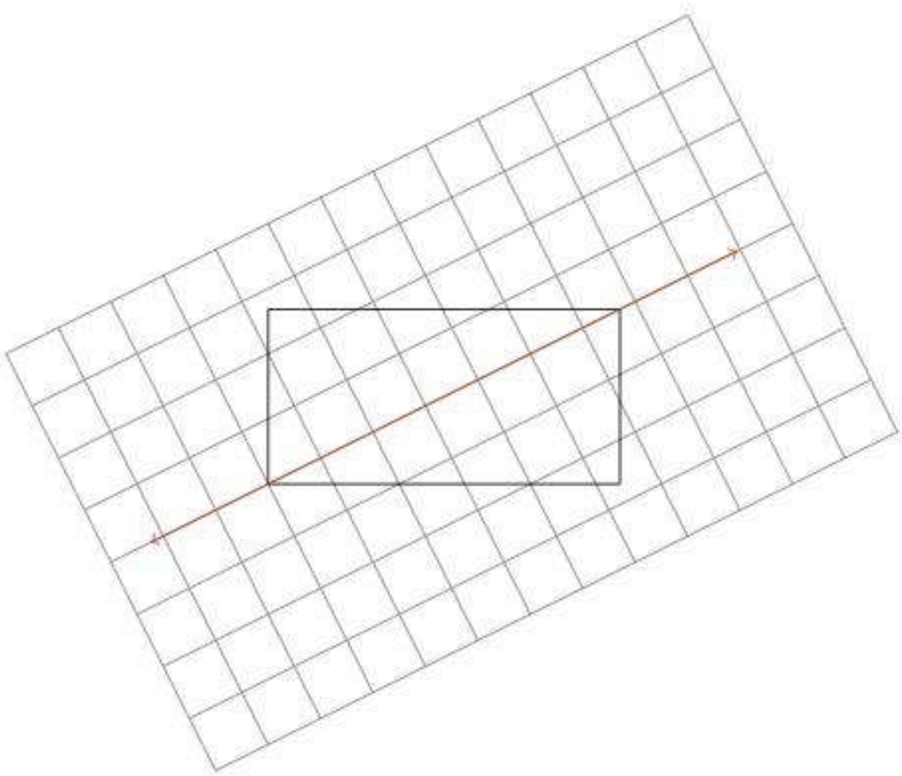
Performance Task 1:
Reflecting a Rectangle over a Diagonal (8.G.A.1)

a. Each picture below shows a rectangle with a line through a diagonal. For each picture, use the grid in the background to help draw the reflection of the rectangle over the line.

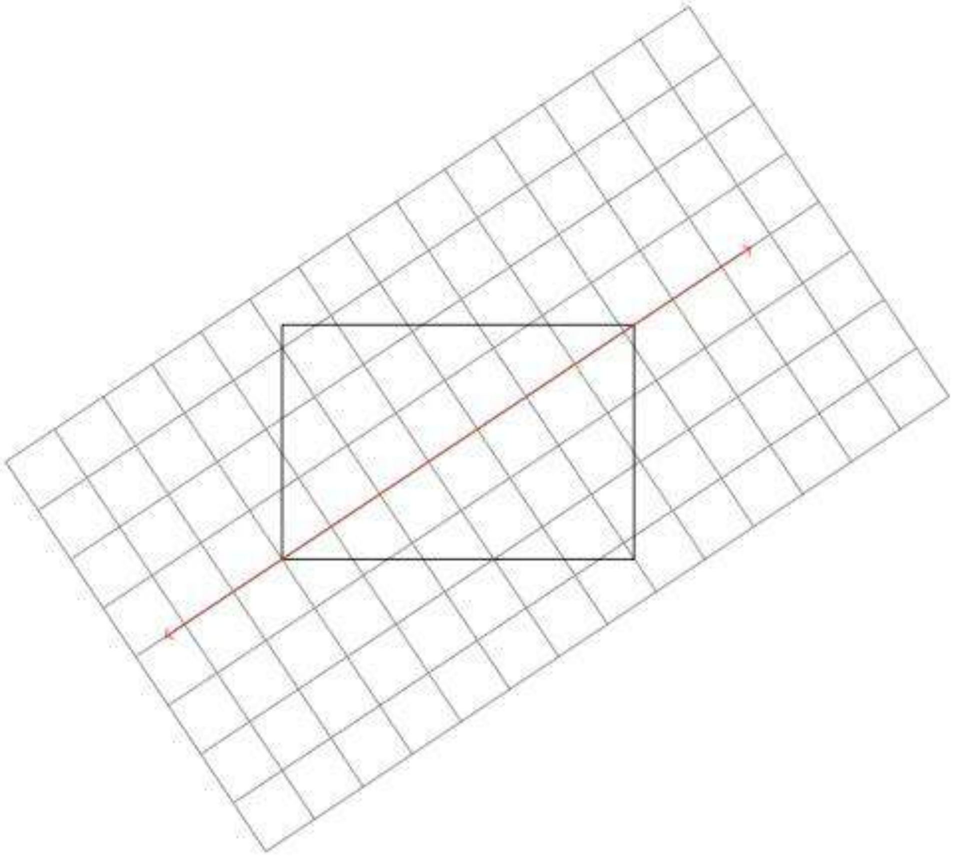
i.



ii.



iii.



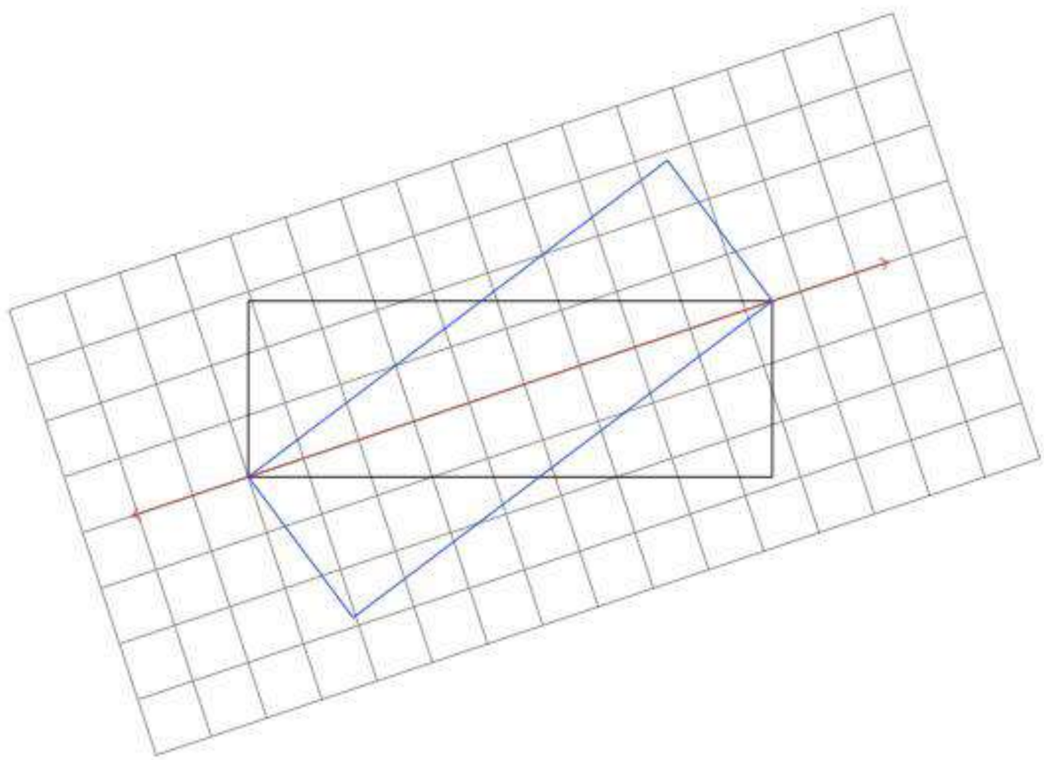
b. Suppose you have a rectangle where the line through the diagonal is a line of symmetry. Using what you know about reflections, explain why the rectangle must be a square.

Solution:

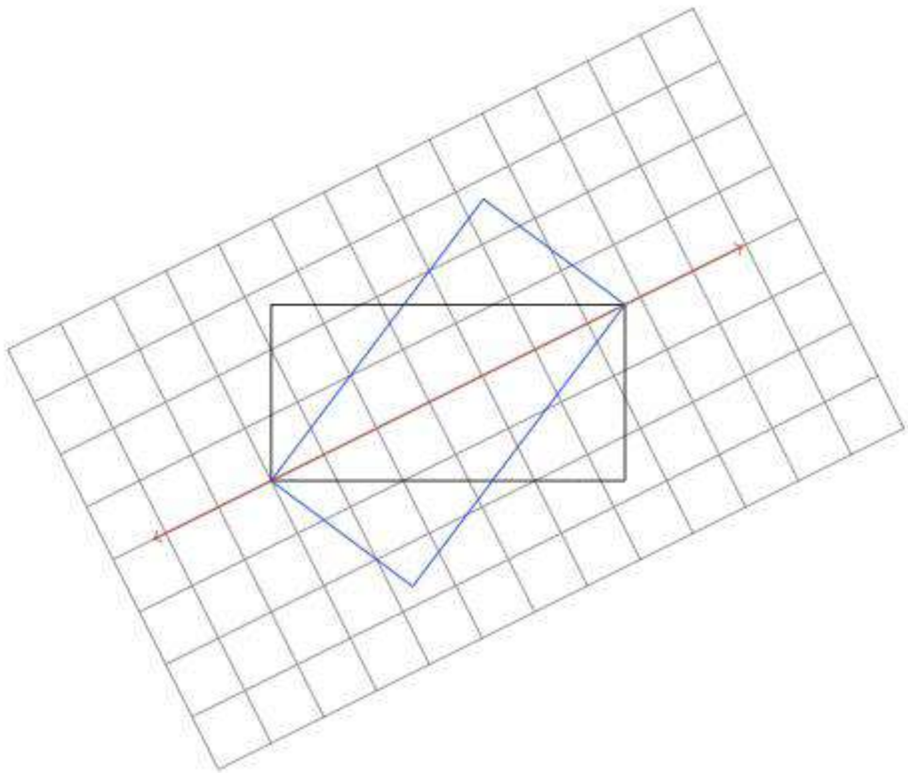
a. The reflections of each rectangle are pictured below in blue. In each case the reflected image shares two vertices with the original rectangle. This is because the line of reflection passes through two vertices and reflection over a line leaves all points on the line in their original position.
Notice that the reflected rectangle is, in each case, still a rectangle of the same size and shape as the original rectangle. Also notice that as the length and width of the rectangles become closer to one another, the two vertices are getting closer and closer to the vertices of the original rectangle. The case where the length and width of the rectangle are equal is examined in part (b).

For each of the pictures below, the same type of reasoning applies: the red line can be thought of as the x-axis of the grid. Reflecting over the x-axis does not change the x-coordinate of a point but it changes the sign of the y-coordinate. In the picture for (i), the rectangle vertex above the red line is just under three boxes above the red line. So its reflection is just under three units below the red line, with the samex coordinate. Similarly in the picture for part (iii), the vertex of the rectangle below the red line is a little more than 3 boxes below the red line: so its reflection will be on the same line through the x-axis but a little over three boxes *above* the red line. Similar reasoning applies to all vertices in the three pictures. Because reflections map line segments to line segments, knowing where the vertices of the rectangles map is enough to determine the reflected image of the rectangle.

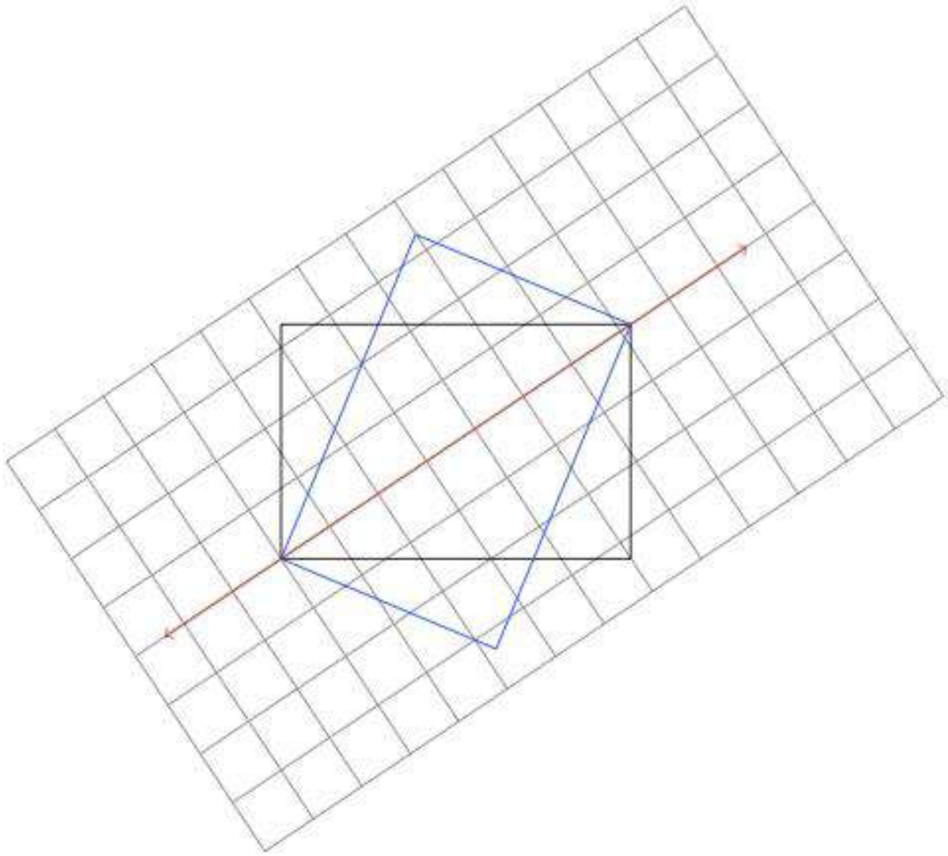
i.



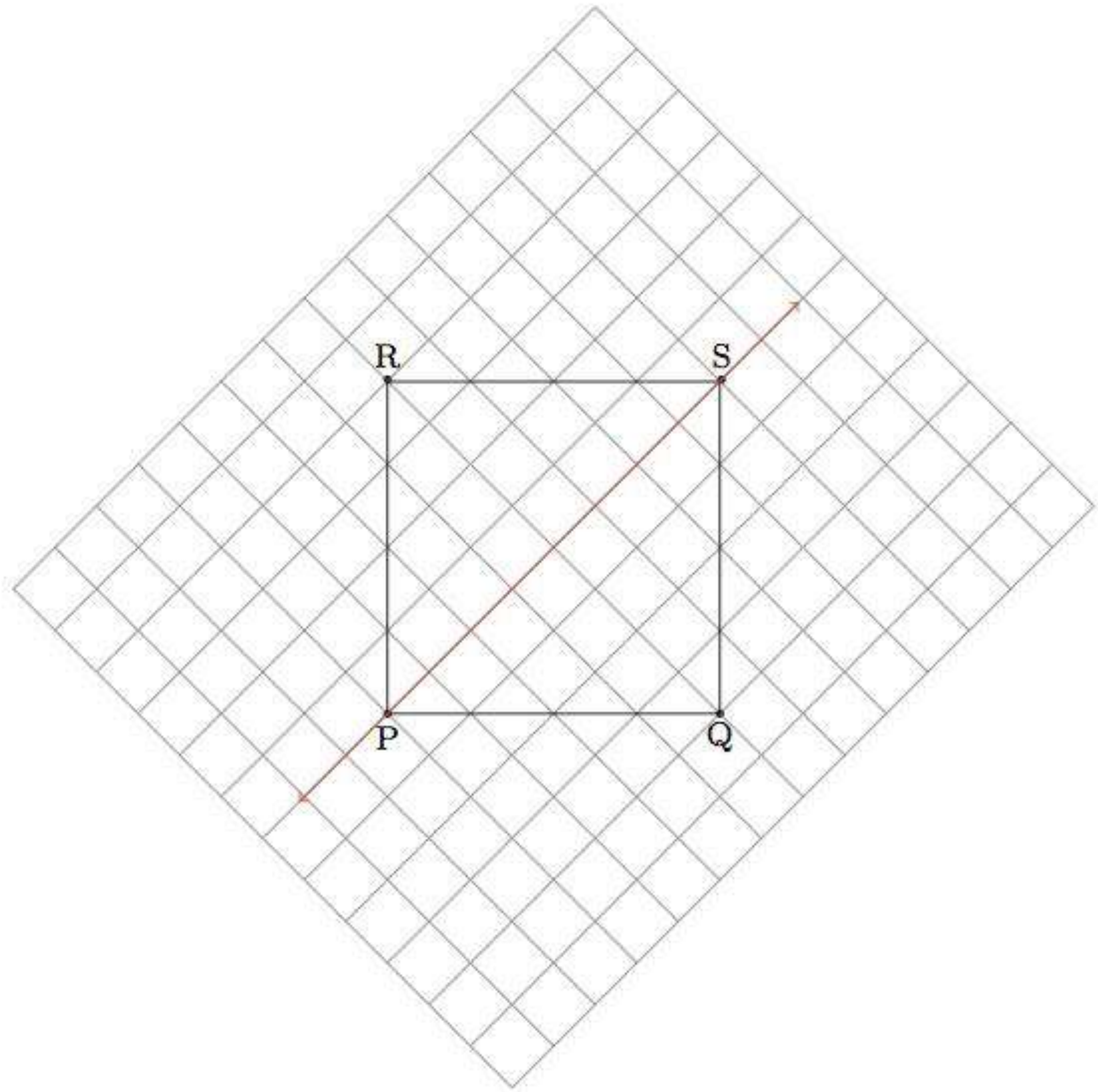
ii.



iii.



b. Below is a picture of a quadrilateral ***PQRS*** with the line $\overleftrightarrow{PS}$ containing the diagonal colored red. We assume ***PQRS*** is mapped to itself by reflection over the line $\overleftrightarrow{PS}$.



Since ***P*** and ***S*** are on the line of reflection, they are mapped to themselves by the reflection. So the fact that the rectangle is mapped to itself means that ***Q*** maps to ***R*** and ***R*** to ***Q***. Since opposite sides of a rectangle are congruent we know that |***RS***|=|***PQ***| and |***PR***|=|***QS***|. Reflection about line $\overleftrightarrow{PS}$ preserves line segments and sends segment $\overline{PR}$ to segment $\overline{PQ}$. Reflections preserve line segment lengths so |***PQ***|=|***PR***|. Putting our equalities together we find

|***RS***|=|***PQ***|=|***PR***|=|***QS***|.

Since ***PQRS*** is a rectangle whose four sides have the same length, it must be a square.

Unit 1 Performance Task 1 PLD Rubric

Reflecting a Rectangle over a Diagonal

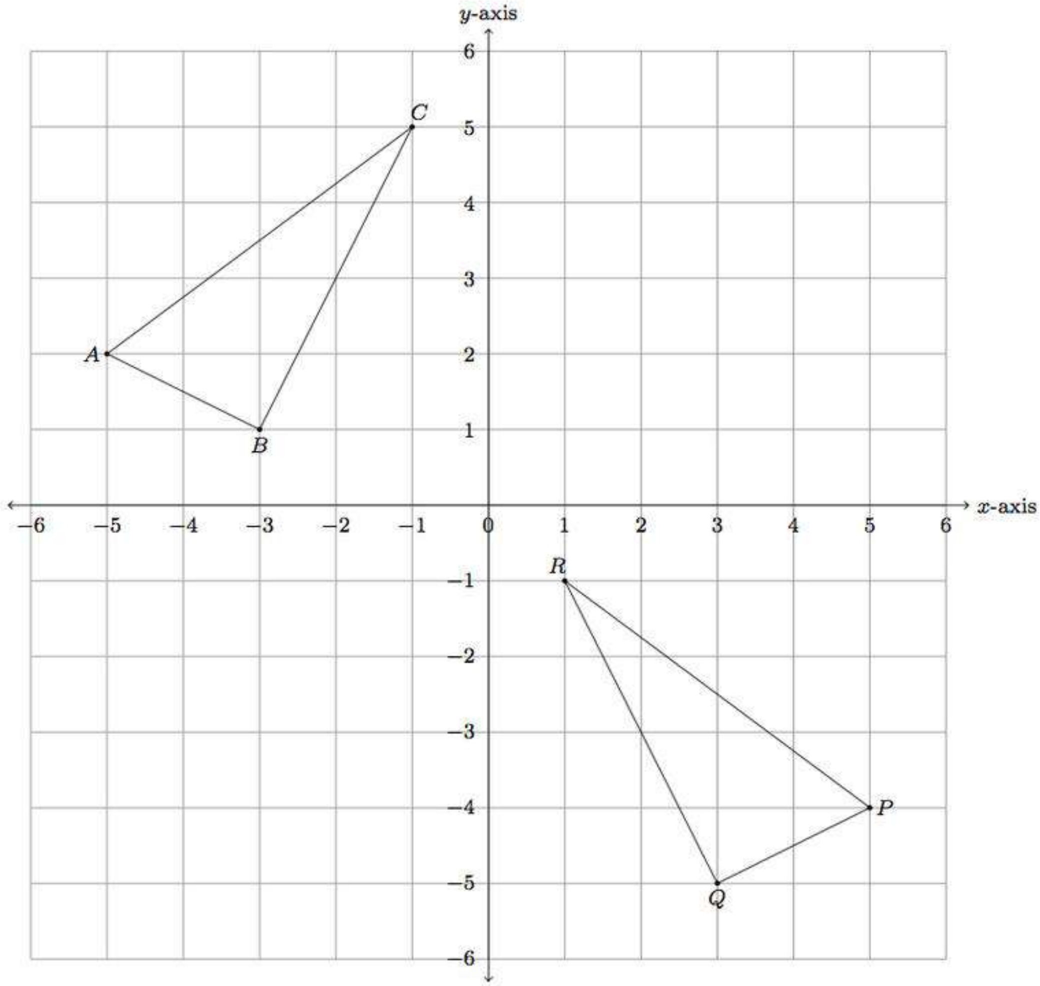
SOLUTION

- **a. i) Student reflects the rectangle through the diagonal line of symmetry.**
- **a. ii) Student reflects the second rectangle through the diagonal line of symmetry.**
- **a. iii) Student reflects the third rectangle through the diagonal line of symmetry.**
- **b. Student correctly explains a rectangle with a diagonal line of symmetry must be a square because the opposite vertices map to themselves by reflection.**

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • majr calculation errors • partial justification of a conclusion	The student shows no work or justification.

Performance Task 2:
Triangle Congruence with Coordinates (8.G.A.2)

Triangles *ABC* and *PQR* are shown below in the coordinate plane:



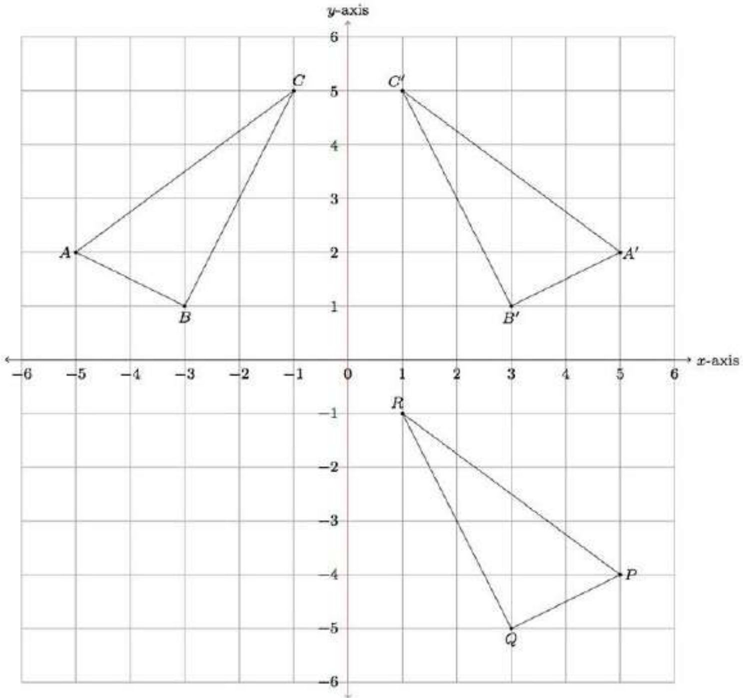
a. Show that △*ABC* is congruent to △*PQR* with a reflection followed by a translation.

b. If you reverse the order of your reflection and translation in part (a) does it still map △*ABC* to △*PQR*?

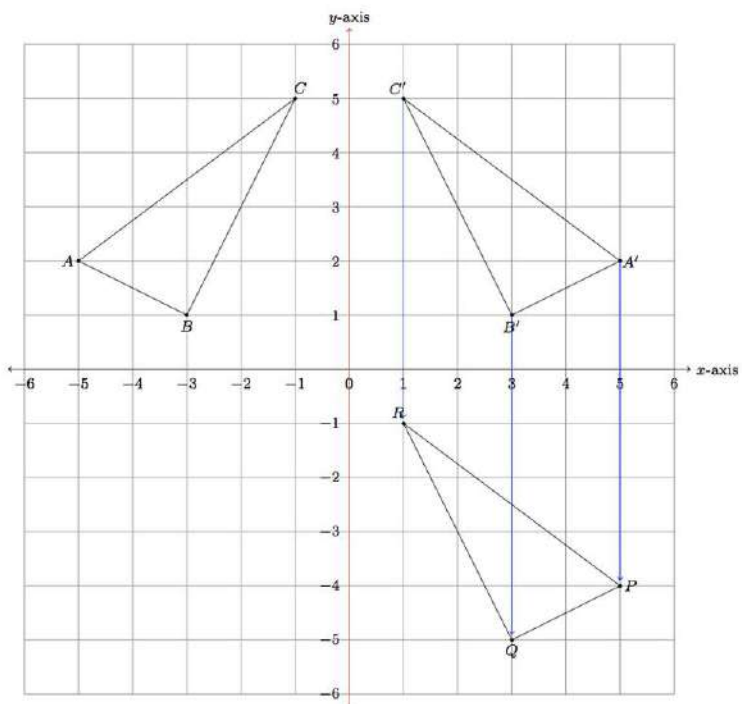
c. Find a second way, different from your work in part (a), to map △*ABC* to △*PQR* using translations, rotations, and/or reflections

Solution:

a. Below the y-axis is shaded red and triangle *ABC* is reflected over the *y*-axis. The image of this reflection is triangle *A′B′C′*. Reflecting about the *y*-axis leaves the *y*-coordinate of each point the same and switches the sign of the *x*-coordinate.

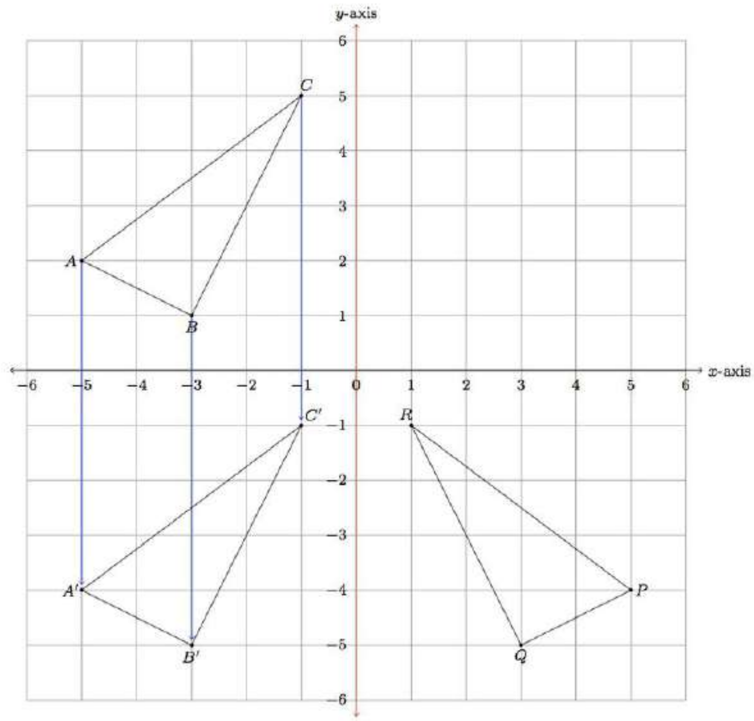


So, for example, $A = (-5,2)$ so $A' = (5,2)$. We can now see that translating triangle *A′B′C′* down by 6 units puts it on top of triangle *PQR*:

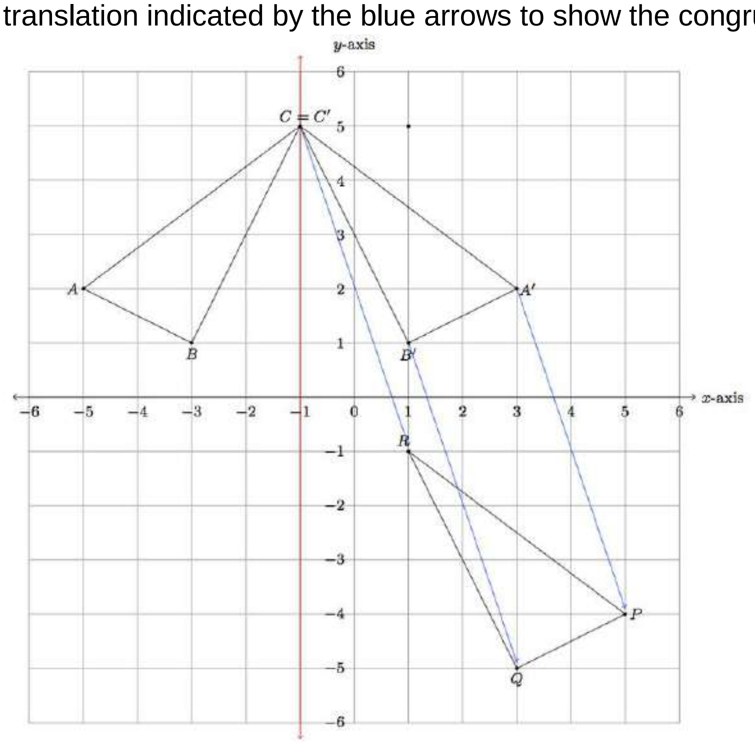


To find the coordinates after applying this translation, the *x*-coordinate stays the same and we subtract 6 from the *y*-coordinate of each point.

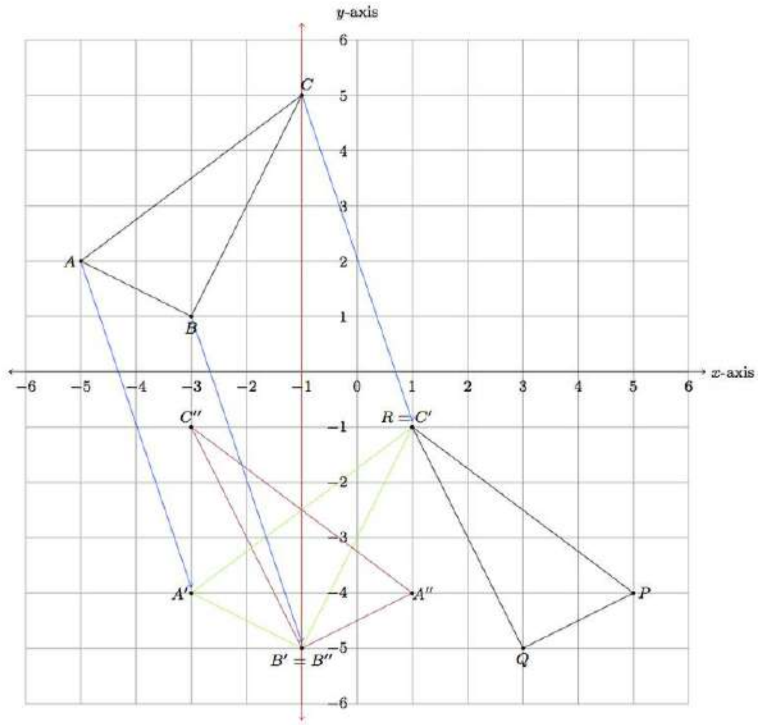
b. The answer here will depend on which reflection and translation have been chosen in part (a). For the reflection and translation chosen above, we reverse the order by first translating △*ABC* by 6 units downward and then reflecting over the *y*-axis. Below, the translated triangle is triangle *A′B′C′* and its reflection over the *y*-axis is △*PQR*:



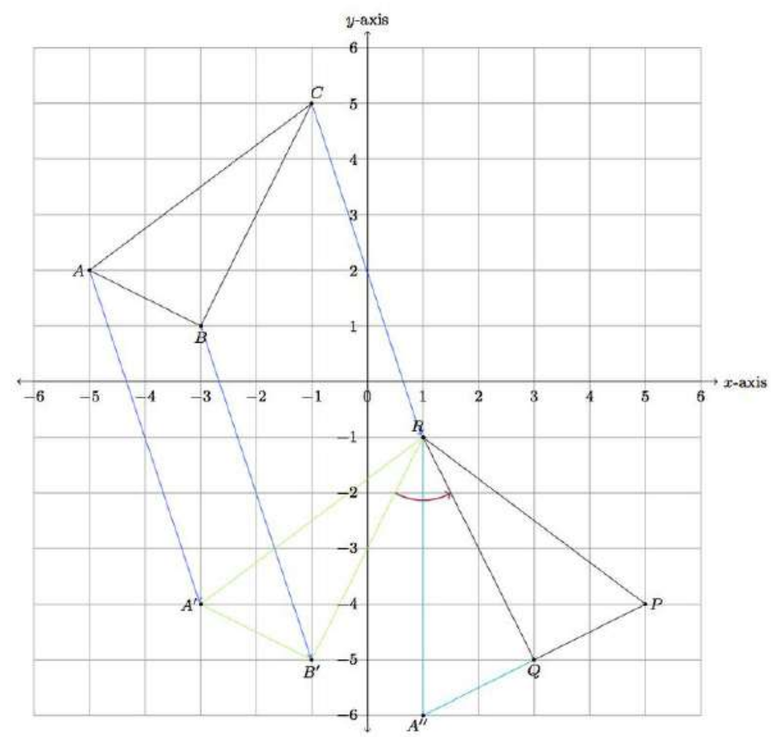
Below is a different reflection through the vertical line through vertex *A*, which can be followed by the translation indicated by the blue arrows to show the congruence of △*ABC* with △*PQR*:



Unlike in the previous case, if we perform the translation first, giving the green triangle *A′B′C′,* and then the reflection, giving the purple triangle *A″B″C″,* this does not produce the triangle *PQR*. So in this case, performing the translation and reflection in a different order produces a different outcome.



c. One way to show the triangle congruence would be to align one vertex at a time. Graphically this is shown below:



First a translation is used to move *C* to *R* with the new triangle shown in green. If *B′* is the image of *B* under this translation, then a rotation, by the directed angle indicated in purple, moves *B′* to *Q*: the triangle after this transformation is indicated in blue, sharing one side with triangle *PQR*. If *A″* is the image of *A* after the translation and rotation, then a reflection across $\overline{QR}$ moves *A″* to *P*.

Unit 1 Performance Task 2 PLD Rubric

Triangle Congruence with Coordinates

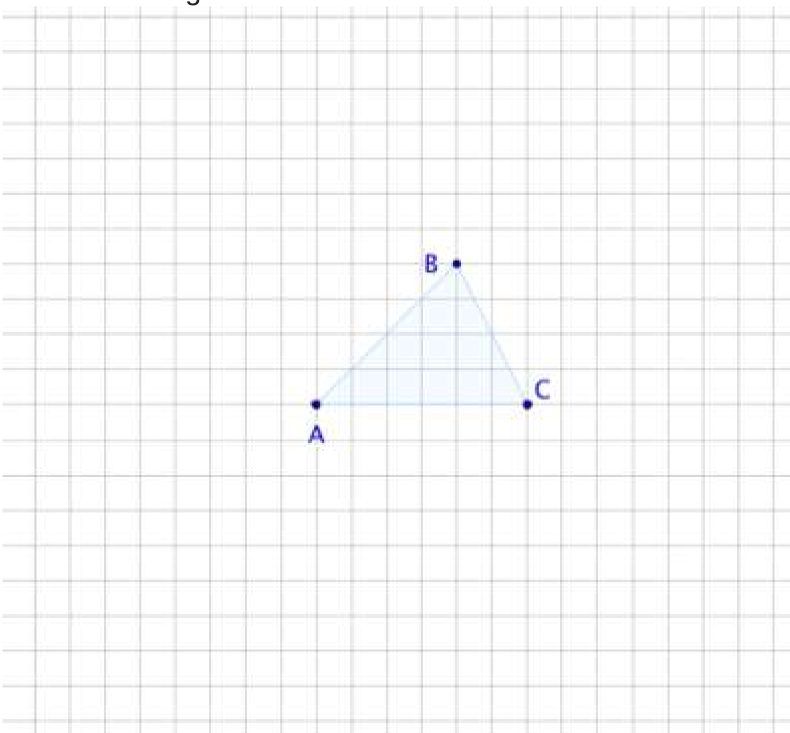
SOLUTION

a. **Student proves the two triangles are congruent with a sequence of reflections and translations.**
b. **Student accurately justifies their reason for yes or no based on the reverse sequence from part (a).**
c. **Student finds a different sequence of transformations than the sequence used in part (a) to prove the two triangles are congruent.**

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • majr calculation errors • partial justification of a conclusion	The student shows no work or justification.

Performance Task 3:
Effects of Dilations on Length, Area, and Angles (8.G.A.3)

Consider triangle ABC.



a. Draw a dilation of ABC with:

- Center A and scale factor 2. 3 points
- Center B and scale factor 3. 3 points
- Center C and scale factor 12. 3 points

b. For each dilation, answer the following questions:

- By what factor do the base and height of the triangle change? Explain. 3 points
- How do the areas of the triangles change in terms of scale factor? Explain. 3 points
- How do the angles of the scaled triangle compare to the original? Explain. 3 points

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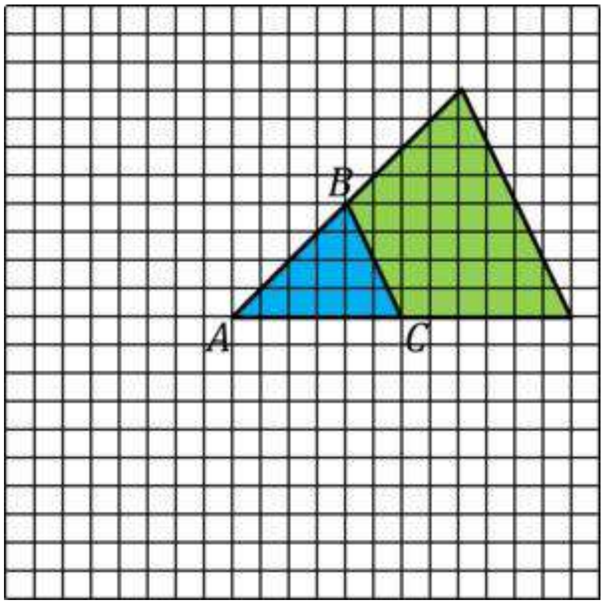
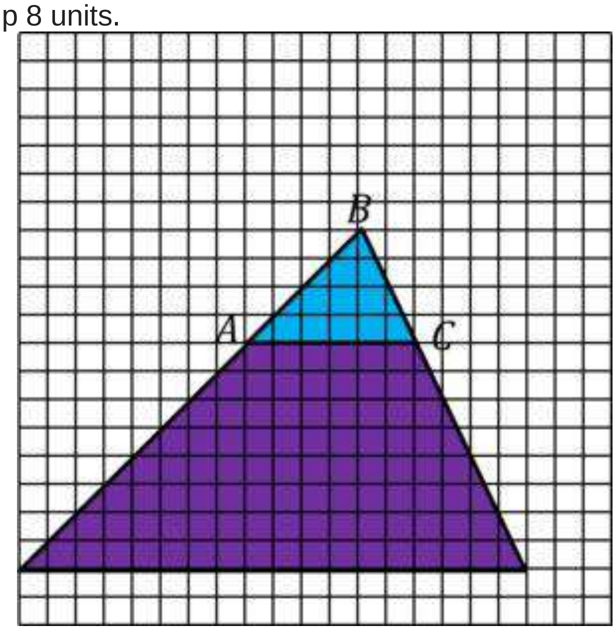
Solution:

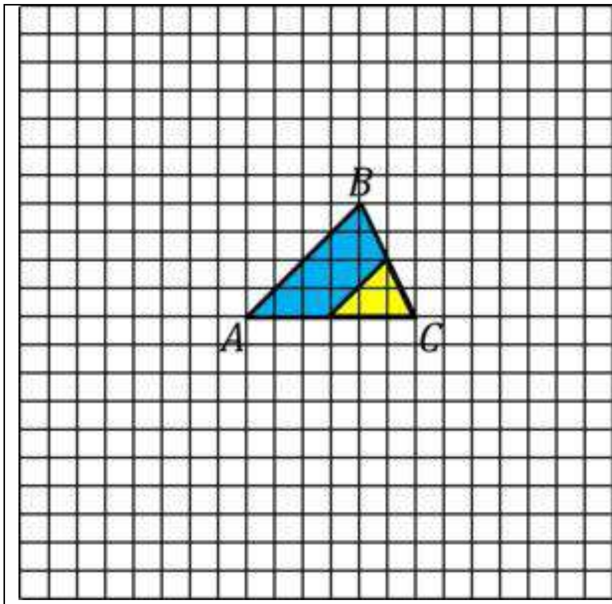
Part a. the three dilations are shown below along with explanations for the pictures: 5 points



The dilation with center A and scale factor 2 doubles the length of segments AB and AC. We can see this explicitly for AC. For AB, this segment goes over 6 units and up 4 so its image goes over 12 units and up 8 units.



The dilation with center B and scale factor 3 increases the length of AB and AC by a factor of 3. The point B does not move when we apply the dilation but A and C are mapped to points 3 times as far from B on the same line.



The scale factor of 1/2 makes a smaller triangle. The center of this dilation (also called a contraction in this case) is C and the vertices A and B are mapped to points half the distance from A on the same line segments.

Part b:

- When the scale factor of 2 is applied with center A the length of the base doubles from 6 units to 12 units. This is also true for the height which was 4 units for △ABC but is 8 units for the scaled triangle. Similarly, when the scale factor of 3 is applied with center B , the length of the base and the height increase by a scale factor of 3 and for the scale factor of 1/2 with center C, the base and height of △ABC are likewise scaled by 1/2.
- The area of a triangle is the base times the height. When a scale factor of 2 with center A is applied to △ABC, the base and height each double so the area increases by a factor of 4: the area of △ABC is 12 square units while the area of the scaled version is 48 square units. Similarly, if a scale factor of 3 with center B is applied then the base and height increase by a factor of 3 and the area increased by a factor of 9. Finally, if a scale factor of 1/2 with center C is applied to △ABC, the base and height are cut in half and so the area is multiplied by 1/4.
- The angle measures do not change when the triangle is scaled. For the first scaling, we can see that angle A is common to △ABC and its scaling with center A and scaling factor 2. Angle B is congruent to its scaled image as we can see by translating △ABC eight units to the right and 4 units up. Finally, angle C is congruent to its scaled image as we verify by translating △ABC 8 units to the right.

Unit 1 Performance Task 3 PLD Rubric

Effects of Dilations on Length, Area, and Angles

SOLUTION

- **Student Indicates dilation with center A and scale factor 2 doubles the length**
- **Student indicates dilation with center B and scale factor 3 increases the length of AB and AC by a factor of 3.**
- **Student Indicates the scale factor of 1/2 makes a smaller triangle.**
- **Student indicates that the original triangle ABC has a height of 4 units and base of 6 units. Dilation by factor of 2 increases the base to 12 units and height to 8units. AND Dilation by factor of 3 increases the height base to 18 units and height to 12 units. And dilation by the factor of ½ decreases the height 2 units and base to 3 units**
- **The student indicates that the area of the Original triangle is 12 square units but after the dilation by factors of 2 the height and base doubles and the Scaled area is 48 square units. Therefore, the area increases by a factor of 2^2=4. When the triangle is dilated by the factor of 3 then the area of scaled triangle is increases by a factor of 3^2=9. When the triangle is dilated by the factor of 1/2 then the area of scaled triangle is increases by a factor of $(1/2)^2$=(1/4)**
- **Student indicates The angle measures do not change when the triangle is scaled.**

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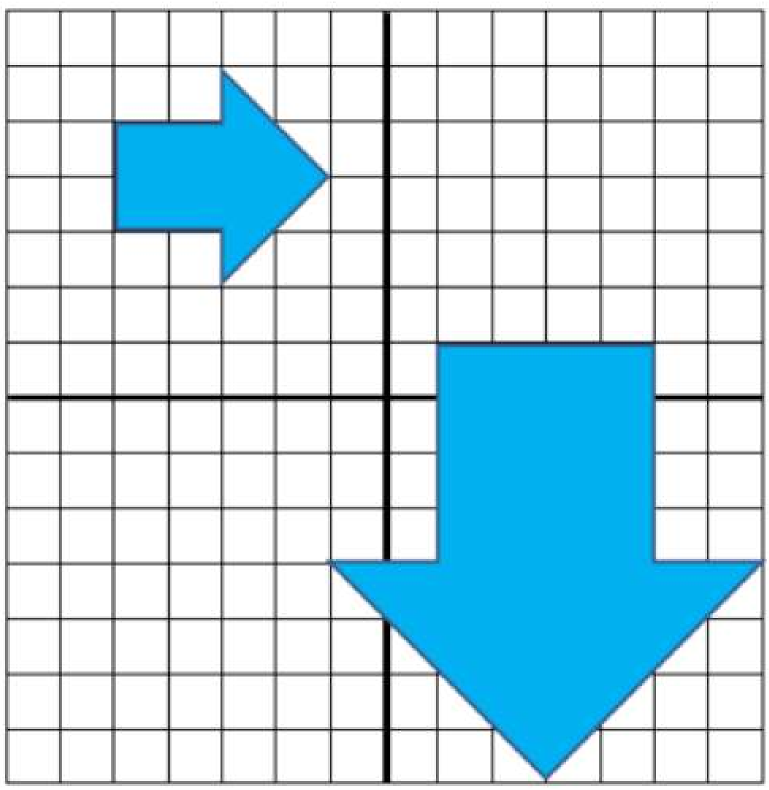
Unit 1 Performance Task Option 1

Are They Similar? (8.G.A.4)

Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar.

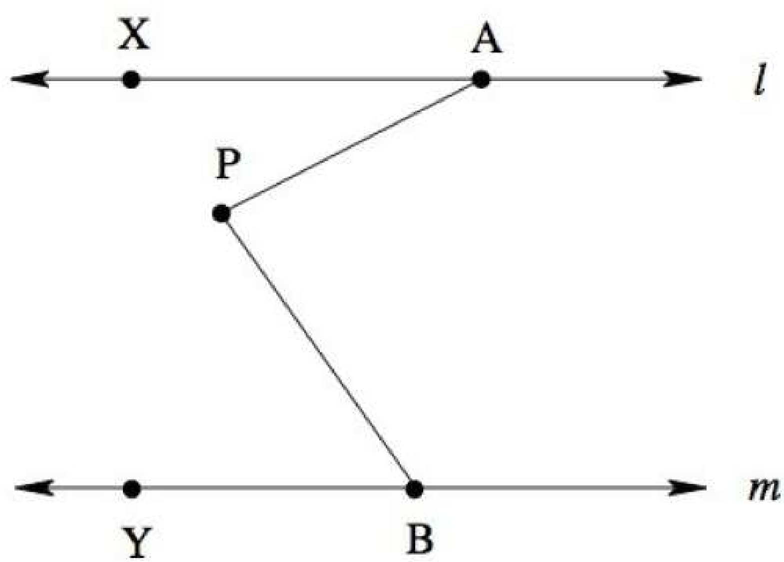


The intersection of the dark lines on the coordinate plane represents the origin (0,0) in the coordinate plane.

Unit 1 Performance Task Option 2

Find the Missing Angle (8.G.A.5)

In the picture below, lines l and m are parallel. The measure of angle $\angle PAX$ is 31°, and the measure of angle $\angle PBY$ is 54°. What is the measure of angle $\angle APB$?



Extensions and Sources

Assessment Resources

http://dashweb.pearsoncmg.com
- Online Connected Math 3 Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

Online Resources

http://www.ixl.com/math/grade-8
Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Tracks student progress, video learning, interactive practice

http://www.doe.k12.de.us/assessment/files/Math_Grade_8.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Special Needs designed tasks, assessment resources

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Document Progressions