Guided Notes – 8.3 Estimating a Population Mean when "σ KNOWN"

1. Inference about population proportions (P) based on Categorical variables.
   - Proporations are %'s.
2. Inference about population means (μ) based on QUANTITATIVE variables.
   - Means are averages.

General form to calculate a confidence interval is on the Green Sheet:
statistic ± (critical value) • (standard deviation of the statistic)

3. What is the formula for a ***1-sample Z- interval for a population mean?***

$$\bar{x} \pm z^* \cdot \left[\frac{\sigma}{\sqrt{n}}\right]$$

ME

   a) What statistic will be used to calculate this confidence interval? $\bar{x}$ sample mean

   b) What is the critical value? z^*

   c) What part of this formula is the margin of error (ME)? $z^* \cdot \left[\frac{\sigma}{\sqrt{n}}\right]$

   d) What conditions are required?

   1. RANDOM – SRS OR RANDOMIZED Experiment
   2. INDEPENDENT – Sampling without replacement must meet 10% condition.
   3. NORMAL – POPULATION WAS NORMAL OR CLT($n \geq 30$)
   4. Plus **you must know** the population standard deviation (σ)

4. In most real world problems, we do NOT know the population mean (μ) or population standard deviation (σ), therefore we do not use the ***Z-statistic*** for inference for means.

But we DO use the Z-statistic to estimate sample size!

5. Describe the three steps for choosing a sample size for a desired margin of error when estimating μ.

(1) GET A REASONABLE VALUE FOR THE Population S.D. (σ) from an earlier pilot study.

(2) Decide on your CONFIDENCE LEVEL (C) and find the critical value z^*.

(3) Decide on your desired Margin of ERROR (ME)

6. Complete the Check Your Undertanding "Monkeys" -- page 501-502.

1) Define population parameter	μ= THE TRUE MEAN CHOLESTEROL OF A SPECIFIC MONKEY
2) Get information to estimate the sample size	CL= 95% z^*= 1.96 σ= 5 mg/dl ME= 1 mg/dl
3) Use formula used to determine the sample size n for a population mean:	$z^* \frac{\sigma}{\sqrt{n}} \leq ME$. Solve for n.
4) Substitute numbers and clearly show all steps to calculate the sample size n	$\sqrt{n}\left(1.96 * \frac{5}{\sqrt{n}} \leq 1\right)\sqrt{n}$ $1.96 * 5 \leq \sqrt{n}$ $(9.8)^2 \leq (\sqrt{n})^2$ $96.04 \leq n$ $n \geq 96.04$ ROUND UP ALWAYS when est. n
5) Always round up to next whole number to ensure ME is met.	We need to sample 97 MONKEYS.

7. It is the size of the Sample size (n) that determines the margin of error. The size of the Population (N) does not influence the sample size we need. This is true as long as the population is much larger than sample (10% Condition)

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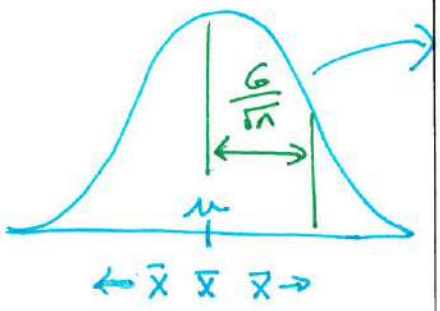
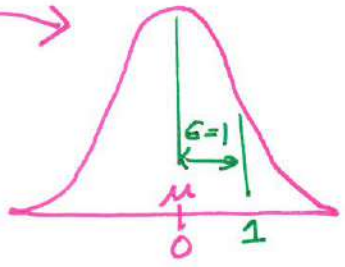
8. What is the standardized value of the z-statistic?

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

a) See Figure 8.11 to understand this new z-statistic

Sketch the sampling distribution of $\bar{x}$ when the normal condition is met and σ is known.	Compare the distributions	Sketch the standard normal distribution . N(0,1)
N 	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	

b) When we don't know "**σ,**" we estimate it using the Sample standard deviation (s_x) creating a **new statistic called the "t-statistic."**

9. SKIP "Bingo" Actvity

10. What is the fromula for the "**t-statistic**"?

$$t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}}$$

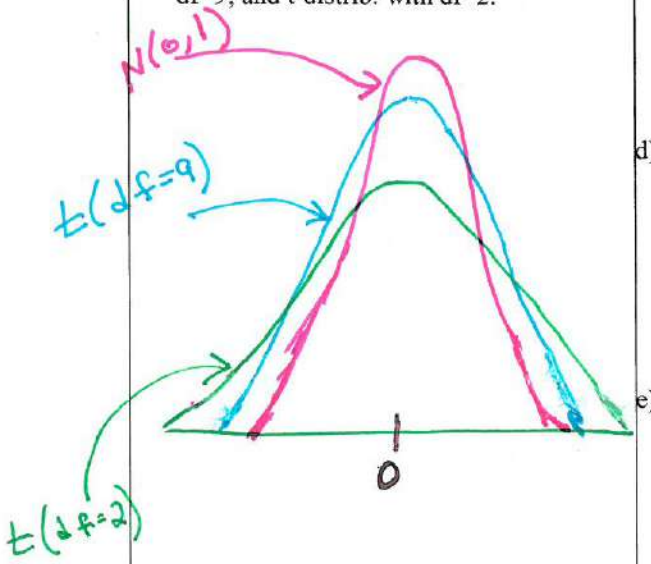
a) How do you calculate the degrees of freedom for a t distribution?

df = degrees of freedom

df = n-1 (sample size minus 1)

See Figure 8.13 to understand the t-statistic

b) Sketch normal distribution;t-distrib. with df=9; and t-distrib. with df=2.



c) Describe the similarities between a standard normal distribution and a t distribution.

t and normal - have they same shape
Symmetric with center at 0

d) Describe the differences between a standard normal distribution and a t distribution.

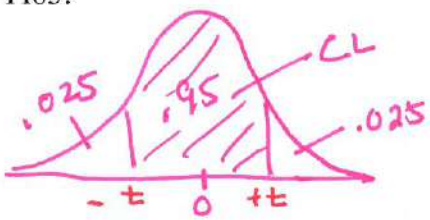
The spread is wider for the t-distrib.
Notice "t" shorter and wider
and the "t" has larger area
in the tails.

e) What happens to the t distribution as the degrees of freedom increase?

As df (and sample size - n)
increase, the distribution for "t"
approaches the standard normal
distribution.

11. How do you find the critical value t* using TI84? You only need to know how to use Table B if you have a TI83.

* Need to sample size (n) and CL.



EXAMPLE: n = 10 CL = 95%

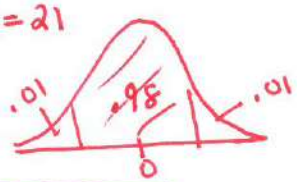
df = n - 1 = 10 - 1 = 9

invT(.025, 9) → $t^* = \pm 2.26$

12. Check Your Understanding -- page 507 (use TI84, sketch the graph, answers in back of book)

a) n = 22 CL = .98%

df = 21

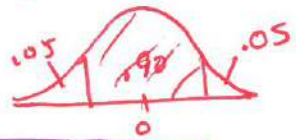


$t^* = \pm 2.52$

invT(.01, 21)

b) n = 10 CL = 90%

df = 9



$t^* = \pm 1.83$

invT(.05, 9)

c) n = 7 CL = 95%

df = 6

$t^* = \pm 2.45$

invT(.025, 6)

13) What is the formula for the *standard deviation* of the sampling distribution of the sample mean $\bar{x}$?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

14) What is the *standard error* of the sample mean $\bar{x}$?

$$SE(\bar{x}) = \frac{s_x}{\sqrt{n}}$$

15. What is the formula for a ***1-sample t- interval for a population mean?***

$$\bar{x} \pm \underbrace{t^*_{n-1} \cdot \left(\frac{s_x}{\sqrt{n}}\right)}_{ME}$$

a) What statistic will be used to calculate this confidence interval? $\bar{x}$

b) What is the critical value? t^* with df= $n-1$

c) What part of this formula is the margin of error (ME)? $t^* \cdot s_x/\sqrt{n}$

16. What conditions are required for a ***1-sample t- interval for a population mean***?

- RANDOM
  - SRS $n =$ OR
  - Randomized Experiment (w/ Randomly assigned TREATMENTS)
- NORMAL
  - THE POPULATION WAS STATED TO HAVE A NORMAL DISTRIBUTION.
  - OR Large Sample - CLT ($n \geq 30$)
- INDEPENDENT

  SAMPLING WITHOUT REPLACEMENT MUST Check 10% Condition - $10n \leq N$ OR $n \leq \frac{1}{10}N$

17. Walk through example "Video Screen Tension."

- You do not need to write the problem.
- Enter the data and use your calculator to replicate all steps. See "Technology Corner" page 514.
- **Your Notes**:

DO PROBLEM CALCULATING CI BY HAND

AND CHECK WITH TI 84

[STAT] TESTS > 8: TInterval

18. "Auto Pollution" example is optional. **Your Notes**:

19. What is a "Robust" procedure?

PROCEDURES THAT ARE NOT STRONGLY AFFECTED WHEN A CONDITION FOR USING THEM IS VIOLATED.

- When are t-procedures NOT robust?
  - WE CAN USE THE T-PROCEDURE AS LONG AS THE SHAPE IS SYMMETRIC WITH NO OUTLIERS OR STRONG SKEWNESS.

20. Describe the 2 different normal conditions when using t-procedures:

- SMALL SAMPLES (n<15 and n<30)
  - YOU MUST GRAPH THE DATA (HISTOGRAM) TO REVIEW THE SHAPE IS ROUGHLY SYMMETRIC WITH NO OUTLIERS OR STRONG SKEWNESS
  - TIP: OVERLAY THE BOX PLOT TO IDENTIFY OUTLIERS.
- LARGE SAMPLES (n≥30)
  - BASED ON CLT, WHEN THE SAMPLE IS SUFFICIENTLY LARGE ($n \geq 30$), THE DISTRIBUTION IS APPROX. NORMAL

21. Walk through example "People, Trees, and Flowers." **Your Notes**: