Calculus Name______________________ ID: 1

Assignment

Date________________ Period____

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = \frac{y}{x^2}$

$\ln|y| = -\frac{1}{x} + C_1$

$y = Ce^{-\frac{1}{x}}$

2) $\frac{dy}{dx} = -\frac{2yx}{\ln y}$

$\frac{(\ln y)^2}{2} = -x^2 + C_1$

$y = e^{\sqrt{-2x^2 + C}}$ or $y = e^{-\sqrt{-2x^2 + C}}$

3) $\frac{dy}{dx} = \frac{-1 + x^2}{y^2}$

$\frac{y^3}{3} = -x + \frac{x^3}{3} + C$

$y = \sqrt[3]{x^3 - 3x + C}$

4) $\frac{dy}{dx} = \frac{3x}{y}$

$\frac{y^2}{2} = \frac{3x^2}{2} + C_1$

$y = \sqrt{3x^2 + C}$ or $y = -\sqrt{3x^2 + C}$

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

5) $\frac{dy}{dx} = \frac{2x}{e^{2y}},\ y(2) = \frac{\ln 9}{2}$

$\frac{e^{2y}}{2} = x^2 + \frac{1}{2}$

$y = \frac{\ln(2x^2 + 1)}{2}$

6) $\frac{dy}{dx} = \frac{x}{y^2},\ y(3) = \frac{\sqrt[3]{132}}{2}$

$\frac{y^3}{3} = \frac{x^2}{2} + 1$

$y = \sqrt[3]{\frac{3x^2}{2} + 3}$

7) $\frac{dy}{dx} = 2x\sqrt{y},\ y(-1) = \frac{9}{4}$

$2\sqrt{y} = x^2 + 2$

$y = \left(\frac{x^2}{2} + 1\right)^2$

8) $\frac{dy}{dx} = xy^2,\ y(-3) = -\frac{1}{5}$

$-\frac{1}{y} = \frac{x^2}{2} + \frac{1}{2}$

$y = -\frac{2}{x^2 + 1}$

Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571	2 : { 1 : integral; 1 : answer }
(b) $H(6) - R(6) = -2.924$, so the level of heating oil is falling at $t = 6$.	1 : answer with reason
(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026	3 : { 1 : limits; 1 : integrand; 1 : answer }
(d) The absolute minimum occurs at a critical point or an endpoint. $H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$. The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$. $125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$ Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.	3 : { 1 : sets $H(t) - R(t) = 0$; 1 : volume is least at $t = 11.318$; 1 : analysis for absolute minimum }

Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3: \begin{cases} 1: \text{sets } Y'(t) = 0 \\ 1: \text{critical } t\text{-value} \\ 1: \text{answer with justification} \end{cases}$