7th Grade Mathematics

Statistics and Probability / Geometry

Unit 4 Curriculum Map: March 21st – June 22nd

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

In this unit, students will ….

- Understand experimental and theoretical probabilities
- Recognize that probabilities are useful for predicting what will happen over the long run
- For an event described in everyday language, identify the outcomes in a sample space that compose the event
- Interpret experimental and theoretical probabilities and the relationship between them and recognize that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers
- Distinguish between outcomes that are equally likely or not equally likely by collecting data and analyzing experimental probabilities
- Realize that the probability of simple events is a ratio of favorable outcomes to all outcomes in the sample space
- Recognize that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
- Determine the fairness of a game
- Explore and develop probability models by identifying possible outcomes and analyze probabilities to solve problems
- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
- Represent sample spaces for simple and compound events and find probabilities using organized lists, tables, tree diagrams, area models, and simulation
- Realize that, just as with simple events, the probability of a compound event is a ratio of favorable outcomes to all outcomes in the sample space
- Design and use a simulation to generate frequencies for simple and compound events

- Analyze situations that involve two or more stages (or actions) called *compound events*
- Use area models to analyze the theoretical probabilities for two-stage outcomes
- Analyze situations that involve binomial outcomes
- Use probability to calculate the long-term average of a game of chance
- Determine the expected value of a probability situation
- Use probability and expected value to make a decision
- Deepen the understanding of the process of statistical investigation and apply this understanding to samples
- Pose questions, collect data, analyze data, and interpret data to answer questions
- Understand that data values in a sample vary and that summary statistics of samples, even same-sized samples, taken from the same population also vary
- Choose appropriate measures of center (mean, median, or mode) and spread (range, interquartile range, or mean absolute deviation) to summarize a sample
- Choose appropriate representations to display distributions of samples
- Compare summary statistics of multiple samples drawn from either the same population or from two different populations and explain how the samples vary
- Understand that simulations can model real-world situations
- Design a model that relies on probability concepts to obtain a desired result
- Use the randomly generated frequencies for events to draw conclusions
- Understand that summary statistics of a representative sample can be used to gain information about a population
- Describe the benefits and drawbacks to various sampling plans
- Use random-sampling techniques to select representative samples
- Apply concepts from probability to select random samples from populations
- Explain how sample size influences the reliability of sample statistics and resulting conclusions and predictions

- Explain how different sampling plans influence the reliability of sample statistics and resulting conclusions and predictions
- Use statistics from representative samples to draw conclusions about populations
- Use measures of center, measures of spread, and data displays from more than one random sample to compare and draw conclusions about more than one population
- Relate area of a circle to covering a figure and circumference to surrounding a figure
- Estimate and calculate areas and circumferences of circles
- Explore the relationship between circle radius (or diameter) and circumference
- Explore the relationship between circle radius (or diameter) and area
- Investigate the connection of π to area calculation by estimating the number of radius squares needed to cover a circle
- Investigate the relationship between area and circumference of a circle
- Solve problems involving areas and circumferences of circles
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure

CMP Pacing Guide

Activity	Common Core Standards	Estimated Time
Unit 4 Diagnostic Assessment	6.EE.7, 4.G.1, 4.MD.3, 6.G.1, 6.G.2, 6.G.4, 6.RP.3, 6.SP.4, 6.SP.5	1 Block
What Do You Expect? (CMP3) Investigation 1	7.SP.6, 7.SP.7a, 7.SP.7b, 7.SP.8, 7.EE.3	3 Blocks
Unit 4 Performance Task 1	7.SP.6	½ Block
What Do You Expect? (CMP3) Investigation 2	7.SP.5, 7.SP.6, 7.SP.7b, 7.SP.8, 7.EE.3	3 Blocks
Assessment: Unit 4 Check Up 1 (CMP3)	7.SP.5, 7.SP.6, 7.SP.7a, 7.SP.7b, 7.SP.8, 7.EE.3	½ Block
What Do You Expect? (CMP3) Investigation 3	7.SP.6, 7.SP.7, 7.SP.7a, 7.SP.7b, 7.SP.8, 7.EE.3, 7.RP.3	3 Blocks
Unit 4 Performance Task 2	7.SP.8	½ Block
What Do You Expect? (CMP3) Investigation 4	7.SP.5, 7.SP.6, 7.SP.8, 7.SP.8a, 7.SP.8b, 7.SP.8c, 7.EE.3, 7.RP.3	4 Blocks
Assessment: Unit 4 Partner Quiz (CMP3)	7.SP.5, 7.SP.6, 7.SP.7, 7.SP.7a, 7.SP.7b, 7.SP.8	½ Block
What Do You Expect? (CMP3) Investigation 5	7.SP.5, 7.SP.7, 7.SP.7a, 7.SP.8	3 Blocks
Unit 4 Assessment 1	7.SP.5, 7.SP.6, 7. SP.7, 7.SP.8	1 Block
Samples and Populations (CMP3) Investigation 2	7.SP.1, 7.SP.2	4 Blocks
Assessment: Unit 4 Partner Quiz (CMP3)	7.SP.1, 7.SP.2	½ Block
Samples and Populations (CMP3) Investigation 3.4	7.SP.1, 7.SP.2	1 Block
Filling and Wrapping (CMP3) Investigation 3	7.G.B.4	4 Blocks
Unit 4 Performance Task 3	7.G.B.4	½ Block
Unit 4 Assessment 2	7.SP.1, 7.SP.2	1 Block
Grade 7 Module 6 Topic A (EngageNY) Lessons 1-4	7.EE.B.4, 7.G.B.5	6 Blocks
Filling and Wrapping (CMP3) Investigation 4	7. G.B.4, 7.G.B.6	5 Blocks
Unit 4 Assessment 3	7.G.B.4, 7.G.B.5, 7.G.B.6	1 Block
Total Time		**43 Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

March

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21 **Unit 4: Stats & Probability** ***Unit 4 Diagnostic***	22	23	24	25 **Good Friday** **District Closed**	26
27	28 ***Performance Task 1 Due***	29 **PARCC Review**	30 **PARCC Review**	31 **PARCC Review**		

April

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1 PARCC Review	2
3	4 *PARCC TESTING (3-8)*	5 *PARCC TESTING (3-8)*	6 *PARCC TESTING (3-8)*	7 *PARCC TESTING (3-8)*	8 *PARCC TESTING (3-8)*	9
10	11 Spring Recess District Closed	12 Spring Recess District Closed	13 Spring Recess District Closed	14 Spring Recess District Closed	15 Spring Recess District Closed	16
17	18	19	20	21 *Assessment: Check Up 1*	22	23
24	25	26	27 *Performance Task 2 Due*	28	29	30

MAY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	***2***	***3***	***4 Assessment: Partner Quiz (What Do You Expect?)***	***5***	***6***	**7**
8	***9***	***10 Assessment: Unit 4 Assessment 1***	***11***	***12***	***13***	**14**
15	***16***	***17 Assessment: Partner Quiz (Samples and Populations)***	***18***	***19***	***20***	**21**
22	**23**	**24**	**25** ***Performance Task 3 is due***	**26** ***Assessment: Unit 4 Assessment 2***	**27**	**28**
29	**30 Memorial Day** **District Closed**	**31 Snow Day Make up** **District Closed**				

June

			Wednesday	Thursday	Friday	Saturday
			1	***2***	***3***	**4**
5	***6*** **Snow Day Make up** **District Closed**	***7***	***8***	***9***	***10***	**11**
12	***13***	***14***	***15***	***16*** ***Assessment: Unit 4 Assessment 3***	***17*** **Solidify Unit 4 Concepts**	**18**
19	**20 Solidify Unit 4 Concepts**	**21 12:30 pm Student Dismissal**	**22 12:30 pm Student Dismissal**	**23 Last day for students**	**24**	**25**
26	**27**	**28**				

Unit 4 Math Background

The terms *chance* and *probability* apply to situations that have uncertain outcomes on individual trials but a regular pattern of outcomes over many trials. For example, when you toss a coin, you are uncertain whether it will come up heads or tails. But you do know that over the long run, if it is a fair coin, you will get about half heads and half tails. This does not mean you won't get several heads in a row or that if you get heads now you are more likely to get tails on the next toss. Uncertainty on an individual outcome but predictable regularity in the long run is a difficult concept for students to grasp. It often takes a significant amount of time and a variety of experiences that challenge prior conceptions before students understand this basic concept of probability.

This unit is both accessible and interesting to Grade 7 students. Students learned about data and statistical measures in Grade 6. The focus of this unit is to extend the concepts developed in Grade 6. From the start of the unit, students gain an understanding of experimental and theoretical probabilities and the relationship between them. Through the examples and problems in the unit, students also make important connections between probability and rational numbers, geometry, statistics, science, and business.

Later in the unit, students use their background knowledge of statistical investigations and probability to draw conclusions about samples and populations. Statistical investigations involve four parts:

- Posing questions
- Collecting data
- Analyzing data distributions
- Interpreting the data and the analysis to answer the questions

At the end of a statistical investigation, students need to communicate the results.

Students will use both data that are provided for them in the student edition and data that they generate. In both cases, students need to consider the process of statistical investigation.

When students collect their own data, they naturally tend to follow through with the process of statistical investigation. When students analyze a data set they have not collected, however, they need to understand the data first in order to complete any analysis.

PARCC Assessment Evidence Statements

CCSS	Evidence Statement	Clarification	Math Practices	Calculator?
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	i) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	4	Yes
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book*; *predict the winner of a school election based on randomly sampled survey data*. *Gauge how far off the estimate or prediction might be*.	None	4	Yes
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is* 10 *cm greater than the mean height of players on the soccer team, about twice the variability* (*mean absolute deviation*) *on either team*; *on a dot plot, the separation between the two distributions of heights is noticeable*.	i) Tasks may use mean absolute deviation or range as a measure of variability.	4	Yes
7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh grade science book are generally longer than the words in a chapter of a fourth grade science book*.	None	4	Yes
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	None	4	Yes
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube* 600 *times, predict that a* 3 *or* 6 *would be rolled roughly 200 times, but probably not*	i) Tasks require the students to make a prediction based on long-run relative frequency in data from a chance process. Data can be provided, or if the task is technology-enhanced, the task can simulate a data	4	Yes

	exactly 200 *times*.	gathering process.		
7.SP.7a	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*	i) Simple events only.	4	Yes
7.SP.7b	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies*?	i) Data can be provided, or if the task is technology-enhanced, the task can simulate a data gathering process.	4	Yes
7.SP.8a	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	None	4, 5	Yes
7.SP.8b	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	None	4, 5	Yes
7.SP.8c	Find the probabilities of compound events using organized lists, tables, tree diagrams, and simulation. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question. If* 40% *of donors have type A blood, what is the probability that it will take at least* 4 *donors to find one with type A blood*?	None	4, 5	Yes

7.G.4-1	Know the formulas for the area and circumference of a circle and use them to solve problems.	i) Pool should contain tasks with and without contexts. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks. iii) Tasks may require answers to be written in terms of π.	4, 5	Yes
7.G.4-2	Give an informal derivation of the relationship between the circumference and area of a circle.	i) Tasks require students to identify or produce a logical conclusion about the relationship between the circumference and the area of a circle, e.g., that given three circles with areas $A1 > A2 > A3$, the circumferences satisfy $C1 > C2 > C3$.	2, 5	Yes
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	i) Pool should contain tasks with and without contexts.	5, 6	Yes
7.G.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	i) Pool should contain tasks with and without contexts. ii) Tasks focus on area of two-dimensional objects.	1, 5	Yes

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	- Students make sense of probability and sampling as a rational number, percentage, or as a visual model
2	Reason abstractly and quantitatively
	- Students reason about probabilities as values between 0 and 1 - Students are able to predict an outcome based on given information - Students reason about values in a data display - Students use tables to seek patterns to reason abstractly
3	Construct viable arguments and critique the reasoning of others
	- Students question each other about a prediction or probability - Students use data to make inferences - Students evaluate reasoning of other students
4	Model with mathematics
	- Students model probabilities using area models, rational numbers, and in visual displays - Students construct arguments using given or calculated evidence - Students use concrete models, such as coins and spinners, to model various situations
5	Use appropriate tools strategically
	- Students simulate and approximate probabilities - Students use concrete models, such as spinners or coins, as tools as appropriate - Students use calculators or other tools to find summary statistics more efficiently - Students generate numbers at random using appropriate probability tools
6	Attend to precision
	- Students use precise language to discuss and present probabilities and outcomes - Students focus on the preciseness/accuracy of calculations - Students look at values within one and two mean absolute deviations of the mean of a distribution - Students use price observations to identify if data values are expected or unexpected
7	Look for and make use of structure
	- Students represent probabilities as a rational number, percentage, or as a visual representation - Students examine different methods throughout the unit to determine structure - Students randomly select samples of size 5, 10, and 30
8	Look for and express regularity in repeated reasoning
	- Students use proportional reasoning to predict a population characteristic using random sample - Students have many opportunities to explore situations that address repeated reasoning - Students complete simulations that mimic real-world situations - Students repeat trials multiple times and modify their conclusions after each repetition

Vocabulary

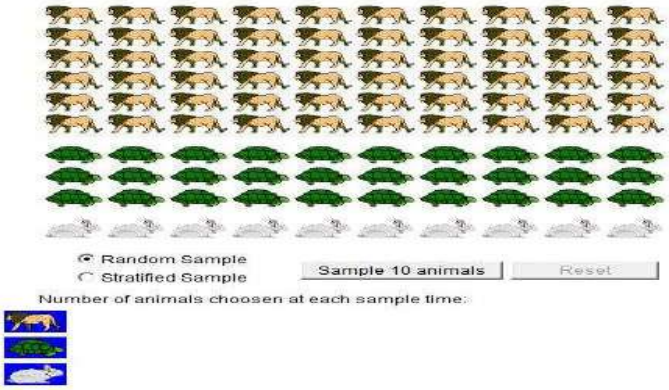
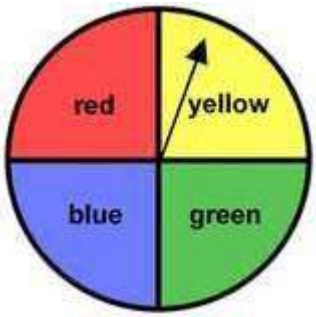
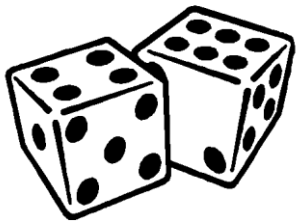
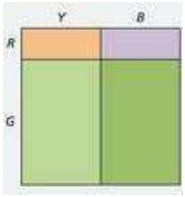
Term	Definition
Compound Event	An event that consists of two or more simple events.
Event	Any possible outcome of an experiment in a probability
Experimental Probability	The ratio of the number of times an outcome actually occurs to the number of trials performed
Mean	A measure of center in a set of numerical data, the sum of the values in a data set divided by the number of values in the data set
Mean Absolute Deviation	The average distance of each data value from the mean. The MAD is a gauge of "on average" how different the values are from the mean value
Outcome	A possible result.
Population	The entire collection of people or objects you are studying.
Probability	It can be listed as a number between 0 and 1
Random Sampling	Choosing a sample in a way that gives every member of a population an equally likely chance of being selected.
Sample	A part of the population that we actually examine to gather information about the whole
Sample Space	All possible outcomes of a given experiment
Systematic Sampling	Choosing a sample in a methodical way.
Theoretical Probability	The expected outcome of an experiment in a probability
Trial	One round of an experiment.
Circumference	The distance around (or perimeter of) a circle. It takes slightly more than three diameters to match the circumference of a circle. More formally, the circumference of a circle is pi (π) times the diameter of the circle.
Complimentary Angles	Complementary angles are a pair of angles whose measures add to 90∘.
Cylinder	A three-dimensional shape with two opposite faces that are parallel and congruent circles. The side (lateral surface) is a rectangle that is "wrapped around" the circular faces at the ends.

Degree	A unit of measure of angles is also equal to 1360 of a complete circle. The angle below measures about 1 degree (1∘); 360 of these would just fit around a point and fill in a complete circle; 90 of them make a right angle.
Dependent Variable	One of the two variables in a relationship. Its value depends upon or is determined by the other variable called the *independent variable*.
Diameter	A segment that goes from one point on a circle through the center of the circle to another point on the circle. Also, diameter is used to indicate the length of this segment. In this circle, segment AB is a diameter.
Radius	A radius of a circle is the distance from the center of the circle to any point on the circle.
Equation	A mathematical sentence formed by setting two expressions equal.
Exterior Angles	An angle at a vertex of a polygon where the sides of the angle are one side of the polygon and the extension of the other side meeting at the vertex. In the pentagons below, angles a, b, c, d, e, f, g, h, i, and j are exterior angles.
Independent Variable	One of the two variables in a relationship. Its value determines the value of the other variable called the dependent variable.
Interior Angles	The angle inside a polygon formed by two adjacent sides of the polygon. In the pentagon below, a, b, c, d, and e are interior angles.
Numerical Expression	An expression consisting of numbers and operations

Potential Student Misconceptions

- Students assume that all events have an equally likely chance of occurring.
- Students assume that sample size is irrelevant.
- Students struggle to make connections between graphs and other representations.
- Students are often unable to distinguish between a histogram and bar graph.
- Students simply apply a formula but do not understand concepts fully and cannot articulate. For example, the area of a triangle is ½ the area of a rectangle with height and width dimensions identical to the height and base dimensions of the triangle.

Teaching Multiple Representations

Concrete Representations	
Population Simulation (Manipulatives) Survey Questions Data Collections	
Spinners	
Random Sampling Activities	
Number Cubes and Dice	
Area Models	

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<tr><td>y</td><td>2.5</td><td>5</td><td>7.5</td><td>10</td></tr>
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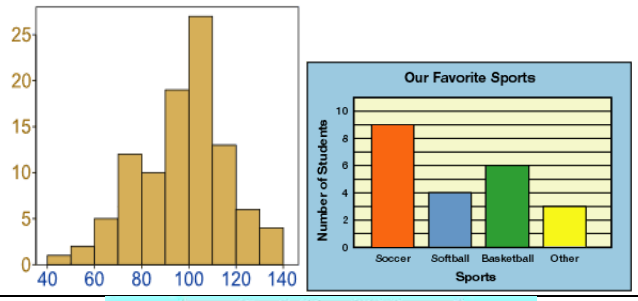
Bar Graph</td><td> 
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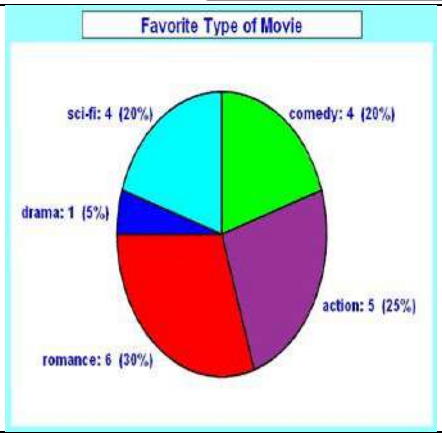
<tr><td>Pie Graphs/Charts</td><td> 
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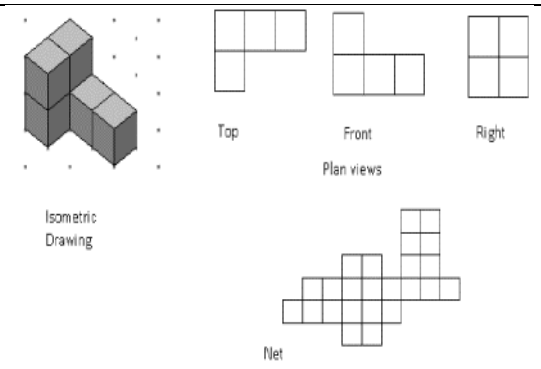
<tr><td>Experimental and Theoretical Probability Algorithm</td><td>$\frac{\text{\# of favorable outcomes}}{\text{\# of total outcomes}}$</td></tr>
<tr><td>3-Dimensional scale drawings of polygons</td><td> 
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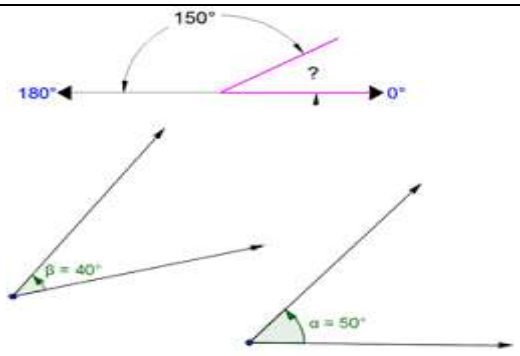
<tr><td>Angles drawn to Measure (supplementary and complementary)</td><td> 
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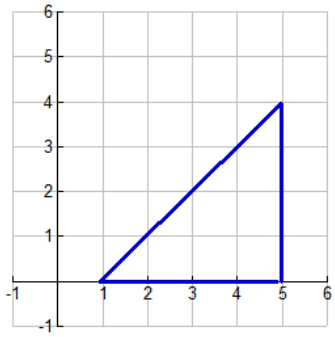
Coordinate Graphs	

Abstract Representations
Refer to the 7th grade reference sheet for formulas for surface area and volume Supplementary angles: pairs of angles that add up to 180 degrees. Complementary angles: angles whose measures sum to 90°

Assessment Framework

Unit 4 Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 4 Diagnostic Assessment (Beginning of Unit)	6.EE.7, 4.MD.3, 4.G.1, 6.G.1, 6.G.2, 6.G.4, 6.RP.3, 6.SP.4, 6.SP.5	1 Block	Individual	Yes (Score will not have weight in Genesis)
Unit 4 Check Up 1 (After Investigation 2) *What Do You Expect?*	7.SP.5,7.SP.6, 7.SP.7, 7.SP.8, 7EE.3	½ Block	Individual	Yes
Unit 4 Partner Quiz (After Investigation 4) *What Do You Expect?*	7.SP.5, 7.SP.6, 7.SP.7, 7.SP.8	½ Block	Group	Yes
Unit 4 Assessment 1 (After Investigation 5) *What Do You Expect?*	7.SP.5, 7.SP.6, 7.SP.7, 7.SP.8	1 Block	Individual	Yes
Unit 4 Partner Quiz (After Investigation 2) *Samples and Populations*	7.SP1, 7SP.2	½ Block	Group	Yes
Unit 4 Assessment 2 (After Investigation 3) Filling and Wrapping	7.SP.1, 7.SP.2	1 Block	Individual	Yes
Unit 4 Assessment 3 (After Investigation 4) Filling and Wrapping	7.G.B.4, 7.G.B.5, 7.G.B.6	1 Block	Individual	Yes

Unit 4 Performance Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 4 Performance Task 1 (Late March) *Rolling Dice*	7.SP.C.6	½ Block	Group	Yes; Rubric
Unit 4 Performance Task 2 (Late April) *Red, Green or Blue*	7.SP.C.8	½ Block	Group	Yes, Rubric
Unit 4 Performance Task 3 (Late May) *Eight Circles*	7.G.B.4	½ Block	Individual	Yes, Rubric

Performance Tasks

Rolling Dice (7.SP.C.6)

Task:

Roll two dice 10 times. After each roll, note whether any sixes were observed and record your results in the table below.

Roll	Any Sixes? (Y/N)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

a. What fraction of the 10 rolls resulted in at least one six?

b. Combine your results with those of your classmates. What fraction of all the rolls in the class resulted in at least one six?

c. Make a list of all the different possible outcomes that might be observed when two dice are rolled. (Hint: There are 36 different possible outcomes.)

d. What fraction of the 36 possible outcomes result in at least one six?

Solution:

a. Students will need two dice to roll and the answers will vary. Possible answer can be 2/10, 3/10, or 4/10. It should be centered around 11/36.
For example:

Roll	Any Sixes? (Y/N)
1	No
2	Yes
3	No
4	No
5	No
6	Yes
7	No
8	No
9	Yes
10	no

There were sixes 3/10 times

b. Combing results from the entire class should produce a result closer to the theoretical value of 11/36 = 0.305

For example: P(Rolling sixes) for 4 students = 11/40 = 0.275

Roll	Any Sixes? (Y/N)			
	Student 1	Student 2	Student 3	Student 4
1	No	N	Y	N
2	Yes	Y	N	N
3	No	Y	N	N
4	No	N	N	N
5	No	N	N	Y
6	Yes	N	Y	N
7	No	N	N	N
8	No	Y	N	Y
9	Yes	N	Y	N
10	no	N	Y	N

c. The table below gives all possible outcomes of rolling two dice. Rolls resulting in at least one six are bold.

Dice	1	2	3	4	5	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	**(1,6)**
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	**(2,6)**
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	**(3,6)**
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	**(4,6)**
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	**(5,6)**
6	**(6,1)**	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	**(6,6)**

P(Rolling six) = 11/36

Unit 4 Performance Task 1 PLD Rubric

SOLUTION

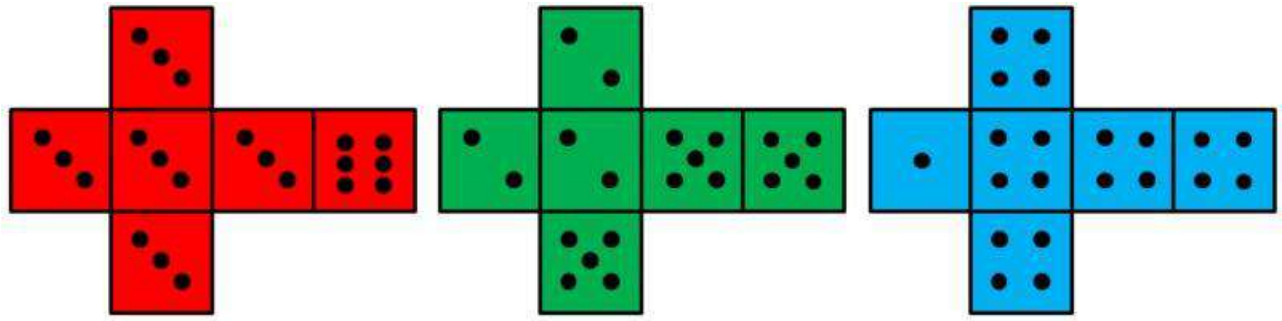
- **Student completes the table and indicates how many times he/she got sixes out of the 10 times. Answer will vary.**
- **Student completes the table with their classmates data and gets a probability that is close 0.305**
- **Student completes the theoretical table with 36 possible outcome**
- **Students write 11/36 as the probability of getting a six out of the 36 possible outcomes.**

Level 5: Distinguish Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error.	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

Red, Green or Blue (7.SP.C.8)

You have three dice; one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The numbers of dots on each side are shown in the table and picture below.

Red	3	3	3	3	3	6
Green	2	2	2	5	5	5
Blue	1	4	4	4	4	4



To play the game, each person picks one of the three dice. However, they have to pick different colors.

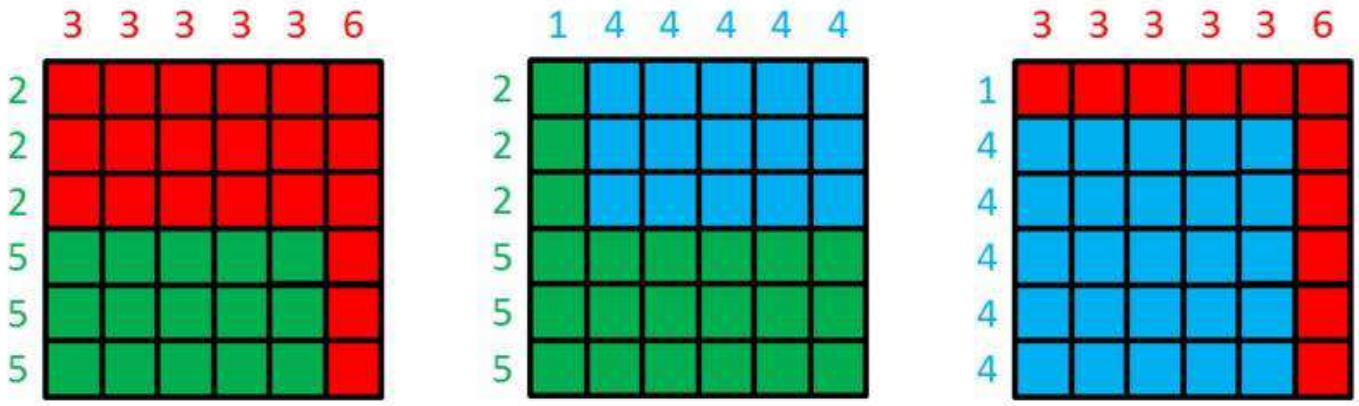
- The two players both roll their dice. The highest number wins the round.
- The players roll their dice 30 times, keeping track of who wins each round.
- Whoever has won the greatest number of rounds after 30 rolls wins the game.

a. Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red?

b. Would you rather be the first person to pick a die or the second person? Explain.

Solution:

a. There are three tables that are color-coded to see who will win which rolls in each of the three possible pairings:



When red plays green, the probability that red will win is

$$21/36 \approx 0.58$$

so red is more likely to beat green after playing many rounds. Likewise, when green plays blue, the probability that green will win is

$$21/36 \approx 0.58$$

so green is more likely to beat blue after playing many rounds. Similarly, when red plays blue, the probability that blue will win is

$$25/36 \approx 0.69$$

so blue is more likely to beat red after playing many rounds.

b. It is better to choose your die second because if your opponent goes first, you can always choose a die that is more likely to beat the die he or she chose. If your opponent chooses red, you can choose blue. If your opponent chooses blue, you can choose green. If your opponent chooses green, you can choose red.

Unit 4 Performance Task 2 PLD Rubric

SOLUTION

- **Student shows a combination of scores for Red and Green, Green and Blue and Blue and red. Student reasons with the probability and states that red is more likely to win against Green, Green is more like to win against red, and Blue is more likely to win against red**
- **Student mentions that it is better to choose your die second because if your opponent goes first, you can always choose a die that is more likely to beat the die he or she chose.**

Level 5: Distinguish Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
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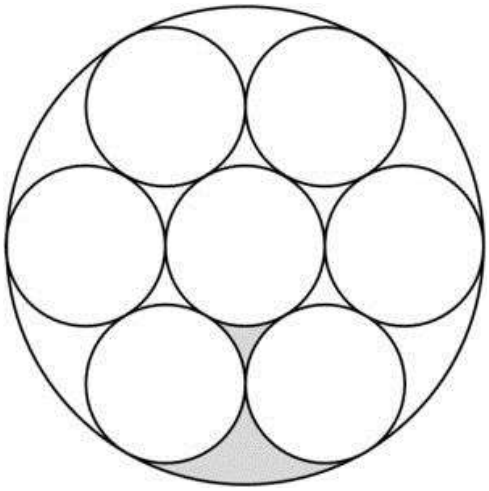
Eight Circles (7.G.B.4)

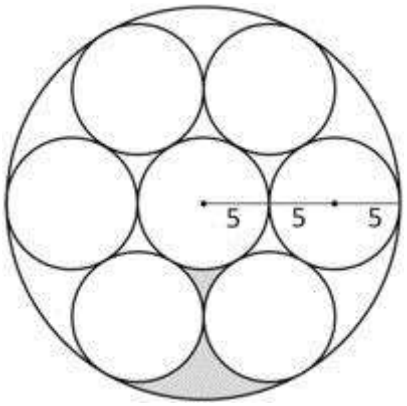
The figure below is composed of eight circles, seven small circles and one large circle containing them all. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.



Calculate:

a. The area of the large circle.
b. The area of the shaded part of the figure.

Solution



The radius of each of the seven small circles is 5 cm. This makes the radius of the large circle $3 \cdot 5 = 15$ cm.

$$\text{Area of a Circle} = \pi r^2$$

The area of the large circle is $\pi(15\text{ cm})^2 = 225\pi\text{ cm}^2$ or 706.858 cm^2.

The area of each small circle is $\pi(5\text{ cm})^2 = 25\pi\text{ cm}^2$

There are seven small circles in all, so the area of all the small circles together is

$$7 \times 25\pi\text{ cm}^2 = 175\pi\text{ cm}^2 \text{ or } 549.779\ \text{cm}^2.$$

If we take the area of the large circle and subtract the area of the seven small circles, we will be left with all of the area contained in the large circle that is not contained in a small circle, that is, the area around the small circles. This area is $225\pi\text{ cm}^2 - 175\pi\text{ cm}^2 = 50\pi\text{ cm}^2$.

or $706.858\ \text{cm}^2 - 549.779\ \text{cm}^2 = 157.079\ \text{cm}^2$.

Notice that the exact shape of the shaded region is repeated six times in the large circle. This makes the shaded region $\frac{1}{6}$ of the area that is contained in the large circle that is not contained in a small circle. Thus the shaded region has an area of

$$\frac{1}{6} \times 50\pi\text{ cm}^2 = \frac{50\pi}{6}\ \text{cm}^2 = \frac{25\pi}{3}\ \text{cm}^2.$$

or $157.079\ \text{cm}^2 / 6 = 26.18\ \text{cm}^2$.

Unit 4 Performance Task 3 PLD Rubric

SOLUTION

- **Students indicates that the area of the large circle is 225π cm^2 or 706.858 cm^2 and indicates the radius as 15 cm and shows/explains all the work**
- **Student indicates that area of the 7 circles is 175π cm^2 or 549.779 cm^2 and indicates the area of the gaps between the circles is**
  $(225\pi - 175\pi)$cm^2 = 50π cm^2 and indicates that the shaded region repeats six times in the circle hence the area of shaded region is
  50π cm^2/6 = 25π cm^2/3 or 157.079 cm^2/6 = 26.179 cm^2

Level 5: Distinguish Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
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Extensions and Sources

Online Resources

http://dashweb.pearsoncmg.com
- CMP3 content online

https://www.engageny.org/
- Common Core aligned curriculum

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

http://www.ixl.com/math/grade-6
- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Interactive, tracks student points, objective descriptive videos, allows for hints

http://www.doe.k12.de.us/assessment/files/Math_Grade_6.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Common core assessments and tasks designed for students with special needs

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Progressions of Expressions and Equations from grades 6-8