Guided Notes -- 8.2 Estimating a Population Proportion

1. Give 2 examples of a ***population proportion: p*** ?

① WHAT PROPORTION OF U.S. ADULTS ARE UNEMPLOYED?

② WHAT PROPORTION OF CELL PHONE USERS OWN AN IPHONE?

2. How do you calculate a ***sampling proportion:*** $\hat{p}$?

$$\hat{p} = \frac{x}{n} = \frac{\text{NUMBER WITH CHARACTERISTIC}}{\text{SAMPLE SIZE}}$$

3. Describe the ***"sampling distribution of a sample proportion*** $\hat{\mathbf{p}}$***"*** "as learned in section 7.2. **Use the correct variable notations.**

- Shape (and state conditions)

  Approximately Normal if the condition IS MET: $np \geq 10$ AND $n(1-p) \geq 10$

- Center (mean of the sampling distribution of $\hat{p}$)

  $\mu_{\hat{p}} = p$

- Spread (standard deviation of the sampling distribution of $\hat{p}$)

  $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

  - What condition is requred to calculate the standard deviation?

    10% Condition = $10n \leq N$

General form to calculate a confidence interval is on the **Green Sheet**:

statistic ± (critical value) • (standard deviation of the statistic)

4. What ***statistic*** will be used to calculate the cofidence interval for proportions?

Statistic is $\hat{p}$

5. How does the standard deviation differ to to standard error for the sampling distribution of $\hat{p}$?

Formula for **standard deviation** of sampling distribution of $\hat{p}$:	Formula for **standard error** OF THE Sample proportion $\hat{p}$:
$\mu_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

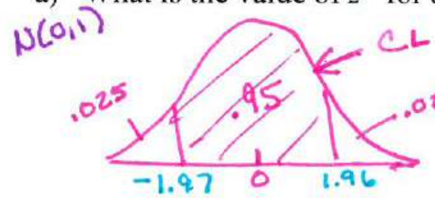
a) Define ***standard error*** of a statistic

WHEN THE STANDARD DEVIATION OF A STATISTIC (ie $\hat{p}$) IS ESTIMATED FROM DATA, THE RESULT IS CALLED THE STANDARD ERROR OF THE STATISTIC EXAMPLE $SE(\hat{p})$

b) (in context) The <u>STANDARD ERROR $SE(\hat{p})$</u> describes how close the <u>Sample proportion</u> ($\hat{p}$) will be, on <u>average</u>, to the <u>population proportion</u> (p) in repeated <u>SRS's</u> of size <u>n</u>.

6. How do you get the ***critical value*** ***(** z^***)**? **Hint**: follow steps outlined on pages 487-488. Use the graphing calculator. Do **not** use Table A.

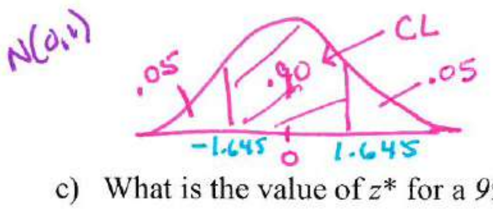
a) What is the value of z^* for a *95% confidence interval*? Include a sketch (see figure 8.8).



$\boxed{Z^* = \pm 1.96}$

INV Norm (.025, 0, 1)

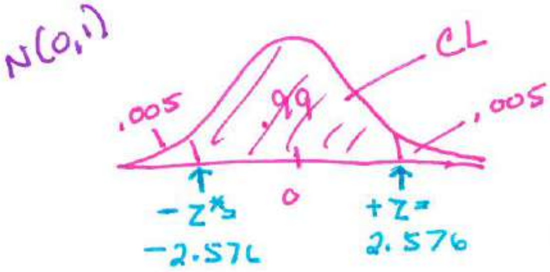
b) What is the value of z^* for a *90% confidence interval*? Include a sketch (see figure 8.8).



$\boxed{Z^* = \pm 1.645}$

INV NORM (.05, 0, 1)

c) What is the value of z^* for a *99% confidence interval*? Include a sketch (see figure 8.8).



$\boxed{Z^* = \pm 2.576}$

INV NORM (.005, 0, 1)

7. What is the formula for a *one-sample z interval for a population proportion*?

$$\hat{p} \pm \underbrace{z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{ME}$$

a) Describe z*

z^* is the critical value for the standard Normal curve N(0,1) with the area C (CL) between $-z^*$ and z^*.

b) What part of this formula is the margin of error (ME)?

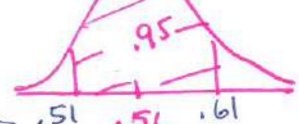
$$ME = z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

c) What conditions are required?

① Normal – $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$

② 10% Condition – $10n \leq N$

8. The 4 step process (simplified) to contruct and interpret a confidence interval.

Follow these required steps:	Example "Teens Say Sex Can Wait" Complete these steps to construct the 95% CI for p.
1) Define population parameter	p= actual proportion of all 13-17 in U.S who say they will wait for sex till marriage
2) State the inference method	1-Sample Z-Interval for a proportion
3) Check conditions $\hat{p} = \frac{246}{439} = .56$ $SE(\hat{p}) = .0237$	Random SRS of 439 teens Normal $n\hat{p} = 439(.56) = 246 \geq 10$ ✓ $n(1-\hat{p}) = 439(.44) = 193 \geq 10$ ✓ Independent Sampling without replacement. It is reasonable 10 (439) = 4,390 teens in the U.S.
4) Sketch graph	95% CL → $z^* = \pm 1.96$ 
5) Show calculations with numbers	$.56 \pm 1.96\sqrt{\frac{(.56)(.44)}{439}}$ $.56 \pm .046$ [.514, .606]
6) Answer in contex	We are 95% confident that the true proportion who say they will wait for sex is between .51 and .61.

9. What formula is used to determine the sample size necessary for a given margin of error?

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

Where, $\hat{p} = .5$ if not given

10. Refer to thr Example "Customer Satisfaction," to complete the table below. Clearly show the steps to determine the sample sizes.

a) ***Use the*** $\hat{p}$ ***to produce the largest sample size in this example.***	***b)*** ***Now, find the sample size if you are told use*** $\hat{p} = .31$
$\hat{p}$ = .5 95% CL $\rightarrow z^* = \pm 1.96$ ME = 3% = .03 $\sqrt{n}$ $\left(1.96\sqrt{\frac{(.5)(.5)}{n}}\right) \leq (.03)\cdot\sqrt{n}$ $\frac{1.96(.5)}{.03} \leq \frac{.03\sqrt{n}}{.03}$ $(32.667)^2 \leq (\sqrt{n})^2$ $n \geq 1067.11$ (Round up) Must Sample 1,068 or more	$\hat{p} = .31$ *CL=95%* *ME=.03* z^*= 1.96 $\sqrt{n}\left(1.96\sqrt{\frac{(.31)(.69)}{n}}\right) \leq (.03)\cdot\sqrt{n}$ $\frac{.9065}{.03} \leq \frac{.03\sqrt{n}}{.03}$ $(30.22)^2 \leq (\sqrt{n})^2$ $n \geq 913.02$ must sample 914 or more

11. What is the rounding rule for determining sample sizes?

ALLWAYS ROUND UP TO ENSURE THE ME IS SATIFIED FOR THE SPECIFIED CONFIDENCE LEVEL