7.1 Ratios and Proportions

Recall

- Can you remember the number of …

Inches in a foot? **12**

Feet in a mile **5280 (5 tomatoes)**

mm. in a cm **10**

Feet in a yard **3**

cm. in an inch **≈ 2.54**

cm. in a meter **100**

Ounces in a pint **16**

km. in a mile **≈ 1.61**

Ounces in a pound **16**

Ratio

- **A ratio is the comparison of two numbers by division.**

- Example: A classroom has 16 boys and 12 girls.

- Boy:Girl Ratio: $\dfrac{16}{12}$ **or** **16:12 or "16 to 12"**
  - (The girl:boy ratio is the reciprocal)

- Ratios are written in lowest terms (not mixed #s)

$$\dfrac{16}{12} \begin{array}{c} \div 4 \\ \div 4 \end{array} = \dfrac{4}{3}$$

Part to Part ratio

- Ratios can be written out of the total. (16+12 = 28 students)

$\dfrac{16}{28}$ **or** $\dfrac{4}{7}$ **are boys, and** $\dfrac{12}{28}$ *or* $\dfrac{3}{7}$ **are girls**

Part to Whole ratio

Ratio, continued

- If a ratio is part-to-whole, you can divide and find a decimal or a percent.

  - $\dfrac{16 \text{ boys}}{28 \text{ students}}$

  $16 \div 28 \approx .5714$, or $\approx 57.1\%$ are boys

Ratio, continued

- Ratios can compare two unlike things:
  - Joe earned $40 in five hours
  - The ratio is $\underline{40 \text{ dollars}}$ or $\underline{8 \text{ dollars}}$
     5 hours 1 hour
  - This is read "*8 dollars per hour*."
  - When the denominator is one, this is called a **unit rate.**

Write the ratio of the 1ˢᵗ measurement to the 2ⁿᵈ

(Make sure they are in the same units!!)

- Example: Weight of a cupcake: 2 oz.

  Weight of a cake: 2lbs. 4oz.

  Cake:
  2 lb 4oz. =
  2(16) + 4 oz. =
  32 + 4 = 36oz.

  $$\frac{2}{36} = \frac{1}{18}$$

- You Try!   Length of a candy bar: 3cm

  Length of a ruler: 12 in

  Ruler:
  12 (2.54) =
  30.48 cm.

  $$\frac{3\,00}{30.48} = \frac{300}{3048} = \frac{\div 12}{\div 12} = \frac{25}{254}$$

More Ratios

Example:

- The perimeter of a rectangle is 280 cm. The ratio of the width to the length is 3:4. What is the length of the rectangle? (TIP: P = 2L + 2W)

Use the formula:

Part + Part + Part + … = Whole

- The ratio of hip hop albums to pop albums is 2:3. If the music collection has 45 albums, how many albums are hip hop?   H= 2x, P = 3x $\frac{45}{5} = \frac{5x}{5}$

45= 2x + 3x

$x = $ **9**

Hip Hop = 2(9) = 18 albums

Extended Ratios

An <u>extended ratio</u> compares three (or more) ratios!

Example:

- The lengths of three <u>sides</u> of a triangle are in the extended ratio 3 : 6 : 8. The triangle's perimeter is 5.1m. What are the lengths of the sides in cm?

100 cm = 1 m
5.1(100) = 510 cm.

$$3x + 6x + 8x = 510$$
$$17x = 510$$
$$x = 30$$

Plug it in!
3(30) = 90 cm.
6(30) = 180 cm.
8(30) = 240 cm.

Even More Ratios

You Try!

- The lengths of three angles of a triangle are in the extended ratio 1 : 2 : 3. What are the measures of each angle?

$$1x + 2x + 3x = 180$$
$$6x = 180$$
$$x = 30$$

Plug it it!
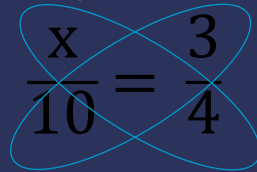$1(30) = 30°$
$2(30) = 60°$
$3(30) = 90°$

Proportion

- Proportion is a statement that says two ratios are equal.
  - Example: $\dfrac{x}{10} = \dfrac{3}{4}$

  - Use Cross-Products Property (a.k.a. Cross Multiply):
    - *"The product of the extremes is equal to the product of the means."*

    If $\dfrac{a}{b} = \dfrac{c}{d}$, then ad = bc

    $$\dfrac{x}{10} = \dfrac{3}{4}$$

    $$\dfrac{4x}{4} = \dfrac{30}{4} \qquad x = \dfrac{15}{2} \text{ or } 7.5$$

Proportion Properties

$$\frac{5}{10} = \frac{1}{2}$$

1. Proportions can be inverted! $\left(\frac{10}{5} = \frac{2}{1}\right)$

2. The means can be switched! $\frac{5}{1} = \frac{10}{2}$

3. You can add each denominator to each numerator!

$$\frac{5 + 10}{10} = \frac{1 + 2}{2}$$

$$\frac{15}{10} = \frac{3}{2}$$

Proportion

Example

- In an election, Damon got three votes for each two votes that Shannon got. Damon got 72 votes. How many votes did Shannon get?

$$\frac{\text{Damon}}{\text{Shannon}} = \begin{array}{c}\rightarrow \\ \rightarrow\end{array} \quad \frac{3}{2} = \frac{72}{s}$$

Cross multiply $3s = 72(2)$

$$\frac{3s}{3} = \frac{144}{3}$$

$s =$ **48 votes**

Proportion, continued

Example:

- Tires cost two for $75. How much will five tires cost?

$$\frac{\text{Tires}}{\$\$\$} = \frac{\rightarrow}{\rightarrow} \qquad \frac{2}{75} = \frac{5}{c}$$

$2c = 75(5)$

$\dfrac{2c}{2} = \dfrac{375}{2}$

c = $187.5

You Try!

- Three cans of soup costs $5. How much will 11 cans cost?

$$\frac{\text{Cans}}{\$\$\$} = \frac{\rightarrow}{\rightarrow} \qquad \frac{3}{5} = \frac{11}{c}$$

$3c = 5(11)$

$\dfrac{3c}{3} = \dfrac{55}{3}$

c = $18.33

More proportions

Example:

3+2 = 5

● The soccer team's win-loss ratio is 3:2, if the team played 20 games, how many did they win?

Notice: This problem involves both parts <u>and</u> the total! (win-loss-total)

$$\frac{Win}{Total} = \frac{\rightarrow}{\rightarrow} \qquad \frac{3}{5} = \frac{w}{20} \qquad \frac{60}{5} = \frac{5w}{5}$$

You Try!

5+6 = 11

$$w = \text{12 wins}$$

● A class has a girl-boy ratio of 5:6. If there were 18 boys, how many students are in the class?

$$\frac{boy}{total} = \frac{\rightarrow}{\rightarrow} \qquad \frac{6}{11} = \frac{18}{c} \qquad \frac{6c}{6} = \frac{198}{6}$$

$$c = \text{33 students}$$

Example:

Solve the proportion.

$$\frac{x}{2x+1} = \frac{16}{40}$$

Don't forget ( )

$40x = (2x+1)16$

$40x = 32x + 16$

$\underline{-32x \quad -32x}$

$\dfrac{8x}{8} = \dfrac{16}{8}$

$x = 2$

You Try!:

Solve the proportion.

$$\frac{x}{6} = \frac{4x - 5}{22}$$

$22x = 6(4x - 5)$

$22x = 24x - 30$

$\underline{-24x \quad -24x}$

$\dfrac{-2x}{-2} = \dfrac{-30}{-2}$

$x = 15$