Answer Key

Lesson 8.2

Challenge Practice

1. $(-1, 5), (7, 5)$

2. $(2, 1), (2, 3)$

3. $(a + 2, b + 3), (a + 6, b + 3)$

4. $(a^2, b^2), (a + a^2, b^2)$ **5.** 2

6. All four vertices of a parallelogram can only have at most two equal x-values and two equal y-values. So the third and fourth vertices of the parallelogram cannot have the same x-values or y-values as the other vertices if there are already two equal x-values or two equal y-values.

7. *Sample answer:* Draw the diagonal of one vertex to the opposite vertex to create two congruent triangles. This can be done in two ways.

8. *Sample answer:* Draw a diagonal connecting two opposite vertices and then draw another diagonal connecting the other two opposite vertices.

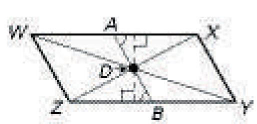
9. *Sample answer:* First Way: Connect one vertex to the midpoint of the opposite side. Repeat for the opposite vertex. Next draw the diagonal connecting the remaining opposite vertices. Second Way: Repeat the first method but connect the other opposite vertices with a diagonal.

10. No.

11. *Sample answer:*

Statements	Reasons
1. *RSTU* and *WXYZ* are parallelograms.	**1.** Given
2. $\overline{RS} \parallel \overline{UT}$, $\overline{RU} \parallel \overline{ST}$, $\overline{WX} \parallel \overline{ZY}$, and $\overline{WZ} \parallel \overline{XY}$	**2.** Definition of a parallelogram
3. $\angle RWX \cong \angle YXW$	**3.** Alternate Interior Angles Theorem
4. $\angle YXW \cong \angle WZY$	**4.** Opposite angles of a parallelogram are congruent.
5. $\angle WZY \cong \angle TYZ$	**5.** Alternate Interior Angles Theorem
6. $\angle RWX \cong \angle TYZ$	**6.** Substitution Prop. of Equality
7. $\angle XRW \cong \angle UZR$	**7.** Alternate Interior Angles Theorem
8. $\angle SXT \cong \angle YTZ$	**8.** Alternate Interior Angles Theorem
9. $\angle XRW \cong \angle SXT$	**9.** Corresponding Angles Postulate
10. $\angle XRW \cong \angle YTZ$	**10.** Substitution Prop. of Equality
11. $\triangle RWX \sim \triangle TYZ$	**11.** Angle-Angle Similarity Post.

12.



Given: *WXYZ* is a parallelogram and $\overline{WY}$ and $\overline{XZ}$ are diagonals of *WXYZ*.

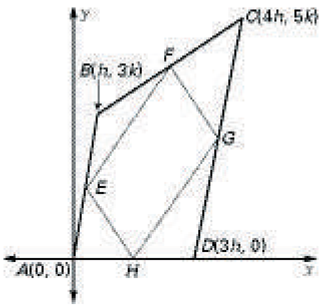
Prove: *D* is the midpoint of the segment with endpoints on opposite sides passing through the point of intersection *D*.

Answer Key

Statements	Reasons
1. *WXYZ* is a parallelogram, $\overline{WX} \parallel \overline{YZ}$, and $\overline{WZ} \parallel \overline{XY}$.	**1.** Given
2. $\overline{DW} \cong \overline{DY}$ and $\overline{DX} \cong \overline{DZ}$	**2.** Theorem 8.10
3. $\overline{WX} \cong \overline{YZ}$, and $\overline{WZ} \cong \overline{YX}$	**3.** Theorem 8.7
4. $\triangle WDX \cong \triangle YDZ$, $\triangle WDZ \cong \triangle YDZ$	**4.** SSS Cong. Post.
5. Altitude of $\triangle WDX =$ alt. of $\triangle YDZ$, and alt. of $\triangle WDZ =$ alt. of $\triangle YDZ$.	**5.** If two triangles are congruent, their altitudes are equal.
6. Let $\overline{AB}$ pass through *D* and have its endpoints on $\overline{WX}$ and $\overline{YZ}$.	**6.** Assume
7. The angle formed by $\overline{AD}$ and the altitude of $\triangle WDX \cong$ the angle formed by $\overline{BD}$ and the alt. of $\triangle YDZ$.	**7.** Vertical Angles Cong. Theorem
8. $\angle 1$ and $\angle 2$ are right angles.	**8.** Definition of an altitude
9. $\angle 1 \cong \angle 2$	**9.** Right Angle Congruence Thm.
10. The triangles formed by the altitudes and $\overline{AD}$ and $\overline{BD}$ are cong.	**10.** ASA Congruence Postulate
11. $\overline{AD} \cong \overline{BD}$	**11.** Corresponding parts of cong. triangles are congruent.
12. *D* is the midpoint of $\overline{AB}$.	**12.** Def. of a midpoint

13. *Sample answer:*

Given: *ABCD* is a quadrilateral and *E*, *F*, *G*, and *H* are midpoints of their respective segments.Prove: When the midpoints of adjacent sides are connected by segments, a parallelogram is formed.Step 1: Place *ABCD* and assign coordinates. Let *E*, *F*, *G*, and *H* be midpoints of their respective segments. Find the coordinates of the midpoints.



E: $\left(\frac{0+h}{2}, \frac{0+3k}{2}\right) = \left(\frac{h}{2}, \frac{3k}{2}\right)$;

F: $\left(\frac{h+4h}{2}, \frac{3k+5k}{2}\right) = \left(\frac{5h}{2}, 4k\right)$

G: $\left(\frac{4h+3h}{2}, \frac{5k+0}{2}\right) = \left(\frac{7h}{2}, \frac{5k}{2}\right)$;

H: $\left(\frac{3h+0}{2}, \frac{0+0}{2}\right) = \left(\frac{3h}{2}, 0\right)$

Step 2: Connect the midpoints of adjacent sides with segments.

Step 3: Prove $\overline{EH} \parallel \overline{FG}$ and $\overline{EF} \parallel \overline{HG}$ by showing their slopes are congruent.

$$m\overline{EH} = \frac{0 - \frac{3k}{2}}{\frac{3h}{2} - \frac{h}{2}} = \frac{\frac{-3k}{2}}{h} = -\frac{3k}{2h}$$

$$m\overline{FG} = \frac{\frac{5k}{2} - \frac{8k}{2}}{\frac{7h}{2} - \frac{5h}{2}} = \frac{-\frac{3k}{2}}{h} = -\frac{3k}{2h}$$

$$m\overline{EF} = \frac{\frac{8k}{2} - \frac{3k}{2}}{\frac{5h}{2} - \frac{h}{2}} = \frac{\frac{5k}{2}}{2h} = \frac{5k}{4h};$$

$$m\overline{HG} = \frac{\frac{5k}{2} - 0}{\frac{7h}{2} - \frac{3h}{2}} = \frac{\frac{5k}{2}}{2h} = \frac{5k}{4h}$$

Because the slopes are the same, $\overline{EH} \parallel \overline{FG}$ and $\overline{EF} \parallel \overline{HG}$.

Step 4: Prove *EFGH* is a parallelogram. Because both pairs of opposite sides are parallel, *EFGH* is a parallelogram. Because *EFGH* is a parallelogram and *E*, *F*, *G*, and *H* are midpoints of the respective sides of *A*, *B*, *C*, and *D*, a parallelogram is formed when the midpoints of adjacent sides are connected by segments.