

6th Grade Mathematics

Expressions & Equations: Extend arithmetic understandings to algebraic equations, reason about equations & inequalities, and analyze quantitative relationships

Unit 3 Curriculum Map - Math in Focus



ORANGE PUBLIC SCHOOLS
OFFICE OF CURRICULUM AND INSTRUCTION
OFFICE OF MATHEMATICS

A STORY OF UNITS

	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
K										
1										
2										
3										
4										
5										
6	The Number System			Ratios & Proportional Relationships			Expressions & Equations		Geometry	
7										



The Number System:
Division of fractions,
computation of
multi-digit numbers,
and the system of
rational numbers



**Ratios &
Proportional
Relationships:**
Understand ratio
concepts and use
ratio reasoning to
solve problems



**Expressions &
Equations:**
Arithmetic with
algebraic
expressions, solve
simple equations/
inequalities, and
analyze relationships



Geometry: Solve
real-world and
mathematical
problems involving
area, surface area,
and volume

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Unit Overview

In this unit students will ...

- Represent repeated multiplication with exponents
- Evaluate expressions containing exponents to solve mathematical and real world problems
- Translate verbal phrases and situations into algebraic expressions
- Identify the parts of a given expression
- Use the properties to identify equivalent expressions
- Use the properties and mathematical models to generate equivalent expressions
- Determine or create an equation or inequality that is appropriate for a real world situation
- Solve mathematical and real world problems with equations and inequalities
- Represent real-world situations as equations or inequalities
- Graph solutions to inequalities on a number line

Enduring Understandings

- Variables can be used as unique unknown values or as quantities that vary.
- Exponential notation is a way to express repeated products of the same number.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations
- Properties of numbers can be used to simplify and evaluate expressions.
- Algebraic properties can be used to create equivalent expressions
- Two equivalent expressions form an equation.
- Use values from a specific set to make an equation or inequality true

Pacing Guide

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 7		
Chapter 7 Recall Prior Knowledge / Pre-Test (MIF)	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3	2 Blocks
Chapter 7 (MIF) Transition Lesson	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3	1 Block
Chapter 7 (MIF) Lesson 7.1	6.EE.2a, 6.EE.2b	2 Blocks
Chapter 7 (MIF) Lesson 7.2	6.EE.2, 6.EE.2c,	2 Blocks
Chapter 7 (MIF) Lesson 7.3	6.EE.3, 6.EE.4, 6.EE.6	3 Blocks
Chapter 7 (MIF) Lesson 7.4	6.EE.2, 6.EE.3, 6.EE.4	3 Blocks
Chapter 7 (MIF) Lesson 7.5	6.EE.6	3 Blocks
Chapter 7 (MIF) Wrap-Up / Review	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3, 6.EE.4, 6.EE.6	2 Blocks
Chapter 7 Assessment (MIF) *Optional*	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3, 6.EE.4, 6.EE.6	1 Block *Optional*
Unit 3 Assessment 1		1 Block
Total Time		20 Blocks

Major Work Supporting Content Additional Content

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 8		
Chapter 8 Recall Prior Knowledge / Pre-Test (MIF)	6.EE.2a, 6.EE.5, 6.EE.8, 6.EE.9	1 Block
Chapter 8 (MIF) Transition Lesson	6.EE.2a, 6.EE.5, 6.EE.8, 6.EE.9	1 Block
Chapter 8 (MIF) Lesson 8.1	6.EE.2c, 6.EE.5	2 Blocks
Chapter 8 (MIF) Lesson 8.2	6.EE.7, 6.EE.9	2 Blocks
Chapter 8 (MIF) Lesson 8.3	6.EE.5, 6.EE.7	2 Blocks
Chapter 8 (MIF) Lesson 8.4	6.EE.7, 6.EE.8	2 Blocks
Chapter 8 (MIF) Wrap-Up / Review	6.EE.2a, 6.EE.2c, 6.EE.5, 6.EE.7 6.EE.8, 6.EE.9	2 Blocks
Chapter 8 Assessment (MIF) *Optional*	6.EE.2a, 6.EE.2c, 6.EE.5, 6.EE.7 6.EE.8, 6.EE.9	1 Block *Optional*
Unit 3 Assessment 2		1 Block
Total Time		14 Blocks

Major Work Supporting Content Additional Content

Unit 3 Overview		
Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 7 (MIF)	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3, 6.EE.4, 6.EE.6	20 Blocks
Chapter 8 (MIF)	6.EE.2a, 6.EE.2c, 6.EE.5, 6.EE.7 6.EE.8, 6.EE.9	14 Blocks
Solidify Unit 3 Concepts / Project Based Learning		5 Blocks
Total Time		39 Blocks

Major Work Supporting Content Additional Content

Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (see *Pacing Guide* on pages 3-4).

FEBRUARY						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

MARCH

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

APRIL						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Math Background

Chapter 7: Algebraic Expressions

In this chapter, students will

- Learn how to write algebraic expressions to represent situations in the world around them.
- Students will understand that algebraic expressions are sometimes called variable expressions because they contain one or more variables.
- Students learn to use variables to represent unknown quantities.
- Students learn to identify terms in algebraic expressions.
- Students learn how to evaluate, simplify, expand and factor algebraic expressions.
- Students recognize equivalent algebraic expressions.
- Students solve real-world problems using algebraic expressions.
- Relate knowledge of bar models to algebraic expressions (part-part-whole model).

Chapter 8: Equations & Inequalities

In this chapter, students will

- Use substitution to evaluate simple equations.
- Solve real-world problems by writing and solving both equations and inequalities.
- Use inverse operations to “get the variable alone” on one side of an equal sign to solve an equation.
- Students learn to think of the symbol $>$ and $<$ as meaning that two expressions are unbalance, or have different values.
- Students are introduced to the symbols $\geq$ and $\leq$, expanding their conception of how two quantities, or expressions, may compare.
- Students will then explore inequality with an unbalance scale.
- Students will also solve and graph one-step inequality on a number line.
- Learn that solutions to linear equations and inequalities may not be infinite.

PARCC Assessment Evidence Statements

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
<u>6.EE.1-1</u>	Write numerical expressions involving whole-number exponents.	i) Tasks involve expressing b-fold products $a \cdot a \cdot a \dots \cdot a$ in the form a^b , where a and b are non-zero whole numbers. ii) Tasks do not require use of the laws of exponents.	MP. 8	No
<u>6.EE.1-2</u>	Evaluate numerical expressions involving whole-number exponents	i) Tasks may involve simple fractions raised to small whole-number powers, e.g., $(\frac{1}{2})^3$, $(\frac{2}{3})^2$. ii) Tasks may involve nonnegative decimals raised to whole-number powers. iii) Tasks do not have a context.	MP. 8	Yes
<u>6.EE.2a</u>	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.	i) Tasks do not have a context. ii) Numerical values in these expressions may include whole numbers, fractions, and decimals. iii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP. 8	Yes
<u>6.EE.2b</u>	Write, read, and evaluate expressions in which letters stand for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i>	i) Tasks do not have a context. ii) Numerical values in these expressions may include whole numbers, fractions, and decimals. iii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP. 7	Yes

6th Grade Unit 3: Expressions and Equations

6.EE.2c- <u>1</u>	Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	i) Tasks do not have a context. ii) Numerical value in these expressions may include whole number, fractions, and decimals. iii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP. 7	Yes
6.EE.2c- <u>2</u>	Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their variables. <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.</i>	i) Tasks are simple applications of formulas that are provided in the prompt. ii) Tasks do not require the student to manipulate the formula or isolate variables to solve an equation. iii) Tasks have “thin context” or no context. iv) Numerical values in these expressions may include whole numbers, fractions, and decimals. v) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP. 7	Yes
6.EE.3	Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent</i>	None	MP. 7 MP. 8	No

	<i>expression $3y$.</i>			
<u>6.EE.4</u>	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</i>	None	MP.7	No
<u>6.EE.5-1</u>	Understand solving an equation as a process of answering a question: Which values from a specified set, if any, make the equation true?	i) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP.5 MP.6	Yes
<u>6.EE.5-2</u>	Use substitution to determine whether a given number in a specified set makes an inequality true.	i) 80% of tasks involve values from an infinite set of nonnegative numbers (e.g., even numbers; whole numbers; fractions). 20% of tasks involve values from a finite set of nonnegative numbers e.g., {2, 5, 7, 9}. ii) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP.5 MP.6	Yes
<u>6.EE.6</u>	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	i) Tasks may require students to write an expression to represent a real-world or mathematical problem. Tasks do not require students to find a solution. ii) Tasks may require students to interpret a variable as a specific unknown number, or, as a number that could represent any number in a specified set.	MP.2 MP.6 MP.7	No

6th Grade Unit 3: Expressions and Equations

<u>6.EE.7</u>	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q , and x are nonnegative rational numbers.	i) Problem situations are of “algebraic” type, not “arithmetic” type. ii) 50% of tasks involve fraction or decimal value of p, q, and/or x. Fractions and decimals should not appear in the same problem. iii) A valid equation and the correct answer are both required for full credit. iv) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP.1 MP.2 MP.6 MP.7	Yes
<u>6.EE.8</u>	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	i) Constraint values (denoted c in standard 6.EE.8) are not limited to integers.	MP.2 MP.6 MP.7	No
<u>6.EE.9</u>	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$</i>	i) The testing interface can provide students with a calculation aid of the specified kind for these tasks.	MP.2 MP.4 MP.6 MP.8	Yes

6th Grade Unit 3: Expressions and Equations

	<i>to represent the relationship between distance and time.</i>			
<u>6.NS.4-1</u>	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.	i) Tasks do not have a context. ii) Tasks require students to find the greatest common factor or the least common multiple only.	-	No
<u>6.NS.4-2</u>	Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.	i) Tasks do not have a context.	MP.7	No
<u>6.G.1</u>	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	i) The testing interface can provide students with a calculation aid of the specified kind for these tasks. ii) A trapezoid is defined as "A quadrilateral with at least one pair of parallel sides."	MP.1 MP.2 MP.5 MP.7	Yes



Differentiated Instruction

Chapter 7

Assessment and Intervention

	ASSESSMENT	 STRUGGLING LEARNERS
DIAGNOSTIC	- Quick Check in Recall Prior Knowledge in Student Book A, pp. 219–220 - Chapter 7 Pre-Test in Assessments	Skills 28–30 in <i>Transition Guide, Course 1</i>
ON-GOING	- Guided Practice - Lesson Check - Ticket Out the Door	- Reteach worksheets - Extra Practice worksheets - Activity Book, Chapter 7
END-OF-CHAPTER	- Chapter Review/Test - Chapter 7 Test, Mid-Course Test in Assessments  ExamView® Assessment Suite CD-ROM Course 1	Reteach worksheets

ELL ENGLISH LANGUAGE LEARNERS

Review the terms *variable*, *algebraic expression*, and *bar model*.

Say You can use the letter n to stand for a number you do not know, or a quantity that can have different values. The letter n is called a **variable**. (Write $n + 2$ on the board.) This expression contains a variable and a number. It is called an *algebraic expression*.

Model Draw a bar model to show the algebraic expression.

For definitions, see Glossary, page 272, and  Online Multilingual Glossary.

ADVANCED LEARNERS

- Students can build visual patterns from any set of identical building blocks, toothpicks, grid paper, or dot paper. For example, students could use building blocks to build perfect cubes or dot paper to form a sequence of triangular numbers.
- Have them list terms in their patterns and write expressions for the n th term in their patterns. They may need help in writing expressions for complex patterns.
- Patterns in two colors can be used to write expressions for each color, and then for the two colors combined as an application of combining like terms.

To provide additional challenges use:

- Enrichment*, Chapter 7
- Student Book A, Brain@Work problem



Differentiated Instruction

Chapter 8

Assessment and Intervention

	ASSESSMENT	 STRUGGLING LEARNERS
DIAGNOSTIC	- Quick Check in Recall Prior Knowledge in Student Book B, pp. 1–4 - Chapter 8 Pre-Test in Assessments	Skills 31–34 in <i>Transition Guide, Course 1</i>
ON-GOING	- Guided Practice - Lesson Check - Ticket Out the Door	- Reteach worksheets - Extra Practice worksheets <i>Activity Book</i>, Chapter 8
END-OF-CHAPTER	- Chapter Review/Test - Chapter 8 Test in Assessments - ExamView® Assessment Suite CD-ROM Course 1	Reteach worksheets


ENGLISH LANGUAGE LEARNERS

Review the terms *equation* and *inequality*.

Say An *equation* is a mathematical statement that two quantities are *equal*, that they have the same value.

Model Write the equation $x + 2 = 5$. Then model the equation using a balance scale. Point out that the scale is balanced, so the amounts on the two sides of the scale must be the same. They are equal: $x + 2 = 5$.

Say An *inequality* is a mathematical statement that two quantities are *not equal*, that they do *not* have the same value.

Model Write the inequality $x + 3 > 8$. Then model the inequality using a balance scale. Point out that the scale is unbalanced, so the amounts on the two sides of the scale must be different. The amount on the left side is heavier than the amount on the right, so $x + 3 > 8$.

For definitions, see Glossary, page 301, and  Online Multilingual Glossary.

ADVANCED LEARNERS

- Have students write their own real-world problems that involve inequalities. Challenge them to come up with problems where the real-world situation places limitations on the solution set, such as excluding non-integers and/or negative numbers. For example:
 Simone bought a 1-gallon container of milk. She put it in her refrigerator and used the milk. Write an inequality that best describes the amount of milk in the container, c , while it was in Simone's refrigerator. ($0 \leq c \leq 1$.) Draw a number line to represent the inequality.
- As needed, provide direction for students. Demonstrate compound inequalities. Also suggest a list of questions to consider: Is there a lower limit to the solution set? An upper limit? Can the solutions include fractions or decimals? Negative numbers?

To provide additional challenges use:

- Enrichment*, Chapter 8
- Student Book A, Brain@Work problem

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	- Students make sense of expressions and formulas by connecting them to real world contexts when evaluating. - Students create the appropriate representations for equations or inequalities
2	Reason abstractly and quantitatively
	- Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. - Students represent ideas and concepts in inequalities, equations, graphs, and table
3	Construct viable arguments and critique the reasoning of others
	- Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations. - Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables.
4	Model with mathematics
	- Students form expressions from real world contexts. Students use algebra tiles to model algebraic expressions. - Students model real world problems in equations, expressions, and inequalities
5	Use appropriate tools strategically
	- Students determine which algebraic representations are appropriate for given contexts. - Students use number lines to graph equations and inequalities
6	Attend to precision
	- Students use the language of real-world situations to create appropriate expressions. - Students accurately define variables in the context of a problem
7	Look for and make use of structure
	- Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a “term” in an expression. - Students seek patterns or structures to model problems using tables and inequalities

	Look for and express regularity in repeated reasoning
8	- Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$. - Students find processes for representing equations and inequalities

Vocabulary

Term	Definition
<i>Addition Property of Equality</i>	Adding the same number to each side of an inequality or equation produces an equivalent expression
<i>Algebraic Expression</i>	A mathematical phrase involving at least one variable and sometimes numbers and operation symbols
<i>Associative Property of Addition</i>	The sum of a set of numbers is the same no matter how the numbers are grouped.
<i>Associative Property of Multiplication</i>	The product of a set of numbers is the same no matter how the numbers are grouped.
<i>Coefficient</i>	A number multiplied by a variable in an algebraic expression.
<i>Commutative Property of Addition</i>	The sum of a group of numbers is the same regardless of the order in which the numbers are arranged.
<i>Commutative Property of Multiplication</i>	The product of a group of numbers is the same regardless of the order in which the numbers are arranged
<i>Constant</i>	A quantity that does not change its value.
<i>Dependent Variable</i>	One of the two variables in a relationship. Its value depends upon or is determined by the other variable called the <i>independent variable</i> . For example, the distance you travel on a car trip (dependent variable) depends on how long you drive (independent variable).
<i>Distributive Property</i>	The sum of two addends multiplied by a number is the sum of the product of each addend and the number.
<i>Division Property of Equality</i>	States that when both sides of an inequality or equation are divided by the same number, the remaining expressions are still equal.
<i>Exponent</i>	The number of times a number or expression (called base) is used as a factor of repeated multiplication. Also called the power.
<i>Equation</i>	A mathematical sentence that contains an equal sign.
<i>Equivalent Expressions</i>	Expressions that represent the same quantity. For example, $2+5$, $3+4$, and 7 are equivalent expressions. You can apply the Distributive Property to $2(x+3)$ to write the equivalent expression $2x+6$. You can apply the Commutative Property to $2x+6$ to write the equivalent expression $6+2x$.

6th Grade Unit 3: Expressions and Equations

<i>Expression</i>	A mathematical phrase containing numbers, variables, and operation symbols.
<i>Inequality</i>	A mathematical sentence that compares quantities with $<$, $>$, $\leq$, and $\geq$ symbols.
<i>Independent Variable</i>	One of the two variables in a relationship. Its value determines the value of the other variable called the <i>dependent variable</i> . If you organize a bike tour, for example, the number of people who register to go (independent variable) determines the cost for renting bikes (dependent variable).
<i>Inverse Operation</i>	A mathematical process that combines two or more numbers such that its product or sum equals the identify
<i>Like Terms</i>	Terms in an algebraic expression that have the same variable raised to the same power. Only the coefficients of like terms are different.
<i>Multiplication Property of Equality</i>	States that when both sides of an equation are multiplied by the same number, the remaining expressions are still equal
<i>Order of Operations</i>	The rules to be followed when simplifying expressions
<i>Rate of Change</i>	The amount of change in the dependent variable produced by a given change in the independent variable.
<i>Solution</i>	The set of all values which, when substituted for unknowns, make an equation true.
<i>Substitution</i>	The process of replacing a variable in an expression with its actual value.
<i>Term</i>	A number, a variable, or a product of numbers and variables.
<i>Variable</i>	A letter or symbol used to represent a number or quantities that vary.

Potential Student Misconceptions

- The mnemonic PEMDAS can mislead students into thinking that addition must come before subtraction and multiplication must come before division.
- Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating $3x$ as 35 when $x = 5$ instead of 3 times 5 = 15. Also, students may rewrite $8 - 2a$ as $6a$.
- Students also miss the understood “1” in front of a lone variable like a or x or p . For example, not realizing that $4a + a$ is $5a$.
- Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3 , $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $(x)(x)(x)$ which is x times x times x , not $3x$ or 3 times x ; $4x$ means 4 times x or $x + x + x + x$, not forty-something.

Teaching Multiple Representations

CONCRETE REPRESENTATIONS																
Algebra Tiles	1 x x² -1 -x -x²															
PICTORIAL REPRESENTATIONS																
Graphic Organizers i.e. input/output charts, tables, etc.	<table><tr><th>Input (days)</th><th>Output (\$)</th></tr><tr><td>1</td><td>15</td></tr><tr><td>2</td><td>20</td></tr><tr><td>4</td><td>30</td></tr><tr><td>6</td><td>40</td></tr><tr><td>9</td><td>55</td></tr><tr><td>11</td><td>65</td></tr></table>	Input (days)	Output (\$)	1	15	2	20	4	30	6	40	9	55	11	65	
Input (days)	Output (\$)															
1	15															
2	20															
4	30															
6	40															
9	55															
11	65															
Pan Balance																
ABSTRACT REPRESENTATIONS																
• Order of Operations • Properties of Addition and Multiplication • Standard algorithms for addition, subtraction, multiplication, and division																
	<table><tr><th></th><th>Word Phrases</th><th>Expression</th></tr><tr><td>+</td><td> • a number plus 5 • add 5 to a number • sum of a number and 5 • 5 more than a number • a number increased by 5 </td><td>$n + 5$</td></tr><tr><td>-</td><td> • a number minus 11 • subtract 11 from a number • difference of a number and 11 • 11 less than a number • a number decreased by 11 </td><td>$x - 11$</td></tr><tr><td>×</td><td> • 3 times a number • 3 multiplied by a number • product of 3 and a number </td><td>$3m$</td></tr><tr><td>÷</td><td> • a number divided by 7 • 7 divided into a number • quotient of a number and 7 </td><td>$\frac{a}{7}$ or $a \div 7$</td></tr></table>		Word Phrases	Expression	+	• a number plus 5 • add 5 to a number • sum of a number and 5 • 5 more than a number • a number increased by 5	$n + 5$	-	• a number minus 11 • subtract 11 from a number • difference of a number and 11 • 11 less than a number • a number decreased by 11	$x - 11$	×	• 3 times a number • 3 multiplied by a number • product of 3 and a number	$3m$	÷	• a number divided by 7 • 7 divided into a number • quotient of a number and 7	$\frac{a}{7}$ or $a \div 7$
	Word Phrases	Expression														
+	• a number plus 5 • add 5 to a number • sum of a number and 5 • 5 more than a number • a number increased by 5	$n + 5$														
-	• a number minus 11 • subtract 11 from a number • difference of a number and 11 • 11 less than a number • a number decreased by 11	$x - 11$														
×	• 3 times a number • 3 multiplied by a number • product of 3 and a number	$3m$														
÷	• a number divided by 7 • 7 divided into a number • quotient of a number and 7	$\frac{a}{7}$ or $a \div 7$														

Assessment Framework

Unit 3 Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Chapter 7 Pretest (Beginning of Unit) <i>Math in Focus</i>	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3	½ Block	Individual	Yes (No Weight)
Unit 3 Assessment 1 (After Chapter 7) <i>District Assessment</i>	6.EE.1, 6.EE.2, 6.EE.3, 6.EE.4	1 Block	Individual	Yes
Chapter 8 Pretest (After Unit 3 Assessment 1) <i>Math in Focus</i>	6.EE.2a, 6.EE.5, 6.EE.8, 6.EE.9	½ Block	Individual	Yes (No Weight)
Unit 3 Assessment 2 (Conclusion of Unit) <i>District Assessment</i>	6.EE.5, 6.EE.6, 6.EE.9	1 Block	Individual	Yes
Chapter 7 Test (Optional) <i>Math in Focus</i>	6.EE.2a, 6.EE.2b, 6.EE.2c, 6.EE.3, 6.EE.4, 6.EE.6	1 Block	Individual or Group	Yes, if administered
Chapter 8 Test (Optional) <i>Math in Focus</i>	6.EE.2a, 6.EE.2c, 6.EE.5, 6.EE.7 6.EE.8, 6.EE.9	1 Block	Individual or Group	Yes, if administered

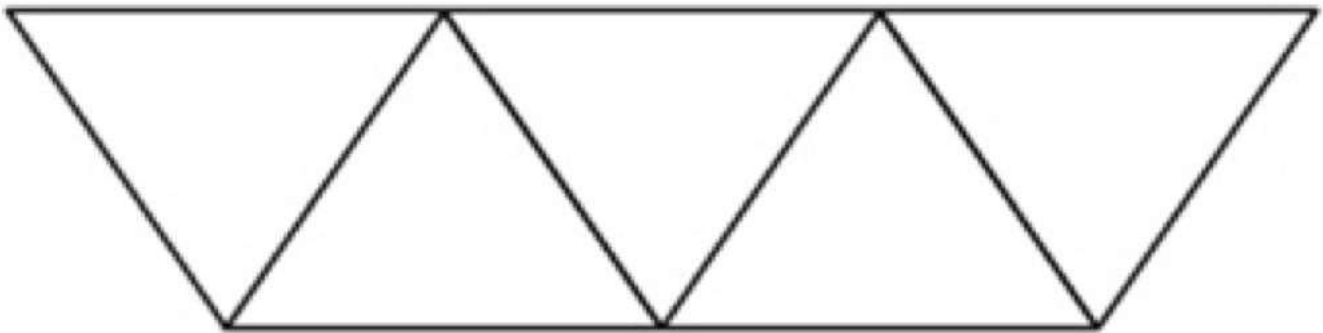
Unit 3 Performance Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Unit 3 Performance Task 1 (Early March) <i>Triangle Tables</i>	6.EE.B.5 6.EE.B.6	1 Block	Individual	Yes; Rubric
Unit 3 Performance Task 2 (Late March) <i>The School Dance</i>	6.EE.B.7	1 Block	Individual	Yes; Rubric
Unit 3 Performance Task 3 (Early April) <i>Height Requirement</i>	6.EE.B.8, 6.RP.A.2	1 Block	Individual	Yes; Rubric
Unit 3 Performance Task Option 1 (Optional) <i>Firefighter Allocation</i>	6.EE.B.6 6.EE.B.7	Teacher Discretion	Teacher Discretion	Yes, if administered
Unit 3 Performance Task Option 2 (Optional) <i>Morning Walk</i>	6.EE.B.7	Teacher Discretion	Teacher Discretion	Yes, if administered

Performance Tasks

Unit 3 Performance Task 1

Triangular Tables (6.EE.B.5, 6.EE.B.6)

A classroom has triangular tables. There is enough space at each side of a table to seat one child. The tables in the class are arranged in a row (as shown in the picture below).



(You may use dots to represent children.)

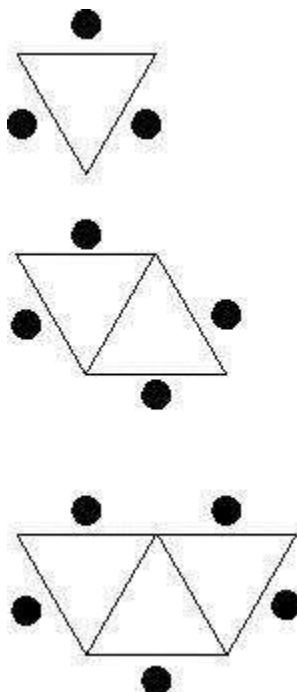
- A. How many children can sit around 1 table? Around a row of two tables? Around a row of three tables? Explain.
- B. Find an algebraic expression that describes the number of children that can sit around a row of n tables. Explain in words how you found your expression.
- C. If you could make a row of 125 tables, how many children would be able to sit around it?
- D. If there are 26 children in the class, how many tables will the teacher need to seat all the children around a row of tables? Explain your answer and how you derived at your answer.

Solution

a. Since one table has three sides and each seats one child, it follows that 3 children can sit around 1 table.

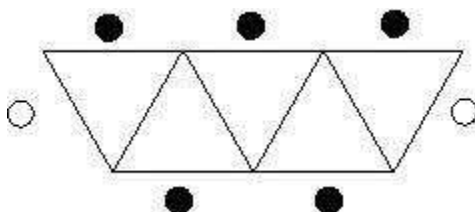
When two tables are put together in a row as pictured, then we can count the number of open sides around the perimeter of the two tables together, since an open side means one child can sit there. There are 4 sides that are open around the table, and so 4 children can sit around a row of 2 tables.

Using the same method as above, we see that when 3 tables are put into a row we will have 5 open sides around the tables. So, 5 children can sit around a row of 3 tables.



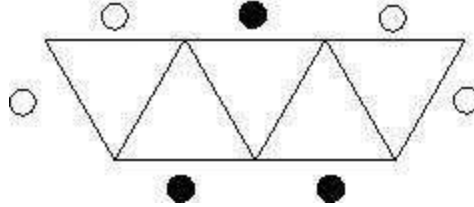
b. To find an expression that describes the number of children that can sit around a row of n tables, we can consider the diagram below. We see that we can fit 1 child at each horizontal table side (black dots) plus 1 child on the left and one on the right (white dots). So we have:

$$\text{children that can sit at } n \text{ tables} = n + 1 + 1 = n + 2$$



Another way to think about counting seats is shown in the picture below. The first table seats two children and the last table seats two children (white dots). All other $n-2$ tables seat one child (black dots). So we have:

children that can sit at n tables $= 2 \cdot 2 + (n-2) \cdot 1 = 4 + (n-2)$.



Other expressions are also possible, which are all equivalent to $n+2$.

c. Using our expression from part (b), with $n=125$, we see that

$$n+2=125+2=127,$$

so 127 children can sit around a row of 125 tables.

d. Using our expression from before we know that a row of n tables seats $n+2$ children. If we want to seat 26 children we need to find n such that $n+2=26$. So we have $n=24$, which means that the teacher needs 24 tables to seat all students in the class.

Unit 3 Performance Task 1 PLD Rubric

SOLUTION

- A) Student models by drawing a picture or table and indicate that 3 children can sit around 1 table, 4 children can sit around 2 tables, and 5 children can sit around 3 tables.
- B) Student indicates the expression $n + 2$ to represent the number children that sit around a table, where n is the number of tables and explains how they arrived to their expression by using the diagram or a pattern that they noticed from the diagram or the table.
- C) Student indicates that 127 students can sit around the table if there are 125 tables and explains how they derived the answer.
- D) Student indicates that 24 tables are needed for 26 students and explains how they derived their answer.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

6th Grade Unit 3: Expressions and Equations
Unit 3 Performance Task 2

The School Dance (6.EE.B.7)

Last year, three 6th grade students, Aliya, Crystal, and Shamika, were selling balloons at the school's Winter Dance to collect money for a class trip to Spain. Aliya sold two more balloons than Crystal, and Shamika sold twice as many balloons as Crystal. Let b represent the total number of balloons Crystal sold. Be sure to show all of your work.

- A. Write an expression to represent the number of balloons Aliya sold.
- B. Write another expression to represent the number of balloons Shamika sold.
- C. If each balloon costs \$2 and altogether Crystal, Aliya and Shamika made \$900, write an equation to represent the total sale.
- D. Solve the equation you wrote in part (c) to find the value of the variable, b .
- E. How many balloons did each student sell?
- Shamika _____ Crystal _____ Aliya _____
- F. How did you determine your answers for part (e)? Use mathematical reasoning to justify your response.

Solution

- a. b represents the number of balloons crystal sold. Aliya sold 2 more than crystal, so she sold $b + 2$ balloons
- b. b represents the number of balloons crystal sold. Shamika sold twice as many balloons as Crystal, so she sold $2*b = 2b$ balloons
- c. Crystal sold b balloons, Aliya sold $2 + b$ balloons, and Shamika sold $2b$ balloons. So all together they sold $b + 2 + b + 2b$ balloons, which simplifies to $4b + 2$ balloons.
If each balloon cost \$2.00 and all together they mad \$900 then equation will look like the following:
Cost*number of balloons together = Total cost

$$\begin{array}{rcl} \$2.00 & (4b + 2) & = \$900.00 \end{array}$$
- d. Solve for b

$$\begin{aligned} 2(4b + 2) &= 900 \\ 8b + 4 &= 900 \\ 8b + 4 - 4 &= 900 - 4 \\ 8b &= 896 \\ 8b/8 &= 896/8 \\ b &= 112 \text{ balloons} \end{aligned}$$
- e. b represents the number of balloons crystal sold, if $b = 112$ then Crystal sold 112 balloons
Aliya sold $b + 2$ so she sold $112 + 2 = 114$ balloons. Shamika sold $2b$, so she sold $2(112) = 224$ balloons.
- f. b represents the number of balloons Crystal sold, so b had to be substituted by 112 balloons in each expression and then evaluate how many balloons each person will sale. Answer needs to be checked by adding the number of balloons each person sold and then multiplying by \$2.00 to get \$900.00.

$$\begin{array}{r} 112 \\ 114 \\ 224 \\ \hline 450 \\ \times 2 \\ \hline \$900 \end{array}$$

Unit 3 Performance Task 2 PLD Rubric

SOLUTION

- **A)** Student indicates that crystal sales b balloons, Aliya sales $b + 2$ balloons and Shamika sales $2b$ balloons and explains the answer.
- **B)** Student indicates the equation $2(b + b + 2 + 2b) = 900$ and shows work.
- **C)** Student solves the equation $b = 112$ and shows work.
- **D)** Student indicates that Crystal sells 112 balloons, Aliya sells 114 balloons and Shamika sells 224 balloons and shows work.
- **E)** Student indicates that he/she substituted the value of b into each expression to find out how many balloons each person sold and shows how to check their answer using the equation they developed in part C.

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Unit 3 Performance Task 3

Height Requirement (6.EE.B.8)

At Sea World San Diego, kids are only allowed into the Air Bounce if they are over 37 inches and less than 61 inches tall. They are only allowed on the Tide Pool Climb if they are under 39 inches:



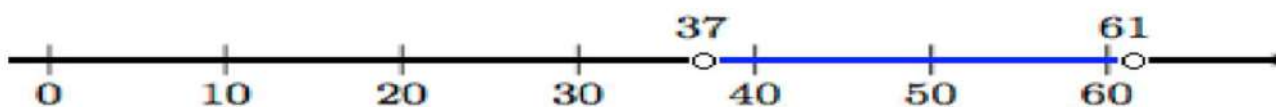
- Represent the height requirements of each ride with inequalities.
- Show the allowable heights for the rides on separate number lines.
- Using inequalities and a number line, describe the height of kids who can go on both the Air Bounce and the Tide Pool Climb.

Solution

a. For the Air Bounce, children must be at over 37 inches tall. If we let h denote the child's height in inches, this means $h > 37$. They also have to be less than 61 inches, so we can say that $h > 37$ and $h < 61$. (This can be written as a compound inequality, but that is not expected at grade 6.)

For the Tide Pool Climb, children are not allowed to be over or equal to 39 inches. Using h for the child's Height, this is represented by $h < 39$. We should also write this as $0 < h$ since a height cannot be zero or Negative.

b. The allowable heights in inches for Air Bounce are shaded blue on the number line below (note that they do not include 37 inches and 61 inches). No negative numbers are included on the number line, as this does not make sense for the context of height.



The allowable heights for Tide Pool Climb are shaded purple on the number line below:



Although it is not possible for a child to be close to 0 inches tall, these numbers are shaded because they fit the inequality $h < 39$. No negative numbers (or 0) are plotted because they do not make sense in the context of height. So the graph shows heights satisfying $0 < h$ and $h < 39$.

c. In order to go on the Tide Pool Climb, a child cannot be over or equal to 39 inches in height. In order to go on the Air Bounce, a child has to be more than 37 inches tall but less than 61 inches tall. So to go on both, a child must be 38 inches tall. The height is in inches and is also plotted on the number line below.



Unit 3 Performance Task 3 PLD Rubric

SOLUTION

A) Student indicates for air bounce the height has to be more than 37 inches but less than 61 inches and writes the inequality as $h > 37$ and $h < 61$ inches. Student also mentions that for tide pool children are not allowed to go over or equal to 39 inches and writes $h < 39$ inches.

B) Student graphs $h > 37$ and $h < 61$ on the same number line. Student graphs < 39 on the other number line. Student also mentions that the height cannot be zero, because it doesn't make any sense in the context.

C) Student indicates that for air bounce the height has to be over 37 and for tide pool the height has to be less than 39. So if a child is 38 inches he or she can go on both rides.

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Unit 3 Performance Task Option 1

Firefighter Allocation (6.EE.B.6, 6.EE.B.7)

A town's total allocation for firefighter's wages and benefits in a new budget is \$600,000. If wages are calculated at \$40,000 per firefighter and benefits at \$20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.

Unit 3 Performance Task Option 2

Morning Walk (6.EE.B.7)

Sierra walks her dog Pepper twice a day. Her evening walk is two and a half times as far as her morning walk. At the end of the week she tells her mom,

I walked Pepper for 30 miles this week!

How long is her morning walk?

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see [21st Century Career Ready Practices](#) .

Extensions and Sources

Online Resources

<http://www.illustrativemathematics.org/standards/k8>

- Performance tasks, scoring guides

<http://www.ixl.com/math/grade-6>

- Interactive, visually appealing fluency practice site that is objective descriptive

<https://www.khanacademy.org/math/arithmetic/fractions>

- Interactive, tracks student points, objective descriptive videos, allows for hints

<https://www.khanacademy.org/math/arithmetic/rates-and-ratios>

- Interactive, tracks student points, objective descriptive videos, allows for hints

http://www.doe.k12.de.us/assessment/files/Math_Grade_6.pdf

- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

<https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx>

- Common core assessments and tasks designed for students with special needs

<http://www.parcconline.org/resources/educator-resources/model-content-frameworks/mathematics-model-content-framework/grade-6>

- PARCC Model Content Frameworks Grade 6