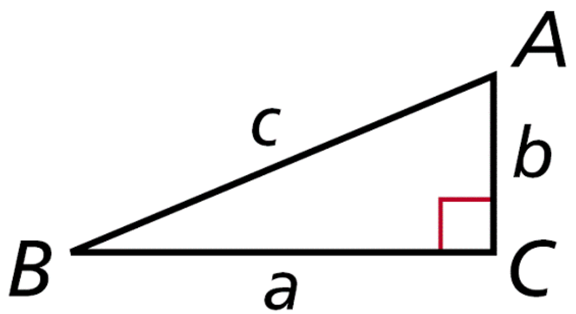
Warm Up

Use ΔABC for Exercises 1–3.



1. If $a = 8$ and $b = 5$, find c. $\sqrt{89}$

2. If $a = 60$ and $c = 61$, find b. 11

3. If $b = 6$ and $c = 10$, find sin B. 0.6

Find AB.

4. $A(8, 10)$, $B(3, 0)$ $5\sqrt{5}$

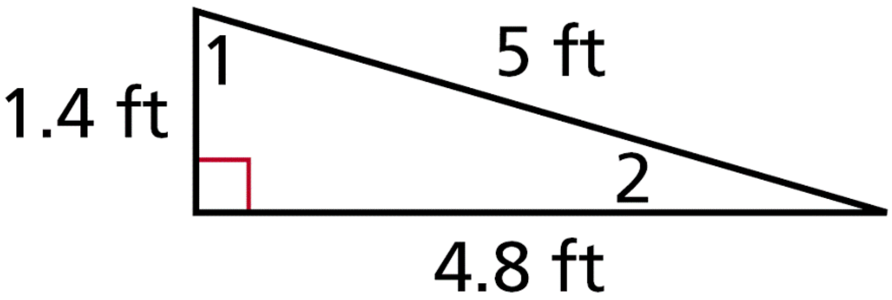
5. $A(1, –2)$, $B(2, 6)$ $\sqrt{65}$

Objective

Use trigonometric ratios to find angle measures in right triangles and to solve real-world problems.

Example 1: Identifying Angles from Trigonometric Ratios

Use the trigonometric ratio $\cos A = \frac{24}{25}$ to determine which angle of the triangle is $\angle A$.



$$\cos A = \frac{\text{adj. leg}}{\text{hyp.}}$$

Cosine is the ratio of the adjacent leg to the hypotenuse.

$$\cos \angle 1 = \frac{1.4}{5} = \frac{7}{25}$$

The leg adjacent to $\angle 1$ is 1.4. The hypotenuse is 5.

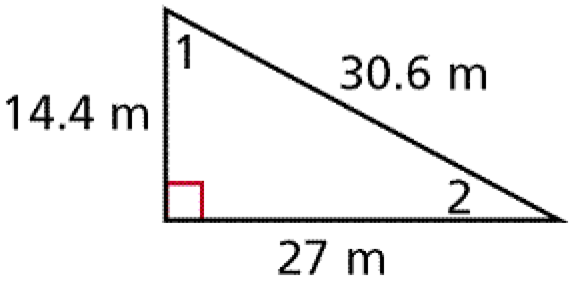
$$\cos \angle 2 = \frac{4.8}{5} = \frac{24}{25}$$

The leg adjacent to $\angle 2$ is 4.8. The hypotenuse is 5.

Since $\cos A = \cos\angle 2$, $\angle 2$ is $\angle A$.

Check It Out! Example 1a

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.



$$\sin A = \frac{8}{17}$$

$$\sin A = \frac{\text{opp. leg}}{\text{hyp.}}$$

Sine is the ratio of the opposite leg to the hypotenuse.

$$\sin\angle 1 = \frac{27}{30.6} = 0.88$$

The leg adjacent to $\angle 1$ is 27. The hypotenuse is 30.6.

$$\sin\angle 2 = \frac{14.4}{30.6} = 0.47$$

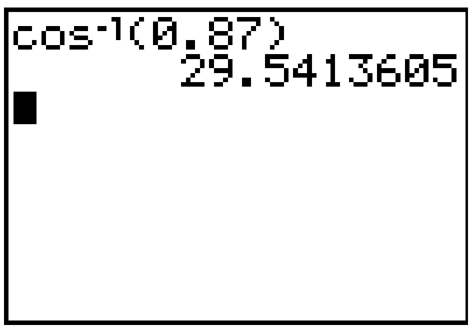
The leg adjacent to $\angle 2$ is 14.4. The hypotenuse is 30.6.

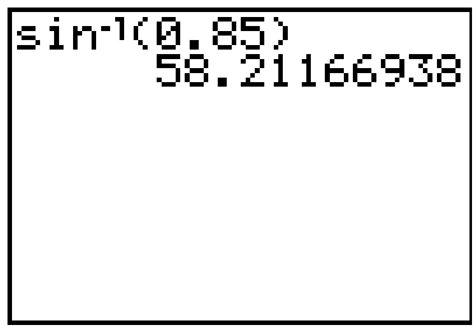
Since $\sin\angle A = \sin\angle 2$, $\angle 2$ is $\angle A$.

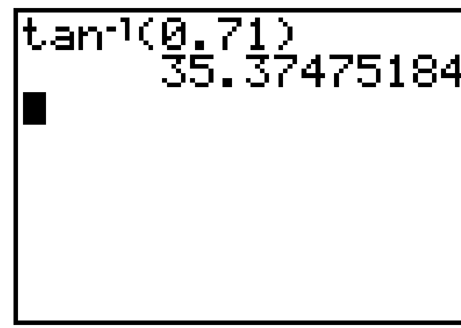
Example 2: Calculating Angle Measures from Trigonometric Ratios

Use your calculator to find each angle measure to the nearest degree.

A. $\cos^{-1}(0.87)$ **B.** $\sin^{-1}(0.85)$ **C.** $\tan^{-1}(0.71)$



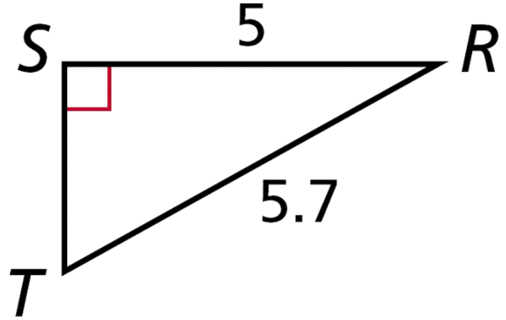




$\cos^{-1}(0.87) \approx 30°$ $\sin^{-1}(0.85) \approx 58°$ $\tan^{-1}(0.71) \approx 35°$

Example 3: Solving Right Triangles

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Method 1: By the Pythagorean Theorem,

$$RT^2 = RS^2 + ST^2$$

$$(5.7)^2 = 5^2 + ST^2$$

So $ST = \sqrt{7.49} \approx 2.74$.

$$m\angle R = \cos^{-1}\left(\frac{5}{5.7}\right) \approx 29°$$

Since the acute angles of a right triangle are complementary, $m\angle T \approx 90° - 29° \approx 61°$.

Example 3 Continued

Method 2:

$$m\angle R = \cos^{-1}\left(\frac{5}{5.7}\right) \approx 29°$$

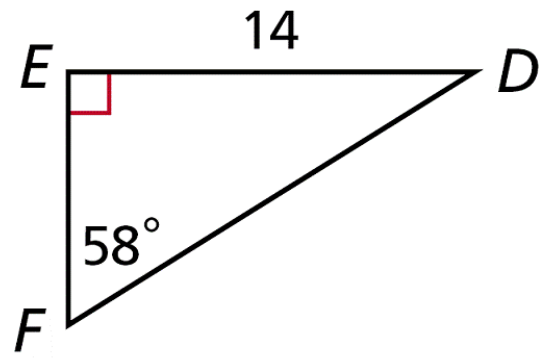
Since the acute angles of a right triangle are complementary, $m\angle T \approx 90° - 29° \approx 61°$.

$\sin R = \dfrac{ST}{5.7}$, so $ST = 5.7 \sin R$.

$$ST \approx 5.7 \sin\left[\cos^{-1}\left(\frac{5}{5.7}\right)\right] \approx 2.74$$

Check It Out! Example 3

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Since the acute angles of a right triangle are complementary, $m\angle D = 90° - 58° = 32°$.

$\tan 32° = \dfrac{EF}{14}$, so $EF = 14 \tan 32°$. $EF \approx 8.75$

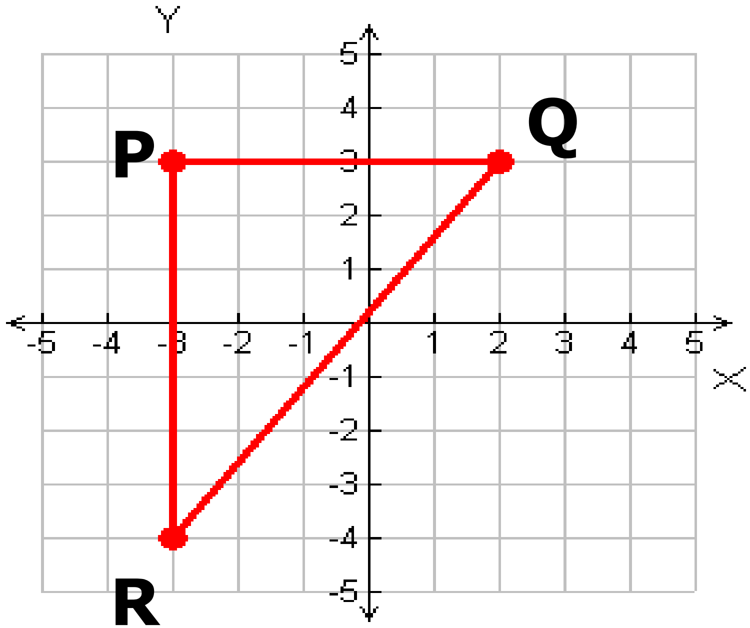
$DF^2 = ED^2 + EF^2$

$DF^2 = 14^2 + 8.75^2$

$DF \approx 16.51$

Example 4: Solving a Right Triangle in the Coordinate Plane

The coordinates of the vertices of ΔPQR are $P(-3, 3)$, $Q(2, 3)$, and $R(-3, -4)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.



Step 1 Find the side lengths.

Plot points P, Q, and R.

$PR = 7$ $\qquad$ $PQ = 5$

By the Distance Formula,

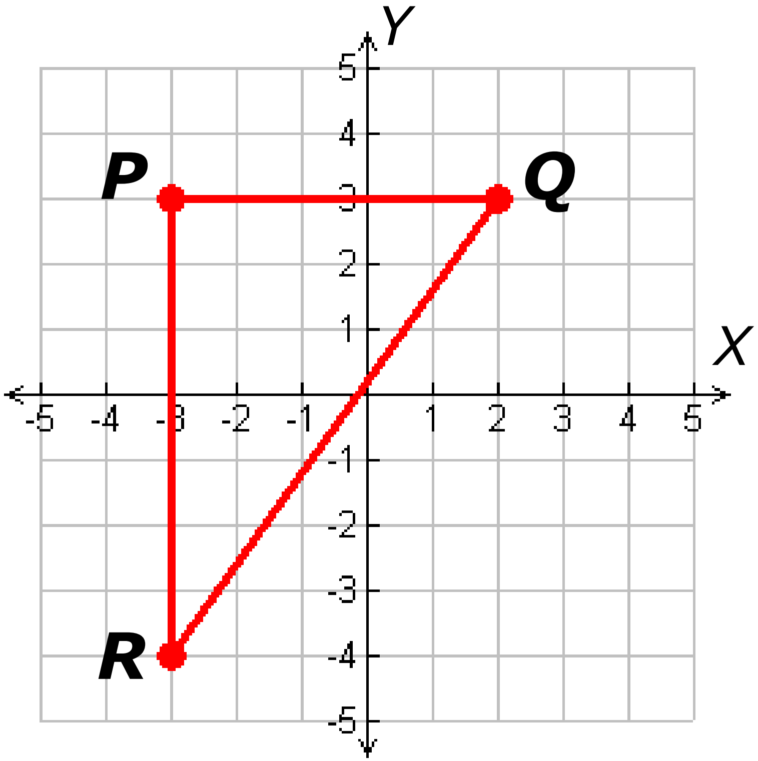
$$QR = \sqrt{(-3-2)^2 + (-4-3)^2}$$

$$= \sqrt{(-5)^2 + (-7)^2}$$

$$= \sqrt{25 + 49} = \sqrt{74} \approx 8.60$$

Example 4 Continued

Step 2 Find the angle measures.



$m\angle P = 90°$ *$\overline{PQ}$ and $\overline{PR}$ are $\perp$.*

$\overline{PR}$ is opp. $\angle Q$,

and $\overline{PQ}$ is adj. to $\angle Q$.

$$m\angle Q = \tan^{-1}\left(\frac{7}{5}\right) \approx 54°$$

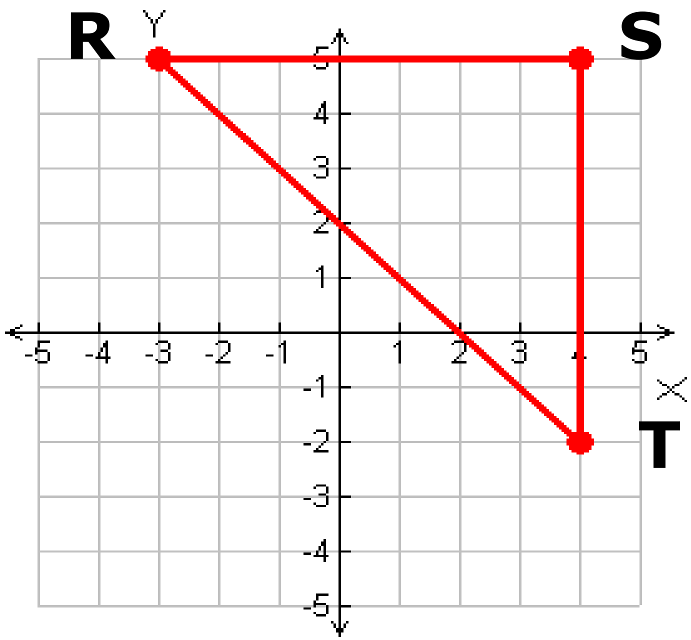
The acute $\angle$s of a rt. Δ are comp.

$m\angle R \approx 90° - 54° \approx 36°$

Check It Out! Example 4

The coordinates of the vertices of ΔRST are $R(-3, 5)$, $S(4, 5)$, and $T(4, -2)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

Step 1 Find the side lengths. Plot points *R*, *S*, and *T*.



$RS = ST = 7$

By the Distance Formula,

$$RT = \sqrt{(4-(-3))^2 + (-2-5)^2}$$

$$= \sqrt{(7)^2 + (-7)^2}$$

$$= \sqrt{49+49} = 7\sqrt{2} \approx 9.90$$

Check It Out! Example 4 Continued

Step 2 Find the angle measures.

$m\angle S = 90°$ — $\overline{RS}$ and $\overline{ST}$ are $\perp$.

$m\angle T = \tan^{-1}\left(\frac{7}{7}\right) = 45°$ — $\overline{RS}$ is opp. $\angle T$, and $\overline{ST}$ is adj. $\angle T$.

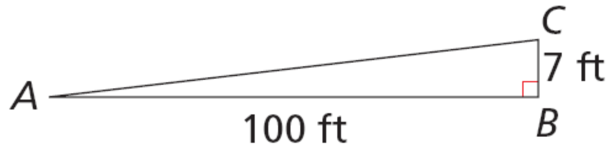
$m\angle R \approx 90° - 45° \approx 45°$ — *The acute $\angle$s of a rt. Δ are comp.*

Example 5: Travel Application

A highway sign warns that a section of road ahead has a 7% grade. To the nearest degree, what angle does the road make with a horizontal line?

$7\% = \frac{7}{100}$ *Change the percent grade to a fraction.*

A 7% grade means the road rises (or falls) 7 ft for every 100 ft of horizontal distance.



Draw a right triangle to represent the road.

$m\angle A = \tan^{-1}\left(\frac{7}{100}\right) \approx 4°$

$\angle A$ is the angle the road makes with a horizontal line.

Lesson Quiz: Part I

Use your calculator to find each angle measure to the nearest degree.

1. $\cos^{-1}(0.97)$ 14°

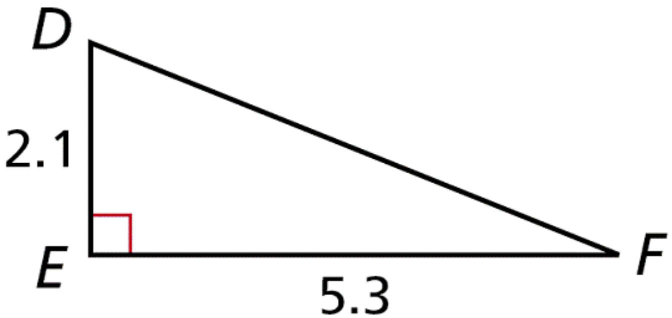
2. $\tan^{-1}(2)$ 63°

3. $\sin^{-1}(0.59)$ 36°

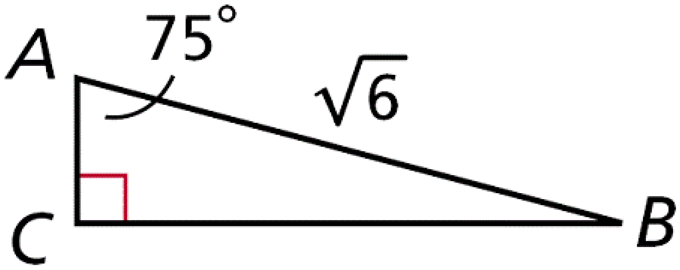
Lesson Quiz: Part II

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.

4.



DF ≈ 5.7; m∠*D* ≈ 68°; m∠*F* ≈ 22°



AC ≈ 0.63; *BC* ≈ 2.37; m ∠*B* = 15°

Lesson Quiz: Part III

6. The coordinates of the vertices of ΔMNP are M (–3, –2), N(–3, 5), and P(6, 5). Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

$MN = 7$; $NP = 9$; $MP \approx 11.40$; $m\angle N = 90°$; $m\angle M \approx 52°$; $m\angle P \approx 38°$