6th Grade Mathematics

Expressions and Equations, Unit 3 Curriculum Map
January 6th – March 7th

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

Unit Overview

In this unit, students will ….

- Represent repeated multiplication with exponents
- Evaluate expressions containing exponents to solve mathematical and real world problems
- Translate verbal phrases and situations into algebraic expressions
- Identify the parts of a given expression
- Use the properties to identify equivalent expressions
- Use the properties and mathematical models to generate equivalent expressions

Enduring Understandings

- Variables can be used as unique unknown values or as quantities that vary.
- Exponential notation is a way to express repeated products of the same number.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations
- Properties of numbers can be used to simplify and evaluate expressions.
- Algebraic properties can be used to create equivalent expressions
- Two equivalent expressions form an equation.

Important Dates and Calendar

Week of …	Monday	Tuesday	Wednesday	Thursday	Friday
1/6-1/10	REVIEW MODULES				
1/13-1/17					
1/21-1/24	No School				Checkpoint
1/27-1/31	UNIT 3 NEW CONTENT				1/2 Day
2/3-2/7					
2/10-2/14					
2/17-2/21	NO SCHOOL				
2/24-2/28	UNIT 3 NEW CONTENT				
3/34-3/7	Assessment Week				

IMPORTANT DATES

Monday, Jan 20th	MLK Day
Friday, Jan 24th	Checkpoint 2 Grades 6-7
Friday, Jan 31st	1/2 Day
Week of Feb 17th	VACATION
Friday, March 14th	Data Due
Friday, March 21st	Data Returned to Principals

CMP3 Pacing Guide

Activity	Common Core Standards/SLO	Estimated Time
Common Core Review Modules	6.RP.1, 6.RP.2, 6.RP.3	12 days
Checkpoint #2 (Friday, January 24th)	SGO Standards	1 day
The Djinni's Choice *Illustrative Mathematics Task*	6.EE.1	1 day
Prime Time Investigations 2, 3, 4 (3.4 Optional)	6.EE.1, 6.EE.2b, 6.EE.3	10 days
Distance to School *Illustrative Mathematics Task*	6.EE.2	1 day
Assessment Check 1	6.EE.1, 6.EE.2	½ day
Variables and Patterns Investigations 3 and 4	6.EE.2, 6.EE.3, 6.EE.4	6 days
Equivalent Expression *Illustrative Mathematics Task*	6.EE.4	1 day
Assessment Check 2	6.EE.4	½ day
Selected Review	*Based on Assessment Checks*	2 days
Unit 2 Assessment	6.EE.1, 6.EE.2, 6.EE.3, 6.EE.4	2 days

CMP2 Pacing Guide

Activity	Common Core Standards/SLO	Estimated Time
Common Core Review Modules	6.RP.1, 6.RP.2, 6.RP.3	12 days
Checkpoint #2 (Friday, January 24th)	SGO Standards	1 day
The Djinni's Choice *Illustrative Mathematics Task*	6.EE.1	1 day
Prime Time Investigations 2, 3	6.EE.2, 6.NS.4	10 days
Distance to School *Illustrative Mathematics Task*	6.EE.2	1 day
Assessment Check 1	6.EE.1, 6.EE.2	½ day
Common Core Investigation CMP Investigation 2: Number Properties and Algebraic Equations	6.EE.2, 6.EE.3, 6.EE.4	5 days
Equivalent Expression *Illustrative Mathematics Task*	6.EE.4	1 day
Assessment Check 2	6.EE.4	½ day
Selected Review	*Based on Assessment Checks*	3 days
Unit 2 Assessment	6.EE.1, 6.EE.2, 6.EE.3, 6.EE.4	2 days

Review Content Overview

Rationale

As referenced in the PARCC Model Content Frameworks, students should build on previous 6th grade work with proportions, unit rates, and graphing in preparation for deeper learning and understanding of expressions and equations. Students have been working with equations informally since kindergarten. Building a strong foundation in earlier grade 6 standards will aid in student success in the expressions and equations standards.

Expressions and Equations questions constitute 25% of the possible points on the Grade 6 New Jersey Assessment of Skills and Knowledge. This unit plan incorporates review using the progressions of Expressions and Equations from grade 6 to grade 8.

To review previous content, Common Core Modules were specially designed. These Modules were developed for use in grades 3 – 8 and Algebra I as response to the need for rich problem solving tasks that satisfy higher levels of cognitive demand. We took cues from the National Mathematics Advisory Panel's Final Report (2008) in recognizing *the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts.*(p. xiv). The Modules contain a variety of well-selected items (offering a triad of conceptual, procedural, real world and mathematical problem solving). Students experience a steady diet of these types of problems throughout each of the Modules. In an effort to provide the District with mathematical content that is narrow, focused and deep, each Module is centered on the Critical Areas specified by the Common Core State Standards for Mathematics (CCSSM) and the Priority Areas denoted within the Partnership for Assessments of Readiness for College and Careers (PARCC) Framework.

Structure of the Modules

The Modules embody 3 integrated frameworks that promote the development of conceptual and problems solving skills and computational fluency. The **conceptual framework** of the Modules builds from the concrete to the pictorial to the abstract (and the constant blending of each) to help students develop a deeper understanding of mathematics. The Modules also reference a **multiple representations framework** that encourages teachers to present content in multiple modalities to support flexible thinking. These frameworks go beyond concrete representation (i.e. manipulatives) to promote the realistic representation of concepts addressed in multiple settings. Lastly, the Modules embody a '**gradual release' framework** that encourages teachers to progress from whole group to collaborative and finally to an independent practice format.

OVERVIEW
Each module begins with an overview. The overview provides the standards, goals, prerequisites, mathematical practices, and lesson progression.

INTRODUCTORY TASKS
The Introductory Tasks serve as the starting point for the referenced standard and are typically either diagnostic, prerequisite or anticipatory in nature.

GUIDED PRACTICE
Serves for additional teacher guided instruction for students who need the additional help. The tasks can be modeled with students.

COLLABORATIVE PRACTICE
Serve as small group, or partnered work. The work should promote student discourse, which allows students to make sense of problems and persevere in solving them (MP.1). Through teacher-facilitated, whole group discussion, students will have the opportunity to critique the reasoning of others (MP.3).

JOURNAL QUESTIONS
Provide the opportunity to individual, independent reflection and practice. This independent format encourages students to construct viable arguments (MP.3) and to reason abstractly/quantitatively (MP.2).

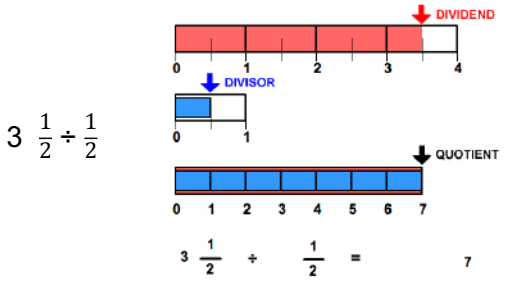
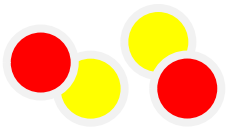
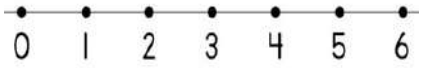
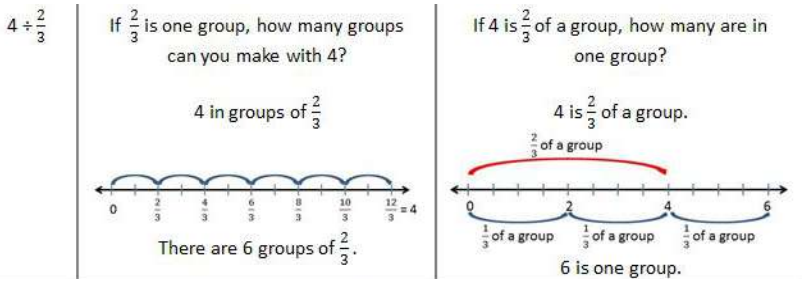
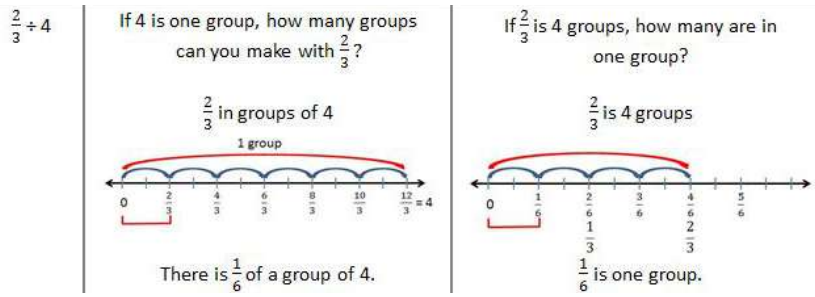
HOMEWORK
Can be used as additional in-class practice, Independent Practice, etc. This work should be reviewed and discussed. Procedural fluencies are reinforced within this section.

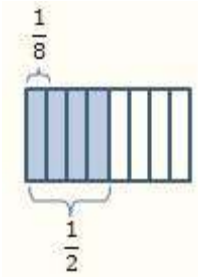
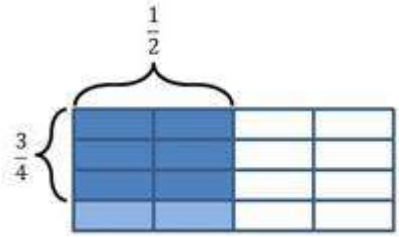
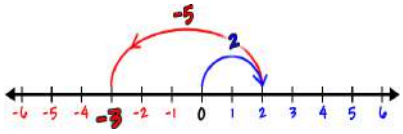
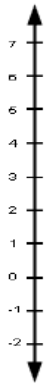
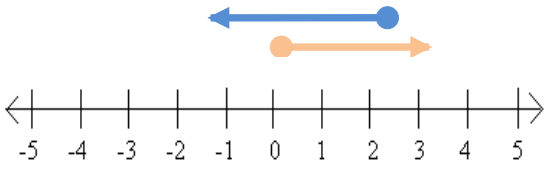
GOLDEN PROBLEM
The Golden Problem is a performance task that reflects an amalgamation of the skills addressed within the Module. The Golden Problem assesses the student's ability to apply the skills learned in a new and non-routine context. More than one-step; problems usually require intermediate values before arriving at a solution (contextual applications). In the US, we see one step problems that require either recall or routine application of an algorithm.

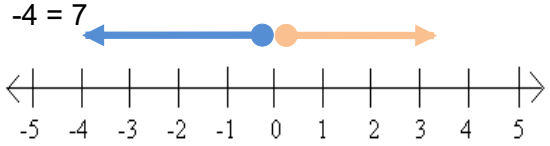
Common Core Standards – Review Content

REVIEW CONTENT	
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Teaching to Multiple Representations – Review Content

CONCRETE REPRESENTATIONS	
• Number Lines	
• 2-color coin counters to represent negatives and positives • Number Lines • Thermometers and other equally partitioned tools	 
PICTORIAL REPRESENTATIONS	
• Number Lines (Division Shown)	
• Rectangular Area Models (Division & Multiplication Shown)	

	$\frac{1}{2} \div \frac{1}{8}$  $\frac{3}{4} \times \frac{1}{2}$ 
• Number Lines (Horizontal) • Number Lines (Vertical)	 Figure 3 - Vertical Number Line 
• Distance / Vector Model	*Adding Integers* Addition is modeled as putting a second vector's tail at the first vector's head and finding where the second vector's head extends to. 3 + -4 = -1  *Subtracting Integers*

	Subtraction can be thought of as comparing the two vectors p, and q, by putting both tails together (starting each from zero) and asking the question: "How would one extend a vector from the head of p to the head of q?" The length and direction of that vector would be the result of the subtraction. 3 - -4 = 7 
ABSTRACT REPRESENTATIONS	
• Applying the Operations • Applying Properties of Numbers • Applying the standard algorithms for addition, subtraction, multiplication, and division	
• Applying Properties of Numbers	p – q = p + (-q) p - -q = p + q

Common Core Standards – Expressions and Equations

GRADE 6 EXPRESSIONS AND EQUATIONS CONTENT		
6.EE.1 (SLO 2)	Write and evaluate numerical expressions involving whole-number exponents.	CALCULATOR ALLOWED
6.EE.2 (SLO 1, SLO 3)	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*	6.EE.2a – CALCULATOR ALLOWED 6.EE.2b – CALCULATOR **NOT** ALLOWED 6.EE.2c – CALCULATOR ALLOWED
6.EE.3 (SLO 4)	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	CALCULATOR **NOT** ALLOWED
6.EE.4 (SLO 4, SLO 5)	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	CALCULATOR **NOT** ALLOWED

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	- Students make sense of expressions and formulas by connecting them to real world contexts when evaluating.
2	Reason abstractly and quantitatively
	- Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3	Construct viable arguments and critique the reasoning of others
	- Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations.
4	Model with mathematics
	- Students form expressions from real world contexts. Students use algebra tiles to model algebraic expressions.
5	Use appropriate tools strategically
	- Students determine which algebraic representations are appropriate for given contexts.
6	Attend to precision
	- Students use the language of real-world situations to create appropriate expressions.
7	Look for and make use of structure
	- Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a “term” in an expression.
8	Look for and express regularity in repeated reasoning
	- Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$.

Vocabulary

Term	Definition
Algebraic expression	A mathematical phrase involving at least one variable and sometimes numbers and operation symbols
Associative Property of Addition	The sum of a set of numbers is the same no matter how the numbers are grouped.
Associative Property of Multiplication	The product of a set of numbers is the same no matter how the numbers are grouped.
Coefficient	A number multiplied by a variable in an algebraic expression.
Commutative Property of Addition	The sum of a group of numbers is the same regardless of the order in which the numbers are arranged.
Commutative Property of Multiplication	The product of a group of numbers is the same regardless of the order in which the numbers are arranged
Constant	A quantity that does not change its value
Distributive Property	The sum of two addends multiplied by a number is the sum of the product of each addend and the number
Exponent	The number of times a number or expression (called base) is used as a factor of repeated multiplication. Also called the power.
Like Terms	Terms in an algebraic expression that have the same variable raised to the same power. Only the coefficients of like terms are different
Order of Operations	The rules to be followed when simplifying expressions
Term	A number, a variable, or a product of numbers and variables.
Variable	A letter or symbol used to represent a number or quantities that vary.

Potential Student Misconceptions

- The mnemonic PEMDAS can mislead students into thinking that addition must come before subtraction and multiplication must come before division.

- Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating 3x as 35 when x = 5 instead of 3 times 5 = 15. Also, students may rewrite 8 – 2a as 6a.

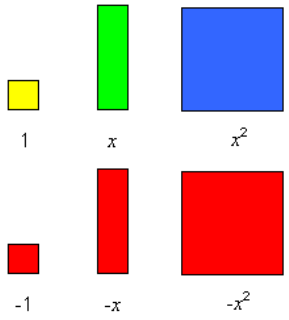
- Students also miss the understood “1” in front of a lone variable like a or x or p. For example, not realizing that 4a + a is 5a.

- Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3, 4x, 3(x + 2y) is critical. The fact that x^3 means (x)(x)(x) which is x times x times x, not 3x or 3 times x; 4x means 4 times x or x + x + x + x, not forty-something.

Teaching Multiple Representations – Major Work

CONCRETE REPRESENTATIONS

Algebra Tiles

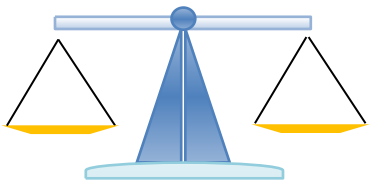


PICTORIAL REPRESENTATIONS

Graphic Organizers i.e. input/output charts, tables, etc.

Input (days)	Output ($)
1	15
2	20
4	30
6	40
9	55
11	65

Pan Balance



ABSTRACT REPRESENTATIONS

- Order of Operations
- Properties of Addition and Multiplication
- Standard algorithms for addition, subtraction, multiplication, and division

	Word Phrases	Expression
+	• a number plus 5 • add 5 to a number • sum of a number and 5 • 5 more than a number • a number increased by 5	$n + 5$
−	•a number minus 11 • subtract 11 from a number • difference of a number and 11 • 11 less than a number • a number decreased by 11	$x - 11$
×	• 3 times a number • 3 multiplied by a number • product of 3 and a number	$3m$
÷	• a number divided by 7 • 7 divided into a number • quotient of a number and 7	$\frac{a}{7}$ or $a \div 7$

Associated Illustrative Math Tasks

The Djinni's Offer (6.EE.1)

After opening an ancient bottle you find on the beach, a Djinni appears. In payment for his freedom, he gives you a choice of either 50,000 gold coins or one magical gold coin. The magic coin will turn into two gold coins on the first day. The two coins will turn into four coins total at the end of two days. By the end or the third day there will be eight gold coins total. The Djinni explains that the magic coins will continue this pattern of doubling each day for one moon cycle, 28 days. Which prize do you choose?

When you have made your choice, answer these questions:

- The number of coins on the third day will be 2×2×2. Can you write another expression using exponents for the number of coins there will be on the third day?
- Write an expression for the number of coins there will be on the 28th day. Is this more or less than a million coins?

Distance to School (6.EE.2)

Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains.

Let d be the distance in miles from a student's home to the school. Write two different expressions that represent how far a student travels by bike in a four week period if there is one rainy day each week.

Equivalent Expressions (6.EE.4)

Which of the following expressions are equivalent? Why? If an expression has no match, write 2 equivalent expressions to match it.

a. $2(x+4)$
b. $8+2x$
c. $2x+4$
d. $3(x+4)-(4+x)$
e. $x+4$

Assessment Checks

Assessment Check 1

1. Simplify the expression below.

   $3^3 - 2^2$
   a. 1
   b. 5
   c. 23
   d. 25

2. Write an expression that is equivalent to 64 using each of the following numbers and symbols only once in the expression.

   7 7 7 ^{2} (exponent of 2) + ÷ ()

3. Mike has x baseball cards. Tyrone has 3 times as many baseball cards as Mike. Frank has 20 baseball cards.

Which expression represents how many cards they have in all?

a. x + 3x + 20
b. 20 + 3x – x
c. x + 3 + 20
d. 20 – 3x + x

4. For the school dance only three members from the student government sold tickets. James sold ten more tickets than Bonnie, and Bonnie sold twice as many tickets as Mike.
   a. Write an algebraic expression to represent how many tickets all three member sold.

   b. A total of 295 tickets were sold for the dance. Using the information above, write an algebraic equation to calculate how many tickets James, Bonnie, and Mike sold individually.

   c. Since party decorations and refreshments for the dance were donated, one hundred percent of all ticket sales from the dance are considered profit. If 295 students attended the dance at a ticket cost of $6, how much money did the student government make from the dance? Show all of your work.

5. Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains.

Let d be the distance in miles from a student's home to the school. Write two different expressions that represent how far a student travels by bike in a four-week period if there is one rainy day each week.

Assessment Check 2

1. Which expression is equivalent to 3x – 3y?
   A. 3xy
   B. 3(x-y)
   C. 3x – y
   D. x – 3y

2. For items a-c, circle Yes or No to indicate whether the pairs are equivalent expressions.

   a. Are 4(3x – y) and 12x – 4y equivalent expressions? Yes No

   b. Are 32 + 16y and 8(4 + 2y) equivalent expressions? Yes No

   c. Are 3(x + 2y) and 3x + 2y equivalent expressions? Yes No

3. Which is equal to 5(2a + 9)?
   A. 10a + 45
   B. 7a + 14
   C. 7a + 45
   D. 10a + 9

4. Two expressions are shown below.

   P: 2(3x – 9)
   Q: 6x – 9

   a. Apply the distributive property to write an expression that is equivalent to expression P.
   b. Explain whether or not expressions P and Q are equivalent for any value of x.

5. Which is equal to 3x + 5 + x + 10 + 2y?
   A. 6x + 15
   B. 3x + 2y + 15
   C. 4x + 2y + 15
   D. 9x + 12y

6. Identify each expression as either equal to 12x + 36y or not equal to 12x + 36y. Write each item letter in the appropriate box below.
   a. (10x + 36y) + (2x + y)
   b. 3(4x + 5y) + 7(3y)
   c. 6(2x + 6y)
   d. 5x + 5y + x + y + 6x + 6y

Expressions Equivalent to $12x + 36y$	Expressions Not Equivalent to $12x + 36y$

Summative Assessment Resources

Summative Assessment Resources

6.EE.1-4

Rectangles

Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

a. Write an expression for Sadie's way of calculating the perimeter. Write an expression for Eric's way as well.

b. Use both of the expressions to find the perimeter of a rectangle with length 30 and width 75.

c. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Watch out for Parentheses!

Evaluate the following numerical expressions.

a. 2(5+(3)(2)+4)

b. 2((5+3)(2+4))

c. 2(5+3(2+4))

Can the parentheses in any of these expressions be removed without changing the value the expression?

Extensions and Sources

Online Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

http://www.ixl.com/math/grade-6
- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/
- Interactive, tracks student points, objective descriptive videos, allows for hints

http://www.doe.k12.de.us/assessment/files/Math_Grade_6.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx

- Common core assessments and tasks designed for students with special needs

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8

http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf
- Progressions of Expressions and Equations from grades 6-8