7th Grade Mathematics

Number Sense
Unit 1 Curriculum Map: September 9th – October 16th

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

<u>In this unit, students will ….</u>

- Explore relationships between positive and negative numbers by modeling them on a number line
- Use appropriate notation to indicate positive and negative numbers
- Compare and order positive and negative rational numbers (integers, fractions, decimals, and zero) and locate them on a number line
- Recognize and use the relationship between a number and its opposite (additive inverse) to solve problems
- Relate direction and distance to the number line
- Use models and rational numbers to represent and solve problems
- Develop understanding of operations with rational numbers and their properties
- Develop and use different models (number line, chip model) for representing addition, subtraction, multiplication, and division
- Develop algorithms for adding, subtracting, multiplying, and dividing integers
- Recognize situations in which one or more operations of rational numbers are needed
- Interpret and write mathematical sentences to show relationships and solve problems
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations
- Use parentheses and the Order of Operations in computations
- Understand and use the Commutative Property for addition and multiplication
- Apply the Distributive Property to simplify expressions and solve problems

Pacing Guide

Activity	Common Core Standards	Estimated Time
Unit 1 Diagnostic Assessment	6.NS.B.3, 6.NS.B.2, 4.NF.C.7, 4.NF.A.2, 5.NF.A.2, 5.NF.4.A 5.NF.B.6, 6.RP.A.2, 6.NS.C.7.A	1 Block
Accentuate the Negative (CMP3) Investigation 1	7.NS.A.1; 7.NS.A.1a; 7.NS.A.2; 7.NS.A.3; 7.EE.B.4b	4 Blocks
Assessment: Check Up 1 (CMP3)	7.NS.A.1; 7.NS.A.1a; 7.NS.A.2; 7.NS.A.3; 7.EE.B.4b	½ Block
Accentuate the Negative (CMP3) Investigation 2	7.NS.A.1; 7.NS.A.1b; 7.NS.A.1c; 7.NS.A.3	3 Blocks
Unit 1 Assessment 1	7.NS.A.1	½ Block
Assessment: Partner Quiz (CMP3)	7.NS.A.1; 7.NS.A.1b; 7.NS.A.1c; 7.NS.A.2; 7.NS.A.3	½ Block
Performance Task 1	7.NS.A.1	1 Block
Accentuate the Negative (CMP3) Investigation 3	7.NS.A.2; 7.NS.A.2a; 7.NS.A.2b; 7.NS.A.2c; 7.NS.A.3	3 Blocks
Unit 1 Assessment 2	7.NS.A.2	½ Block
Accentuate the Negative (CMP3) Investigation 4	7.NS.A.1; 7.NS.A.1d; 7.NS.A.2; 7.NS.A.2a; 7.NS.A.2d; 7.NS.A.3	3 Blocks
Unit 1 Assessment 3	7.NS.3	½ Block
Performance Task 2	7.NS.A.2d	1 Block
Total Time		**18½ Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

SEPTEMBER						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
		1 OPENING DAY **SUP. FORUM**	**2 PD DAY**	**3 PD DAY**	**4 PD DAY 12:30 pm Dismissal**	**5**
6	**7 Labor Day** **No School**	**8 1st Day for students**	**9**	**10 Unit 1: Number System** ***Unit 1 Diagnostic***	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**	**19**
20	**21** ***Assessment: Check Up 1***	**22**	**23**	**24 12:30 pm Student Dismissal**	**25** ***Assessment: Unit 1 Assessment 1***	**26**
27	**28** ***Assessment: Partner Quiz***	**29** ***Performance Task 1 Due***	**30**			

OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5 ***Assessment: Unit 1 Assessment 2***	6	7	8	9 ***Assessment: Unit 1 Assessment 3***	10
11	12 **Columbus Day** **No School**	13 ***Performance Task 2 Due***	14 **Solidify Unit 1 Concepts**	15 **Solidify Unit 1 Concepts**	16 **Unit 1 Complete**	17
18	19	20	21	22 **12:30 pm Student Dismissal**	23	24
25	26	27	28	29 **PD Day 12:30 pm Student Dismissal**	30	31

Math Background

In this unit students use integers to find patterns for adding, subtracting, multiplying, and dividing. Students use the rules they discovered for integers to compute with rational numbers with a specialized focus on using operations with negative rational numbers. Order of operations rules are reinforced with an emphasis on negative numbers. To help students understand the relationship between addition and subtraction and between multiplication and division, students are asked to use fact families. Also, students use the number line to compare integers and as a way to name points to left of 0.

The unit begins with giving students experiences with rational numbers, ordering numbers, and informal operation computation in a variety of contexts. Students use horizontal and vertical number lines when representing positive and negative numbers in the form of integers, fractions, and decimals. They also reinforce skills in graphing inequalities when exploring relationships between rational numbers.

Next students experiment with addition and subtraction by modeling real-world situations representing positive and negative integers and use a more sophisticated model of a number line. These experiences build the foundation for developing algorithms for addition and subtraction with positive and negative rational numbers. Students examine the Commutative Property of addition with rational numbers and then use it to simplify more complicated problems. This is followed by students developing and using algorithms for multiplying and dividing rational numbers. This completes the basic operations with rational numbers.

In the end the concepts of the unit come together as students use properties of operations in situations involving rational numbers. Students examine the Order of Operations and work with the Distributive Property. Students also solve problems in contexts that require them to decide what operations they need and to use the algorithms they have developed to find solutions.

PARCC Assessment Evidence Statements

CCSS	Evidence Statement	Clarification	Math Practices	Calculator ?
7.NS.1a	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	i) Tasks require students to recognize or identify situations of the kind described in standard 7.NA.1a.	5	No
7.NS.1b -1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. b. Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative.	i) Tasks do not have a context. ii) Tasks are not limited to integers. iii) Tasks involve a number line. iv) Tasks do not require students to show in general that a number and its opposite have a sum of 0: this aspect of standard 7.NS.1b may be assessed on the Grade 7 PBA.	5, 7	No
7.NS.1b -2	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. b. Interpret sums of rational numbers by describing real world contexts.	i) Tasks require students to produce or recognize real world contexts that correspond to given sums of rational numbers. ii) Tasks are not limited to integers. iii) Tasks do not require students to show in general that a number and its opposite have a sum of 0.	2, 3, 5	No

7.NS.1c -1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Apply this principle in real-world contexts.	i) Pool should contain tasks with and without contexts. ii) Contextual tasks might, for example, require students to create or identify a situation described by a specific equation of the general form $p - q = p + (-q)$. such as $3 - 5 = 3 + (-5)$. iii) Non-contextual tasks are not computation tasks but rather require students to demonstrate conceptual understanding, for example by identifying a sum that is equivalent to a given difference. iv) Tasks are not limited to integers.	2, 7, 5	No
7.NS.1d	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. d. Apply properties of operations as strategies to add and subtract rational numbers.	i) Tasks do not have a context. ii) Tasks are not limited to integers. iii) Tasks may involve sums and differences of 2 or 3 rational numbers. iv) Tasks require students to represent addition and subtraction on a horizontal or vertical number line, or compute a sum or difference, or demonstrate conceptual understanding for example by producing or recognizing an expression equivalent to a given sum or difference.	7, 5	No
7.NS.2a -1	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers.	i) Tasks do not have a context. ii) Tasks are not computation tasks but rather require students to demonstrate conceptual understanding, for example by providing students with a numerical expression and requiring students to produce or recognize an equivalent expression using properties of operations, particularly the distributive property.	7	No

7.NS.2a -2	Apply and extend previous understanding of multiplication and division and of fractions to multiply and divide rational numbers. a. Interpret products of rational numbers by describing real world contexts.	None	2, 4	No
7.NS.2b -1	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If *p* and *q* are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$.	i) Tasks do not have a context. ii) Tasks are not computation tasks but rather require students to demonstrate conceptual understanding, for example by providing students with a numerical expression and requiring students to produce or recognize an equivalent expression.	7	No
7.NS.2b -2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Interpret quotients of rational numbers by describing real world contexts.	None	2, 4	No
7.NS.2c	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Apply properties of operations as strategies to multiply and divide rational numbers.	i) Tasks do not have a context. ii) Tasks are not limited to integers. iii) Tasks may involve products and quotients of 2 or 3 rational numbers. iv) Tasks require students to compute a product or quotient, or demonstrate conceptual understanding for example by producing or recognizing an expression equivalent to a given expression.	7	No

7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.	i) Tasks are one-step word problems. ii) Tasks sample equally between addition/subtraction and multiplication/division. iii) Tasks involve at least one negative number. iv) Tasks are not limited to integers.	1, 4	No

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them** - Explain and demonstrate rational number operations by using symbols, visuals, words, and real life contexts - Demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.)
2	**Reason abstractly and quantitatively** - Demonstrate quantitative reasoning by representing and solving real world situations using visuals, numbers, and symbols - Demonstrate abstract reasoning by translating numerical sentences into real world situations - Students reason abstractly and quantitatively when they determine whether the product of two or more rational numbers is positive or negative in Problem 3.2 (Accentuate the Negative) and when they use the Distributive Property to compare and verify multiple solution methods in Problem 4.3(Accentuate the Negative).
3	**Construct viable arguments and critique the reasoning of others** - Discuss rules for operations with rational numbers using appropriate terminology and tools/visuals - Apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position - In Problem 1.1(Accentuate the Negative), students find the difference in points scored for two teams. They may justify their answers by finding each team's point difference from zero and then adding.
4	**Model with mathematics** - Model understanding of rational number operations using tools such as algebra tiles, counters, visual, and number lines and connect these models to solve problems involving real-world situations - Students use multiplication number sentences to model a relay race in Problem 3.1(Accentuate the Negative). They use positive and negative numbers to represent running speeds to the right and to the left. They also use positive and negative numbers to represent times in the future and in the past.
5	**Use appropriate tools strategically** - Demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, and calculators) while solving problems with rational numbers - In Problem 1.3(Accentuate the Negative), students use number lines to explore sums of positive and negative numbers in the familiar context of temperature changes.

6	Attend to precision
	- Demonstrate precision by using correct terminology and symbols and labeling units correctly - Use precision in calculation by checking the reasonableness of their answers and making adjustments accordingly - Students attend to precision when they work with the Order of Operations in Problem 4.1(Accentuate the Negative). They use parentheses in different places within expressions to make the greatest and least possible values.
7	Look for and make use of structure
	- Look for structure in positive and negative rational numbers when they place them appropriately on the number line - Use structure in calculation when considering the position of numbers on the number line - Recognize the problem solving structures of word problems and use this awareness to aid in solving - In Problem 2.4(Accentuate the Negative), students examine the structure of fact families as they rewrite addition sentences as subtraction sentences and subtraction sentences as addition sentences.
8	Look for and express regularity in repeated reasoning
	- Use manipulatives to explore the patterns of operations with rational numbers - - Use patterns to develop algorithms - Use algorithms to solve problems with a variety of problem solving structures - Students observe patterns in Problem 2.1(Accentuate the Negative) when they categorize groups of addition sentences.

Vocabulary

Term	Definition
Absolute Value	The distance between a number and zero on the number line. The symbol for absolute value is shown in this equation I-8I=8
Additive Inverse	Two numbers whose sum is 0 are additive inverses of one another. Example: ¾ and ¾ are additive inverse of one another because ¾ + (-3/4) = (-3/4) + ¾ = 0
Algorithm	A set of rules for performing a procedure.
Commutative Property	The order of the addition or multiplication of two numbers does not change the result.
Distributive Property	The Distributive Property states that for any three numbers *a*,*b*, and *c*, $a(b+c)=ab+ac$.
Integers	A number expressible in the form a or –a for some whole number a. The set of whole numbers and their opposites {…, -3, -2, -1, 0, 1, 2, 3 … }
Long Division	Standard procedure suitable for dividing simple or complex multi-digit numbers. It breaks down a division problem into a series of easier steps.
Multiplicative Inverse	Two numbers whose product is 1 are multiplicative inverses of one another. Example: ¾ and 4/3 are multiplicative inverses of one another because $(^3/_4)(^4/_3) = (^4/_3)(^3/_4) = 1$.
Natural Numbers	The set of numbers {1, 2, 3, 4, …}. Natural numbers are also called counting numbers
Negative Numbers	The set of numbers less than zero
Number Sentence	A mathematical statement that gives the relationship between two expressions that are composed of numbers and operation signs.
Opposite Numbers	Two different numbers that have the same absolute value. Example: 4 and 4 are opposite numbers because both have an absolute value of 4
Positive Numbers	The set of numbers greater than zero.
Rational Numbers	The set of numbers that can be written in the form a/b where a and b are integers and b $\neq$ 0.

Potential Student Misconceptions

- When subtracting numbers with positive and negative values, students often subtract the two numbers and use the sign of the larger number in their answer rather than realize they are actually moving up or down the number line depending on the signs of the numbers. They also become very confused when subtracting a negative and often add the numbers and make the answer negative or subtract the numbers and make the answer negative.

- Another common mistake occurs when students attempt to apply the rules for multiplying and dividing numbers to adding and subtracting. For example, if they are subtracting two negative numbers they subtract the numbers and make the answer positive. Similarly, when subtracting a negative and positive value, they subtract the two numbers make the answer negative.

- Students will frequently forget the direction to move when adding on a number line. It is advisable to start with smaller numbers that they are familiar with before giving problems with larger numbers or with fractions, or decimals.

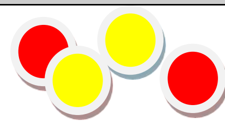
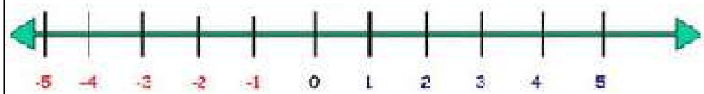
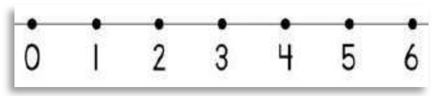
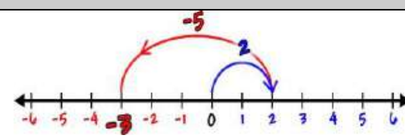
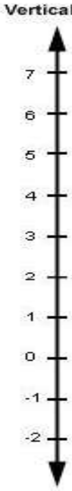
- When interpreting a negative mixed number, the students frequently assume that the whole number part is negative and the fraction part is positive instead of considering the whole mixed number as negative, both the whole number and the fraction part. Just as students are taught that 23 means 20 + 3, and that 2 ¾ means 2 + ¾, teachers should explicitly explain what -2 ¾ means. They should lead the students to understand that it means (-2 + -3/4) and not (-2 + ¾).

- Students often make the mistake of assuming that signed numbers mean only integers. They should be exposed to exercises that include signed fractions and decimals to curb this mistake.

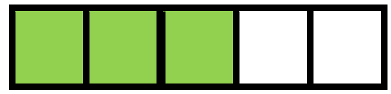
- When dealing with addition and subtraction rules, students often make the mistake of changing the sign of the first number instead of leaving it as it is and then changing the subtraction sign and changing the second number to its additive inverse. Students should spend more time working on addition and subtraction using the number line so that they may have a strong foundation and understanding of the reason that subtraction changes to addition and the second number is changed to its additive inverse

- Students may misread signs of rational numbers. When associated with a rational number, the + sign should be read as “positive.” The – sign should be read as “negative” or “the opposite of.”

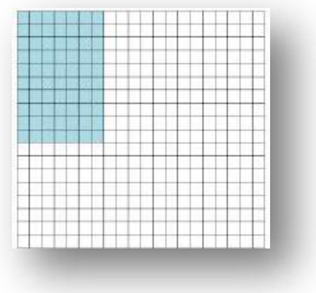
Teaching Multiple Representations

CONCRETE REPRESENTATIONS	
2-color coin counters to represent negatives and positives	
Number Lines	
Thermometer **(other equally partitioned tools)**	
Rectangular Strips	
PICTORIAL REPRESENTATIONS	
Number Lines (Horizontal)	
Number Lines (Vertical)	Figure 3 - Vertical Number Line 

- **Bar/Fraction Models**



- **100's Grid**

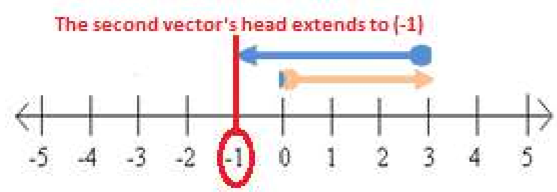


- **Distance / Vector Model**

Adding Integers

Addition is modeled as putting a second vector's tail at the first vector's head and finding where the second vector's head extends to.

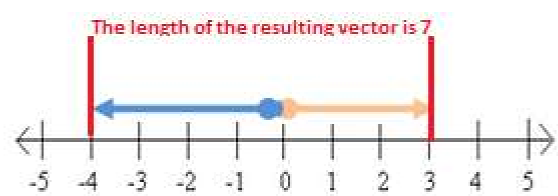
3 + -4 = -1



Subtracting Integers

Subtraction can be thought of as comparing the two vectors p, and q, by putting both tails together (starting each from zero) and asking the question: "How would one extend a vector from the head of p to the head of q?" The length and direction of that vector would be the result of the subtraction.

3 - -4 = 7



ABSTRACT REPRESENTATIONS
Applying Properties of Numbers; p – q = p + (-q); p - -q = p + q Applying Properties of Numbers Applying the standard algorithms for addition, subtraction, multiplication, and division Symbolic Representations

Assessment Framework

Unit 1 Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 1 Diagnostic Assessment (Beginning of Unit)	4.NF.C.7, 4.NF.A.2, 5.NF.A.2, 5.NF.4.A 5.NF.B.6, 6.NS.B.3, 6.NS.B.2, 6.RP.A.2, 6.NS.C.7.A	1 Block	Individual	No
Unit 1 Check Up 1 (After Investigation 1) *CMP3*	7.NS.A.1; 7.NS.A.1a; 7.NS.A.2; 7.NS.A.3; 7.EE.B.4b	½ Block	Individual	Yes
Unit 1 Assessment 1 (After Investigation 2) *Model Curriculum*	7.NS.A.1	½ Block	Individual	Yes
Unit 1 Partner Quiz (After Investigation 2) *CMP3*	7.NS.A.1; 7.NS.A.1b; 7.NS.A.1c; 7.NS.A.2; 7.NS.A.3	½ Block	Group	Yes
Unit 1 Assessment 2 (After Investigation 3) *Model Curriculum*	7.NS.A.2	½ Block	Individual	Yes
Unit 1 Assessment 3 (Conclusion of Unit) *Model Curriculum*	7.NS.A.3	½ Block	Individual	Yes
Unit 1 Check Up 2 (Optional) *CMP3*	7.NS.A.2; 7.NS.A.2a; 7.NS.A.2b; 7.NS.A.2c; 7.NS.A.3	½ Block	Individual or Group	Yes

Unit 1 Performance Assessment Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**	**Graded ?**
Unit 1 Performance Task 1 (Late September) *Comparing Differences and Distances*	7.NS.A.1	1 Block	Group	Yes; Rubric
Unit 1 Performance Task 2 (Early October) *Decimal Expansions of Fractions*	7.NS.A.2 d	1 Block	Individual w/ Interview Opportunity	Yes: rubric
Unit 1Performance Task Option 1 (optional)	7.NS.1	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 1 Performance Task Option 2 (optional)	7.NS.2	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

Unit 1 Performance Task 1

Comparing Differences and Distances (7.NS.A.1)

Task:

Conner and Aaron are working on their homework together to find the distance between two numbers, a and b, on a number line. Conner counts the units between the numbers, while Aaron subtracts the least number from the greatest. While both methods can give the correct answer, Conner and Aaron do not always apply them correctly.

a. In the first question $a = 1\frac{1}{3}$ and $b = 5\frac{1}{4}$.

Conner finds the difference $b - a$.

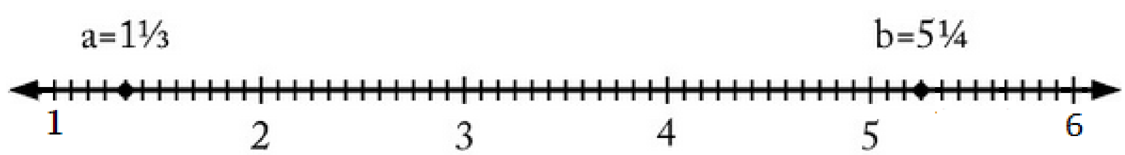
$$\begin{aligned} b - a &= 5\frac{1}{4} - 1\frac{1}{3} \\ &= \frac{21}{4} - \frac{4}{3} \\ &= \frac{63}{12} - \frac{16}{12} = \frac{47}{12} = 3\frac{11}{12} \end{aligned}$$

So Conner says that the distance between the two points is $3\frac{11}{12}$.

Aaron marks the two numbers on the number line and counts 3 whole units between them. Then he adds $\frac{1}{4}$ and $\frac{1}{3}$ to account for the additional fractional distances. Since

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

he says the distance between the two numbers is $3\frac{7}{12}$.



Which, if either of them, is correct? Find and correct any incorrect work.

b. In the second question, $a = -3\frac{1}{3}$ and $b = 2\frac{2}{5}$.

Conner finds the difference $b - a$.

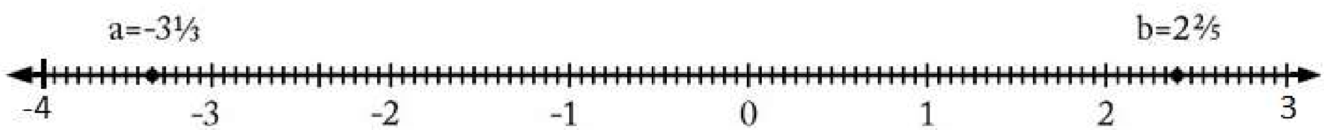
$$\begin{aligned} b - a &= 2\frac{2}{5} - 3\frac{1}{3} \\ &= \frac{12}{5} - \frac{10}{3} \\ &= \frac{36}{15} - \frac{50}{15} = \frac{14}{15} \end{aligned}$$

So Conner says that the distance between the two points is $\frac{14}{15}$.

Aaron marks the two numbers on the number line and counts 5 whole units between them. Then he adds $\frac{1}{3}$ and $\frac{2}{5}$ to account for the additional fractional distances. Since

$$\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$$

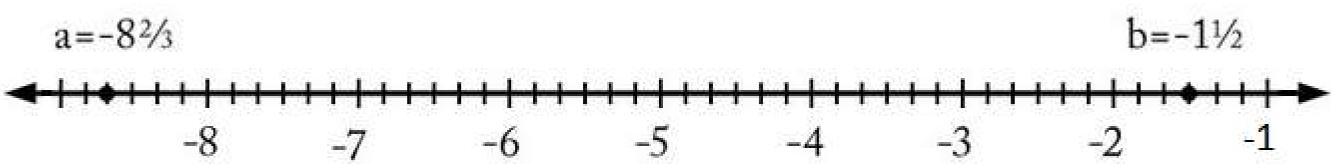
he says that the distance between the two numbers is $5\frac{11}{15}$.



Which, if either of them, is correct? Find and correct any incorrect work.

c. After talking with each other and understanding their mistakes, Aaron and Conner each obtain the correct answer with their preferred method when $a = -8\frac{2}{3}$ and $b = -1\frac{1}{2}$.

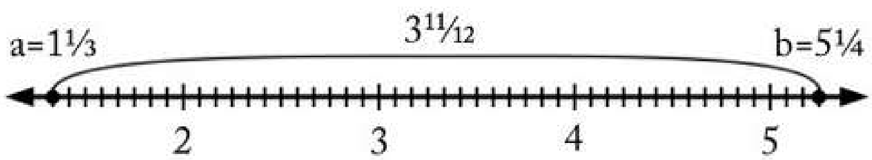
Show how each student might have arrived at his answer.



Solution:

a. Conner's answer - and application of his method - are correct.

Aaron correctly identifies 3 whole units between 2 and 5, along with $\frac{1}{4}$ of a unit between 5 and $5\frac{1}{4}$. But between $1\frac{1}{3}$ and 2 are $\frac{2}{3}$ of a unit, so by his method one should be adding 3 and $\frac{1}{4}$ and $\frac{2}{3}$ to get $3\frac{11}{12}$.

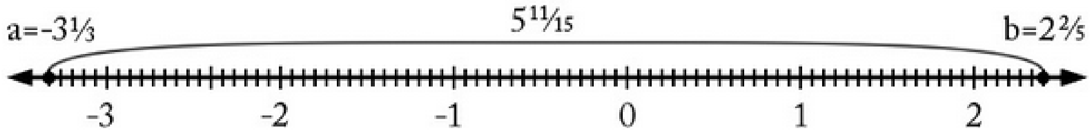


b. Aaron's answer - and application of his method - are correct.

Conner makes two errors in subtraction. The biggest error is taking $\frac{12}{5} - \frac{10}{3}$ rather than $\frac{12}{5} - \frac{-10}{3}$. This correct difference is equal to

$$\frac{12}{5} + \frac{10}{3} = \frac{36}{15} + \frac{50}{15} = \frac{86}{15} = 5\frac{11}{15}.$$

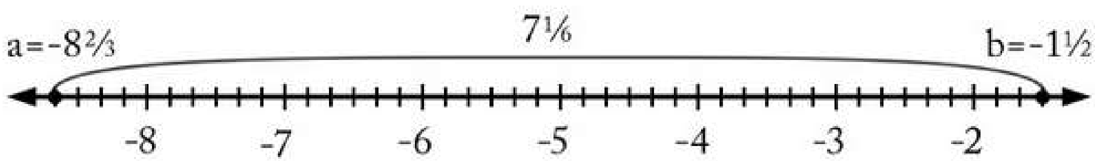
Conner makes a second error after this incorrect set-up, subtracting the smaller number $\frac{36}{15}$ from the larger $\frac{50}{15}$, probably in order to get a positive answer. If he had performed the subtraction appropriately to get a negative number, he might have caught his first error.



c. Conner subtracts the least number from the greatest:

$$\begin{aligned} -1\frac{1}{2} - (-8\frac{2}{3}) &= 8\frac{2}{3} - 1\frac{1}{2} \\ &= \frac{26}{3} - \frac{3}{2} \\ &= \frac{52-9}{6} \\ &= 7\frac{1}{6} \end{aligned}$$

Aaron counts the units between the numbers. There are 6 units between -8 and -2. There are also $\frac{2}{3}$ and $\frac{1}{2}$ of a unit distance between a and b and these integers. The total distance is $6 + \frac{4}{6} + \frac{3}{6} = 7\frac{1}{6}$.



Unit 1 Performance Task 1 PLD Rubric

SOLUTION

- Student indicates that Conner is correct for Part A. Aaron marks two points on the number line correctly. He also identifies the two units correctly and the distance between 5 and 5 ¼ However the distance between 2 and 1 and 1/3 is not 1/3, its 2/3 or 8/12.
- Student indicates that Aron's answer is right for Part B. Conner didn't include the negative sign for 3 and 1/3 when he wrote the number sentence and also he subtracted a big number from a small number, however his answer was positive.
- Student shows both methods for finding the correct answer, which is 7 and 1/6

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Unit 1 Performance Task 2

Decimal Expansion of Fractions (CCSSI 7.NS.A.2d)

Task:

Sarah learned that in order to change a fraction to a decimal, she can use the standard division algorithm and divide the numerator by the denominator. She noticed that for some fractions, like $\frac{1}{4}$ and $\frac{1}{100}$ the algorithm terminates at the hundredths place. For other fractions, like $\frac{1}{8}$, she needed to go to the thousandths place before the remainder disappears. For other fractions, like $\frac{1}{3}$ and $\frac{1}{6}$, the decimal does not terminate. Sarah wonders which fractions have terminating decimals and how she can tell how many decimal places they have.

a. Convert each of the following fractions to decimals to help Sarah look for patterns with her decimal conversions:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{10}, \frac{1}{12}, \frac{1}{15}.$$

b. Which fractions on the list have terminating decimals (decimals that eventually end in 0's)? What do the denominators have in common?

c. Which fractions on the list have repeating decimals? What do the denominators have in common?

d. Make up a fraction that will have repeating decimal.

Solution:

a. We show the long division process on the most difficult of these fractions, namely $\frac{1}{12}$:

$$\begin{array}{r} 0.0833 \\ 12\,\overline{)\,1.0000} \\ 0.96 \\ \hline 0.040 \\ 0.036 \\ \hline 0.0040 \\ 0.0036 \\ \hline 0.0004 \end{array}$$

$\frac{1}{2} = 0.5$ $\frac{1}{3} = 0.\overline{3}$ $\frac{1}{4} = 0.25$ $\frac{1}{5} = 0.2$ $\frac{1}{6} = 0.1\overline{6}$

$\frac{1}{10} = 0.1$ $\frac{1}{11} = 0.\overline{09}$ $\frac{1}{12} = 0.08\overline{3}$ $\frac{1}{15} = 0.0\overline{6}$.

b. The fractions with terminating decimals on the list are:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}.$$

The only prime factors of the denominators for each of these fractions are 2 and/or 5.

Taking $\frac{1}{4}$ as an example, we can see where the terminating decimal comes from by observing that 4 is a factor of 100: specifically we use the fact that $4 \times 25 = 100$.

$$\begin{aligned} \frac{1}{4} &= \frac{1 \times 25}{4 \times 25} \\ &= \frac{25}{100} \\ &= 0.25 \end{aligned}$$

The last equality comes from the fact that dividing by 100 moves the decimal two places to the left.

c. The fractions with repeating decimals on the list are:

$$\frac{1}{3}, \frac{1}{6}, \frac{1}{11}, \frac{1}{12}, \frac{1}{15}.$$

Each of these fractions has a prime factor different from 2 or 5 in the denominator: 3, 6, 12, and 15 have a prime factor of 3 and 11 has a prime factor of 11. Unlike in the cases in part (b), multiplying by a power of 10 will never result in a whole number here because a factor of 3 or 11 will always remain in the denominator. This means that the decimals do not terminate.

d. The examples studied here indicate that the pattern of a decimal expansion is determined by the denominator (though different numerators should be tried to see if the 1 in the numerator of all of these fractions plays an important role). When the only prime factors of the denominator are 2 and 5 the decimal terminates. When the denominator has a prime factor other than 2 or 5 the decimal eventually repeats. More work would be necessary to see if this always holds: this would mean looking at more fractions with different numerators and denominators and eventually thinking carefully about the division algorithm.

Unit 1 Performance Task 2 PLD Rubric

SOLUTION

- Student converts all the fractions into decimals correctly and writes 0.5, 0.33333333…, 0.25, 0.2, 0.16666666…, 0.1, 0.09999999.., 0.0833333333…0.0666666666….
- Student indicates that fractions with terminating decimals are ½, ¼, 1/5, and 1/10. Student indicates the pattern with the denominators for the terminating fractions. All the denominators are factors of 100. Even though 2 and 5 are prime factors, they are still factors of 100.
- Student indicates that the fractions with repeating decimals are 1/3, 1/6, 1/11, 1/12, and 1/15. None of denominators are factors of 100
- Student provides some examples of non-terminating decimals and terminating decimals and provides the reason for it. For example: 2/3, 5/6, 1/13.

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Unit 1 Performance Task Option 1

Freezing Points (7.NS.A.1)

Ocean water freezes at about $-2\frac{1}{2}\,^{0}C$. Fresh water freezes at $0^{o}C$. Antifreeze, a liquid used in the radiators of cars, freezes at $-64^{o}C$. Imagine that the temperature has dropped to the freezing point for ocean water. How many degrees more must the temperature drop for the antifreeze to turn solid?

Unit 1 Performance Task Option 2

Why is Negatives Times a Negative Always Positive (7.NS.A.2)

Some people define $3×5$ as $5+5+5$, which has a value of 15.

a. If we use the same definition for multiplication, what should the value of $3×(−5)$ be?

b. Here is an example of the distributive property:

$$3×(5+4)=3×5+3×4$$

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $3×(5+(−5))$?

If we use the fact that $5+(−5)=0$ and $3×5=15$, what should the value of $3×(−5)$ be?

c. We can multiply positive numbers in any order:

$$3×5=5×3$$

Use what you know from parts (a) and (b). If we can multiply signed numbers in any order, what should the value of $(−5)×3$ be?

If the distributive property works for both positive and negative numbers, what expression would be equivalent to $(−5)×(3+(−3))$?

d. Use what you know from parts (a), (b), and (c). What should the value of $(−5)×(−3)$ be?

Extensions

Online Resources

http://dashweb.pearsoncmg.com
- Core program resources

https://sites.google.com/site/opsmathcontent/
- Resources for content (performance tasks, useful websites, assessment items)

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

http://www.ixl.com/math/grade-7
- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/math/arithmetic/absolute-value
- Interactive, tracks student points, objective descriptive videos, allows for hints

https:// www.insidemathematics.org
- Performance tasks, scoring guides

http://turnonccmath.net
- Progression of the standards, learning trajectories, unpacking of standards

http://achievethecore.org/
- Performance Tasks, Assessments, Lessons, Tools for Planning

http://www.doe.k12.de.us/assessment/files/Math_Grade_7.pdf
- CCSS aligned assessment questions, including Next Generation Assessment Prototypes