



North Carolina Department of Public Instruction

## INSTRUCTIONAL SUPPORT TOOLS

FOR ACHIEVING NEW STANDARDS

### **6<sup>th</sup> Grade Mathematics • Unpacked Contents**

For the new Standard Course of Study that will be effective in all North Carolina schools in the 2018-19 School Year.

This document is designed to help North Carolina educators teach the 6<sup>th</sup> Grade Mathematics Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

#### **What is the purpose of this document?**

The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

#### **What is in the document?**

This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

#### **How do I send Feedback?**

Please send feedback to us at [feedback@dpi.state.nc.us](mailto:feedback@dpi.state.nc.us) and we will use your input to refine our unpacking of the standards. Thank You!

#### **Just want the standards alone?**

You can find the standards alone at <http://www.ncpublicschools.org/curriculum/mathematics/scos/>.

# North Carolina 6<sup>th</sup> Grade Standards

## Standards for Mathematical Practice

| Ratio and Proportional Relationships                                                                                                                                                                      | The Number System                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Expressions & Equations                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Geometry                                                                                                                                                                                                              | Statistics & Probability                                                                                                                                                                                                                                             |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Understand ratio concepts and use ratio reasoning to solve problems.</p> <p><a href="#">NC.6.RP.1</a><br/> <a href="#">NC.6.RP.2</a><br/> <a href="#">NC.6.RP.3</a><br/> <a href="#">NC.6.RP.4</a></p> | <p>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</p> <p><a href="#">NC.6.NS.1</a></p> <p>Compute fluently with multi-digit numbers and find common factors and multiples.</p> <p><a href="#">NC.6.NS.2</a><br/> <a href="#">NC.6.NS.3</a><br/> <a href="#">NC.6.NS.4</a></p> <p>Apply and extend previous understandings of numbers to the system of rational numbers.</p> <p><a href="#">NC.6.NS.5</a><br/> <a href="#">NC.6.NS.6</a><br/> <a href="#">NC.6.NS.7</a><br/> <a href="#">NC.6.NS.8</a><br/> <a href="#">NC.6.NS.9</a></p> | <p>Apply and extend previous understandings of arithmetic to algebraic expressions.</p> <p><a href="#">NC.6.EE.1</a><br/> <a href="#">NC.6.EE.2</a><br/> <a href="#">NC.6.EE.3</a><br/> <a href="#">NC.6.EE.4</a></p> <p>Reason about and solve one-variable equations.</p> <p><a href="#">NC.6.EE.5</a><br/> <a href="#">NC.6.EE.6</a><br/> <a href="#">NC.6.EE.7</a></p> <p>Reason about one variable inequalities.</p> <p><a href="#">NC.6.EE.8</a></p> <p>Represent and analyze quantitative relationships between dependent and independent variables.</p> <p><a href="#">NC.6.EE.9</a></p> | <p>Solve real-world and mathematical problems involving area, surface area, and volume.</p> <p><a href="#">NC.6.G.1</a><br/> <a href="#">NC.6.G.2</a><br/> <a href="#">NC.6.G.3</a><br/> <a href="#">NC.6.G.4</a></p> | <p>Develop understanding of statistical variability.</p> <p><a href="#">NC.6.SP.1</a><br/> <a href="#">NC.6.SP.2</a><br/> <a href="#">NC.6.SP.3</a></p> <p>Summarize and describe distributions.</p> <p><a href="#">NC.6.SP.4</a><br/> <a href="#">NC.6.SP.5</a></p> |

## Standards for Mathematical Practice

| Practice                                                            | Explanation and Example                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Make sense of problems and persevere in solving them.            | In grade 6, students solve real world problems through the application of algebraic and geometric concepts. These problems involve ratio, rate, area and statistics. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”. Students can explain the relationships between equations, verbal descriptions, tables and graphs. Mathematically proficient students check answers to problems using a different method.                                                                                                          |
| 2. Reason abstractly and quantitatively.                            | In grade 6, students represent a wide variety of real world contexts through the use of numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.                                                                                                                                                                                                                                                                                                                                                                                |
| 3. Construct viable arguments and critique the reasoning of others. | In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.                                                                                                                                                            |
| 4. Model with mathematics.                                          | In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and descriptions of variability of data displays (i.e. box plots and histograms) to summarize and describe data. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. |
| 5. Use appropriate tools strategically.                             | Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to understand division and to create dot plots, histograms and box plots to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.                                                                                                                                                                              |
| 6. Attend to precision.                                             | In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.                                                                                                                                                                                                                                                                                                                                                                                                           |
| 7. Look for and make use of structure.                              | Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$ , $2c = 12$ by subtraction property of equality, $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.                                                                                                                                                                 |
| 8. Look for and express regularity in repeated reasoning.           | In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.                                                                                                                           |

Return to: [Standards](#)

## Ratio and Proportional Reasoning

### Understand ratio concepts and use ratio reasoning to solve problems.

**NC.6.RP.1** Understand the concept of a ratio and use ratio language to:

- Describe a ratio as a multiplicative relationship between two quantities.
- Model a ratio relationship using a variety of representations.

#### Clarification

This standard addresses the definition and nature of ratios.

A ratio is a comparison of two or more related quantities.

**For example:** “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.”

“For every vote candidate A received, candidate C received nearly three votes.”

These quantities may:

- be discrete, e.g., 5 cats (can’t have  $\frac{1}{2}$  a cat!)
- be continuous, e.g., 3.5 ft. (can be divided into smaller parts.)
- have the same or different units.

Students should be exposed to all combinations of these quantity types.

Using the concept of a ratio, students write ratios from known quantities in a variety of ways, including writing ratios using an initially unknown quantity. For example, in the ratio of 12 boys to 13 girls in a class, it is possible to describe this situation with a ratio of 12 boys to 25 students even though the total number of students was not directly given in the situation.

#### Describing the multiplicative relationships of ratios.

In elementary school students relied largely on additive reasoning to solve problems. While additive reasoning can be used when solving ratio problems, 6<sup>th</sup> grade students will transition to multiplicative reasoning to solve ratio problems. Students will describe two multiplicative relationships in ratios:

1. **The multiplicative relationship within a ratio.** Students will use the term **rate** to describe these relationships. In ratios, the rate is the multiplicative change from one quantity to the other quantity.
2. **The multiplicative relationship between two ratios.** Students will use the term **scale factor** to describe these relationships. In ratios, the scale factor shows the relative multiplicative change in the magnitude of the quantities from one ratio to another.

**For example:** In a simple salad dressing, a certain amount of olive oil is mixed with vinegar, as seen in the chart below. Describe the multiplicative relationships seen in the ratios.

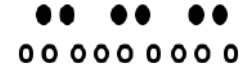
|                   |   |    |
|-------------------|---|----|
| Olive oil (Tbsp.) | 6 | 24 |
| Vinegar (Tbsp.)   | 2 | 8  |

Looking from vinegar to olive oil, this relationship has a rate of 3. Looking from olive oil to vinegar, this relationship has a rate of  $\frac{1}{3}$ .

#### Checking for Understanding

Students recorded the number of fish in an aquarium. They used a filled in circle for guppies and an open circle for goldfish. Below is their recorded count.

- a) What is the ratio of guppies to goldfish?
- b) What is the ratio of guppies to all fish?
- c) A student said that they could write the ratio of goldfish to guppies as 3 to 2. Is this student correct? Demonstrate how you know using the picture.



Ben is working on puzzles. He noticed that he completes puzzles at a steady pace. He recorded the number of puzzles he solved and how many hours it took him in the table.

|         |   |    |
|---------|---|----|
| Puzzles | 2 | 4  |
| Hours   | 6 | 12 |

- Write as many ratios from the table as you can and identify which ratios have the same multiplicative relationships.
- How can these multiplicative relationships be seen in the table?

Using a context, write three ratios that have a rate of 5.

- What other rate can be found in these ratios?
- What are the scale factors between your ratios?

**Understand ratio concepts and use ratio reasoning to solve problems.**

**NC.6.RP.1** Understand the concept of a ratio and use ratio language to:

- Describe a ratio as a multiplicative relationship between two quantities.
- Model a ratio relationship using a variety of representations.

**Clarification**

|                   |   |   |    |
|-------------------|---|---|----|
| Olive oil (Tbsp.) | 6 | 4 | 24 |
| Vinegar (Tbsp.)   | 2 | 4 | 8  |

|                   |   |    |
|-------------------|---|----|
| Olive oil (Tbsp.) | 6 | 24 |
| Vinegar (Tbsp.)   | 2 | 8  |

Looking from the first ratio to the second ratio, this relationship has a scale factor of 4. Looking from the second ratio to the first ratio, this relationship has a scale factor of  $\frac{1}{4}$ .  
*Note: While the relationship from the second ratio to the first may seem easier to describe with division, the focus remains on the multiplicative relationship and that by scaling by a number less than 1 makes the quantities smaller.*

**Different Representations for Ratios**

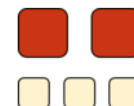
Ratios can be expressed in many forms, including but not limited to:

- Verbal expressions
- Using a colon
- Ratio boxes and tables
- Fraction notation\*
- Double number line
- Coordinate plane

*\*Fraction notation should be used with caution as fractions represent only part to whole relationships while ratios can represent both part to part and part to whole relationships. The over use of fraction notation may lead students to believing that ratios are fraction.*

**Checking for Understanding**

A recipe calls for 2 cups of tomato sauce and 3 tablespoons of oil. We can say that the ratio of cups of tomato sauce to tablespoons of oil in the recipe is 2:3, or we can say the ratio of tablespoons of oil to cups of tomato sauce is 3:2.



For each of the following situations, draw a picture and name two ratios that represent the situation.

- To make papier-mâché paste, mix 2 parts of water with 1 part of flour.
- A farm is selling 3 pounds of peaches for \$5.
- A person walks 6 miles in 2 hours.

*Taken from Illustrative Mathematics: Representing a Context with a Ratio*

**Understand ratio concepts and use ratio reasoning to solve problems.**

**NC.6.RP.2** Understand that ratios can be expressed as equivalent unit ratios by finding and interpreting both unit ratios in context.

**Clarification**

This standard asks for students to understand that unit ratios are any ratio in which one of the quantities being compared in the ratio has the value of 1. For ratios that compare two quantities, two distinct unit ratios are possible to find, unless the ratio is 1:1.

**For example:** In the ratio of 40 dollars for 10 hours of work, the unit ratios are 1 dollar for  $\frac{1}{4}$  hour of work and 4 dollars for 1 hour of work.

It is important for students to understand that:

- Unit ratios are equivalent to the original ratio.
- Finding the unit ratios reveals the two rates.

These understandings allow students to interpret the unit ratio in context.

**Checking for Understanding**

On a bicycle Jack can travel 20 miles in 4 hours. What are the unit ratios in this situation?

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Find the unit ratios for 4 candy bars for 3 dollars.

---

There are 240 students in the 6<sup>th</sup> grade with 12 teachers.

- What are the unit ratios?
- Explain the meaning of each unit ratio.

**Understand ratio concepts and use ratio reasoning to solve problems.**

**NC.6.RP.3** Use ratio reasoning with equivalent whole-number ratios to solve real-world and mathematical problems by:

- Creating and using a table to compare ratios.
- Finding missing values in the tables.
- Using a unit ratio.
- Converting and manipulating measurements using given ratios.
- Plotting the pairs of values on the coordinate plane.

**Clarification**

Students use ratio reasoning to solve problems. Ratio reasoning includes using either of the multiplicative relationships (rate or scale factor) in ratios to think through problems.

For this standard, all initial values should be whole numbers. Numbers formed in the process of working with the ratios and answers to problems may be fractions or decimals. (An exception to starting with whole numbers may occur in some measurement conversions, such as 1 inch to 2.5 cm.)

Students recognize and explain ratio equivalency in multiple ways and with various representations. Students use a variety of models to assist with solving problems. Tables, tape diagrams, double number lines, and the coordinate plane offer ways to approach equivalent ratios. The use of cross-products is not an expectation of this grade level.

**Using Ratio Tables and Unit Ratios**

Tables are a natural way to organize and study equivalent ratios. Students work with vertical and horizontal tables.

Students create ratio tables from a context and then use the multiplicative, and sometimes additive relationships, to find missing values in a table to solve problems. A key understanding, students recognize that in a table of equivalent ratios, the rates of each ratio are also equivalent.

As problems become more complex, students may use the appropriate unit ratio to find the solution.

**Comparing Ratios**

There are multiple ways of comparing ratios. In 6<sup>th</sup> grade, students are expected to use ratio tables to compare the characteristics of the ratios. This can be accomplished by using multiplicative or additive reasoning to make one of the quantities in the ratios the same or using a unit ratio to draw a conclusion based on the values of the other quantity.

**Converting and Manipulating Measurements**

Students know the conversions facts for:

- Distance in the customary system (inches, feet, yards, and miles)
- The metric system units and the prefixes: milli, centi, deci, deca, hecto, kilo
- Time

**Checking for Understanding**

Billy needs to make some lemonade for a bake sale at school. He found two recipes.

The first recipe calls to use 5 lemons for every 2 quarts of water.

The second recipe calls for 2 lemons for every quart of water.

Billy prefers a stronger lemon taste in his lemonade.

- a) Which recipe should Billy use?
- b) Show how you know this in multiple ways.

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StoriesTold.com sales its audio books at the same rate and are currently advertising 3 audio books for \$39.

What would 7 audio books cost?

How many audio books could be purchased with \$54?

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In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts? How much trail mix would be created using this ratio?

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James is making orange juice from concentrated frozen orange juice that he must mix with water. The concentrated juice is in 12 fluid ounce cartons. The ratio of orange juice concentrate to water is 12 fluid ounces to 36 fluid ounces. If James needs 4.5 gallon of orange juice, which is 576 fluid ounces, how many cartons of concentrated orange juice does he need?

**Understand ratio concepts and use ratio reasoning to solve problems.**

**NC.6.RP.3** Use ratio reasoning with equivalent whole-number ratios to solve real-world and mathematical problems by:

- Creating and using a table to compare ratios.
- Finding missing values in the tables.
- Using a unit ratio.
- Converting and manipulating measurements using given ratios.
- Plotting the pairs of values on the coordinate plane.

**Clarification**

All other conversion facts, including those between the customary and metric systems, will be provided.

Students are not expected to use dimensional analysis for conversions or make multiple unit conversions of different quantities in the ratio. For example, students will not be asked to convert feet per second to miles per hour.

**The Coordinate Plane**

Students represent equivalent ratios on a coordinate plane and use the patterns to solve problems.

Students understand that:

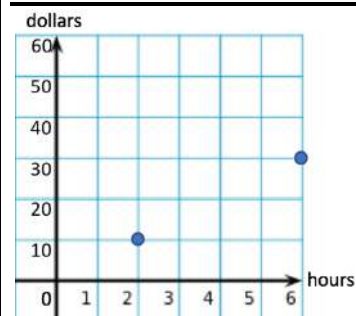
- The origin, (0,0), is an equivalent ratio to all other ratios.
- The coordinates of equivalent ratios form a straight line that is unique to that set of ratios.
- The points that fall between the coordinates that are on the straight line also represent equivalent ratios. However, it is only appropriate to draw a line through the found coordinate(s) if both quantities are continuous.

**Checking for Understanding**

How many centimeters are in 7 feet, given that 1 in.  $\approx$  2.5 cm.

Rima and Eric have earned a total of 135 tokens to buy items at the school store. The ratio of the number of tokens that Rima has to the number of tokens that Eric has is 8 to 7. How many tokens does Rima have?

NAEP – Released Item (2013) **Question ID:** 2013-8M3 #5 M150201



Jacqueline is earning money by babysitting. She graphed how many hours she worked and how much money she made for her last two jobs, one on a weeknight and one on a weekend.

- Using the information from the graph, create a table that shows how much money she earned for each hour listed on the graph.
- Plot the missing points on the graph.
- What patterns do you see on the graph?

Return to: [Standards](#)

**Understand ratio concepts and use ratio reasoning to solve problems.**

**NC.6.RP.4** Use ratio reasoning to solve real-world and mathematical problems with percents by:

- Understanding and finding a percent of a quantity as a ratio per 100.
- Using equivalent ratios, such as benchmark percents (50%, 25%, 10%, 5%, 1%), to determine a part of any given quantity.
- Finding the whole, given a part and the percent.

**Clarification**

In this standard, students will be introduced to percents and use percents to solve basic percent problems.

**Ratio Reasoning**

One of the essential understandings needed for this standard, is that a percent is a part to total ratio. The expectation of this standard is that the concepts and skills learned in the ratio standards will be applied to percents. For this reason, rules and formulaic approaches should be avoided.

As with ratios, the initial values in percent problems should only be whole numbers. The answer, or numbers produced finding the answer, may be a fraction or decimal.

Using ratio reasoning, students should:

- Identify and explain the value of the total in the part to total ratio, as the total may not be explicitly given.
- Understand that percents cannot be directly compared to other percents unless the percents are from the same context (have the same amount associated with 100%). For example, in some cases 20% of something can be a greater amount than 50% of something.

**Benchmark Percents**

The benchmark percents should be conceptually developed and their use encouraged. These percents can be developed using 100s grids and percent bars. Answering questions with benchmark percents often require the use of both multiplicative and additive reasoning.

**Percents in 6<sup>th</sup> grade**

Students will not be asked to work with percents greater than 100 in 6<sup>th</sup> grade. As with all other standards, this standard may be combined with other standards to form more steps. For example, a question may be asked for the students to find the cost of a dinner, given a bill total and a percent being left for a tip. Finding the tip would be covered under this standard while the cost of the dinner, the bill plus the tip, would be covered under 6.NS.3, fluently operating with decimals.

**Checking for Understanding**

Most dogs fail to become service dogs. In a recent training class, only 7 of the 15 dogs were certified as service dogs. What percent of dogs became certified service dogs?

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What is 40% of 30?

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Kendall bought a vase that was priced at \$450. In addition, she had to pay 3% sales tax. How much did she pay for the vase?

*Taken from Illustrative Mathematics: [Kendall's Vase – Tax](#)*

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If 44% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 11 like chocolate ice cream?

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A soccer player scored 12 goals during this season. This player scored on 30% of the shots attempted. How many shots were attempted?

Return to: [Standards](#)

## The Number System

**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

**NC.6.NS.1** Use visual models and common denominators to:

- Interpret and compute quotients of fractions.
- Solve real-world and mathematical problems involving division of fractions.

### Clarification

In 5<sup>th</sup> grade, students divided a whole number by a unit fraction or a unit fraction by a whole number. Students accomplished this division through the use of physical and visual models. In 6<sup>th</sup> grade, students will continue to use models to divide fractions.

It is the expectation of this standard that as students use models to solve division problems involving two fractions, students understand that in order to find the answer, it is necessary to find a common unit. Through repetition and reasoning with the models, students develop an algorithm of using common denominators when dividing fractions. Multiplying by the reciprocal is not the expectation of this standard and is not supported with understanding at this grade level.

**For example:** You are stuck in a big traffic jam on the freeway  $1\frac{1}{2}$  miles away from your exit. You are timing your progress and find that you travel  $\frac{2}{3}$  of a mile in one hour. If you keep moving at this slow rate, how long will it be until you get to your exit?

*Solution using a physical model (number cubes): Find how many  $\frac{2}{3}$  are in  $1\frac{1}{2}$ .*

*Using blocks, we can represent  $\frac{2}{3}$  and  $1\frac{1}{2}$ .*

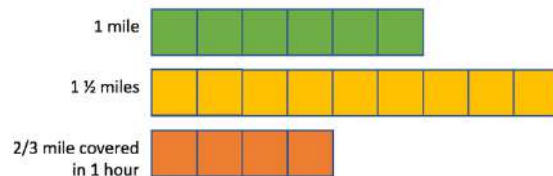
$\frac{2}{3}$ : Using 3 orange blocks to represent 1 mile, 2 blocks represent  $\frac{2}{3}$  mile.

$1\frac{1}{2}$ : Using 2 yellow blocks to represent 1 mile, 3 blocks would represent  $1\frac{1}{2}$  miles.

Notice that each color block represents a different value. Each orange block represents  $\frac{1}{3}$  and each yellow block represents  $\frac{1}{2}$ . In order to see how to find how many  $\frac{2}{3}$  are in  $1\frac{1}{2}$ , we must find a common unit or a common way to represent these numbers so that we can count.

A common unit of 2 and 3 is 6. This means that we can rework the blocks so that 1 mile is represented by 6 blocks.

This means that  $1\frac{1}{2}$  miles are represented with 9 blocks and the  $\frac{2}{3}$  mile covered in 1 hour can be represented with 4 blocks.



### Checking for Understanding

A worker is using a polyurethane spray can to seal and protect several new dinner tables. It takes  $\frac{2}{5}$  of a can to seal and protect each table. The worker has 3 full cans of spray. How many tables can the worker seal and protect?

Evaluate the following expressions using models and common denominators.

a)  $\frac{5}{6} \div \frac{1}{4}$

b)  $\frac{1}{2} \div \frac{3}{5}$

c)  $15\frac{1}{2} \div \frac{3}{4}$

d)  $4\frac{2}{7} \div 1\frac{2}{3}$

Susan has  $\frac{2}{3}$  of an hour left to make cards. It takes her about  $\frac{1}{6}$  of an hour to make each card. About how many can she make?

A rectangular parking lot has an area of  $\frac{2}{3}$  of a square kilometer. The width is  $\frac{1}{2}$  of a kilometer. What is the quotient of  $\frac{2}{3}$  and  $\frac{1}{2}$  and what does it tell us?

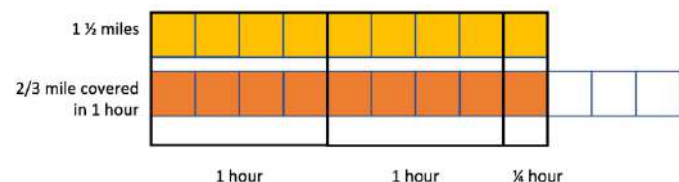
**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

**NC.6.NS.1** Use visual models and common denominators to:

- Interpret and compute quotients of fractions.
- Solve real-world and mathematical problems involving division of fractions.

**Clarification**

*Now with the problems represented, focus back to the question being asked.*



How many  $\frac{2}{3}$  are in  $1\frac{1}{2}$  miles?  
Since the  $\frac{2}{3}$  are represented with 4 blocks, we can repeat the 4 blocks

until we cover the 9 blocks representing the  $1\frac{1}{2}$  miles.

This happens  $2\frac{1}{4}$  times, representing  $2\frac{1}{4}$  hours.

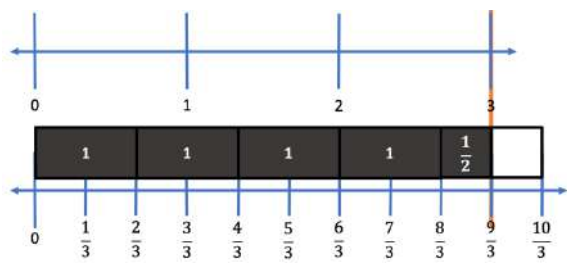
As seen in the problem above, the key understanding of this standard, is that division problems require common units. This leads the students to the concept of using a common denominator to divide fractions.

$$\frac{3}{2} \div \frac{2}{3} \rightarrow \frac{9}{6} \div \frac{4}{6} \rightarrow \frac{9 \div 4}{6 \div 6} = \frac{9/4}{1} = 9/4$$

When finding common denominators, NC.6.NS.4 has a limitation in which neither denominator should be greater than 12.

As these problems involve fractions, the remainder should be represented as a fraction. Students are expected to explain the meaning of the quotient in terms of its context and its relation to the divisor and dividend.

**For example:** Given that  $3 \div \frac{2}{3} = 4\frac{1}{2}$ , what does the  $4\frac{1}{2}$  represent?



*Solution:* The quotient,  $4\frac{1}{2}$ , represents 4 groups of  $\frac{2}{3}$  and  $\frac{1}{2}$  of another group of  $\frac{2}{3}$  in 3 wholes.

*Note:* It is possible to interpret the quotient as how many are in 1 whole. For example, if there are 3 objects in  $\frac{2}{3}$  of a

unit, there would be  $4\frac{1}{2}$  objects in a whole unit. This interpretation is unlikely in 6<sup>th</sup> grade.

**Checking for Understanding**

A recipe requires  $\frac{1}{4}$  lb of onions to make 3 servings of soup. Mark has  $1\frac{1}{2}$  lbs of onions. How many servings can Mark make?

Return to: [Standards](#)

**Compute fluently with multi-digit numbers and find common factors and multiples.****NC.6.NS.2** Fluently divide using long division with a minimum of a four-digit dividend and interpret the quotient and remainder in context.**Clarification**

This standard introduces the long division process, the standard algorithm for division, for the first time. To divide fluently, means to operate flexibly, accurately, efficiently and appropriately. In elementary, students used a variety of methods to divide (repeated subtraction, equal groups, decomposing using place value, finding greatest multiples, etc.).

In order to achieve fluency, the student must understand the meaning of division and its relationship to multiplication and place value. Students are expected to interpret the quotient and remainder in context. Students should choose an appropriate manner to write the remainder, using an R, a decimal, or a fraction.

| Describing the remainder | Example when appropriate                                                                                                                                                     |
|--------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Using R                  | When needing a count of how many will be left. <i>A group of 5 friends are dividing up Halloween treats.</i>                                                                 |
| Using a decimal          | Money, a context using decimals, metric measurements.                                                                                                                        |
| Using a fraction         | For many customary measurements, a fraction is more appropriate.<br><i>The area is <math>14\text{in}^2</math>. The length is <math>4\text{in}</math>. What is the width?</i> |

**Checking for Understanding**

Divide the following:

- |                    |                      |
|--------------------|----------------------|
| a) $2600 \div 25$  | b) $1131 \div 87$    |
| c) $1435 \div 164$ | d) $71,508 \div 531$ |

A group of 32 students have raised money to help pay for a field trip to the Outer Banks Research Park. The trip will cost \$3,200 and they have raised \$2,156. The students have to pay for the remaining cost of the trip. How much will each student have to pay?

Return to: [Standards](#)**Compute fluently with multi-digit numbers and find common factors and multiples.****NC.6.NS.3** Apply and extend previous understandings of decimals to develop and fluently use the standard algorithms for addition, subtraction, multiplication and division of decimals.**Clarification**

Students build off of previous understandings to fluently use the standard algorithms for operations with decimals. Fluently means to operate flexibly, accurately, efficiently and appropriately.

For addition and subtraction, students use reasoning with place value in the base ten number system to understand why numbers are placed to align the decimal points.

For multiplication and division, students can use estimation about products and quotients to determine an algorithm for the placement of the decimal in the quotient or product.

Students use reasoning of the base ten number system and knowledge of multiplying by tens or tenths to understand the placement of the decimal in the product or quotient.

**Checking for Understanding**

Evaluate the following:

- |                  |                      |
|------------------|----------------------|
| a) $32.57 + 7.6$ | b) $14.2 - 3.54$     |
| c) $23.67(5.8)$  | d) $2.248 \div 5.62$ |

A student claims that the number of decimal places in the product will always be the same as the total number of decimal places in the factors. Do you agree or disagree? Explain your reasoning.

Return to: [Standards](#)

**Compute fluently with multi-digit numbers and find common factors and multiples.**

**NC.6.NS.4** Understand and use prime factorization and the relationships between factors to:

- Find the unique prime factorization for a whole number.
- Find the greatest common factor of two whole numbers less than or equal to 100.
- Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.
- Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.

**Clarification**

The standard places focus on the relationship between the factors of numbers to be used as a tool when solving the specific problem types listed in the bullets. In elementary, students learned to identify primes, composites, and factor pairs.

*Note: Students may use their knowledge of multiplication facts to bypass any procedure to obtain the answer.*

**Find the unique prime factorization for a whole number.**

Students are expected to find the prime factorization of a whole number. Students learned to distinguish between prime and composite numbers in elementary. To meet this standard, students:

- Understand that each whole number has a unique prime factorization and that each prime factorization is unique to each whole number.
- Use exponents to write the prime factorization.

Factor trees and upside-down division are a few ways to organize the prime factors.

**For example:** Write the prime factorization of 18.

*Solution: Using upside down division. Start with the smallest usable prime number, 2, to divide the 18. This gives a 9. Since 9 is not prime and not divisible by 2, go the next highest prime number, 3, to divide 9. That produces 3, which is a prime number. Since this number is no longer divisible by another, the pattern is complete. Using exponents, the prime factorization is  $2 \cdot 3^2$ .*

$$\begin{array}{r} 2 \overline{) 18} \\ 3 \overline{) 9} \\ 3 \end{array}$$

**Find the greatest common factor of two whole numbers less than or equal to 100.**

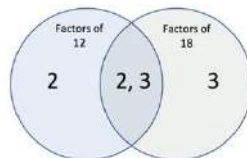
Students find the greatest common factor and defend their answer using the prime factorization of each number.

**For example:** Find the greatest common factor of 12 and 18.

*Solution: The prime factorization of 12 is  $2^2 \cdot 3$ .*

*The prime factorization of 18 is  $2 \cdot 3^2$ .*

*Using a Venn Diagram: GCF is 6 (product of numbers in the intersection).*



**Checking for Understanding**

Write the prime factorization of the following numbers:

- a) 12
- b) 24
- c) 10
- d) 60

Answer the following questions using your work from the question above.

- e) What is the multiplicative relationship between 12 and 24? How do you see this in the prime factorization?
- f) What is the greatest common factor between 12 and 24?
- g) What is the multiplicative relationship between 10 and 60? How do you see this in the prime factorization? What is the GCF of 10 and 60?
- h) What is the greatest common factor between 10 and 24?

Answer the following questions using your work from the question above.

- i) What is the least common multiple of 12 and 10?
- j) What is the GCF of 12 and 10?
- k) A common factor can always be found by multiplying the two numbers, in this case  $12 \cdot 10 = 120$ . However 120 is not the LCM. Is there a relationship between 120, the GCF of 12 and 10, and the LCM of 12 and 10?
- l) Does this relationship work with other numbers? Demonstrate how this works or does not work.

**Compute fluently with multi-digit numbers and find common factors and multiples.**

**NC.6.NS.4** Understand and use prime factorization and the relationships between factors to:

- Find the unique prime factorization for a whole number.
- Find the greatest common factor of two whole numbers less than or equal to 100.
- Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.
- Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.

**Clarification**

**Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.**

This is the students' first exposure to the process of factoring. Students use their skills of finding a GCF to rewrite the sum of two whole numbers using the distributive property. Students can then demonstrate equivalency of the expression by evaluating each expression.

**For example:** Rewrite the following into an equivalent expression, using the GCF of both numbers and the distributive property:  $32 + 54$

Solution:  $2 \cdot 16 + 2 \cdot 27 = 2(16 + 27)$

Check for equivalency:  $32 + 54 = 86$  and  $2(16 + 27) = 2(43) = 86$

**Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.**

Students find the least common multiple and justify their answer using the prime factorization of each number.

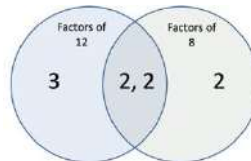
**For example:** Find the least common multiple of 12 and 8.

Solution: The prime factorization of 12 is  $2^2 \cdot 3$ .

The prime factorization of 8 is  $2^3$ .

Using a Venn Diagram:

The LCM is  $3 \cdot 2 \cdot 2 \cdot 2 = 24$ .



In 5<sup>th</sup> grade, students added and subtracted fractions that are related, meaning that one of the denominators is a factor of the other. With this limitation, students were able to change to a like denominator using equivalent fractions, for example knowing that  $\frac{1}{2}$  is equivalent to  $\frac{2}{4}$ .

In 6<sup>th</sup> grade, student will use their new tool, the LCM, to find the least common denominator, allowing them to add and subtract fractions with any denominator less than 12.

**Checking for Understanding**

Rewrite the following into equivalent expressions using the GCF of both numbers and the distributive property. When complete, evaluate the expressions to check for equivalency.

a)  $16 + 22$

b)  $12 + 18$

c)  $36 + 84$

d)  $13 + 65$

Evaluate the following expressions:

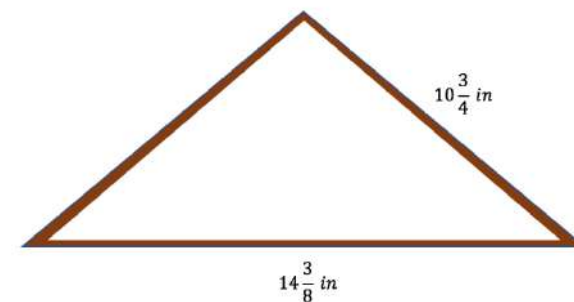
a)  $\frac{6}{7} + \frac{1}{2}$

b)  $\frac{3}{4} - \frac{7}{10}$

c)  $2\frac{3}{8} + 5\frac{5}{6}$

d)  $6\frac{1}{3} - 2\frac{5}{9}$

Simon is building a triangular picture frame. What length of wood must he buy to construct the isosceles triangle frame seen in the picture?



Return to: [Standards](#)

**Apply and extend previous understandings of numbers to the system of rational numbers.**

**NC.6.NS.5** Understand and use rational numbers to:

- Describe quantities having opposite directions or values.
- Represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- Understand the absolute value of a rational number as its distance from 0 on the number line to:
  - Interpret absolute value as magnitude for a positive or negative quantity in a real-world context.
  - Distinguish comparisons of absolute value from statements about order.

**Clarification**

This standard introduces students to the concept of negative values.

**Describe quantities having opposite directions or values.**

Students recognize real-world contexts that have positive and negative values. For example, students understand that some quantities can be measured in negative, or opposite values, such as temperature. For other quantities, this would not be appropriate, such as the number of students in a classroom.

**Represent quantities in real-world contexts, explaining the meaning of 0 in each situation.**

Students understand the meaning of 0 in each context. Understanding the meaning of zero and positive and negative values in context is crucial to create and interpret graphs.

Students understand that integers are whole numbers and their opposites.

In 6<sup>th</sup> grade, students can describe rational numbers as integers, fractions and decimals. It is not an expectation to define the complete real number system.

**Understand the absolute value of a rational number as its distance from 0 on the number line**

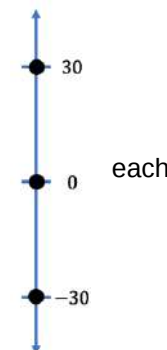
This is the students' first exposure to absolute value. Students are expected to describe absolute value as the distance of a number from zero. Students explain the differences between comparisons of the absolute value of numbers and comparisons of the numbers themselves. For example,  $-7 < 2$  but  $|-7| > |2|$ .

**Checking for Understanding**

Name three measurements that can have both positive and negative values and provide an example of a positive and negative value in each context.

Answer the following questions about the three points plotted on the number line to the right.

- a) If the number line represented temperature measured in degrees Celsius, what does each point represent and describe how it would feel if that was the temperature outside.
- b) If the number line represents your bank account, what would each point mean?



One morning the temperature is  $-28^{\circ}\text{F}$  in Anchorage, Alaska, and  $65^{\circ}\text{F}$  in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

Describe the following as true or false. If it is false, correct the statement.

- a) The farther a number is from zero, the value of the number decreases.
- b) The farther a number is from zero, the absolute value of the number increases.

If  $-3.5 < -3$ , why is  $|-3.5| > |-3|$ ?

**Apply and extend previous understandings of numbers to the system of rational numbers.**

**NC.6.NS.6** Understand rational numbers as points on the number line and as ordered pairs on a coordinate plane.

- a. On a number line:
  - Recognize opposite signs of numbers as indicating locations on opposite sides of 0 and that the opposite of the opposite of a number is the number itself.
  - Find and position rational numbers on a horizontal or vertical number line.
- b. On a coordinate plane:
  - Understand signs of numbers in ordered pairs as indicating locations in quadrants.
  - Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  - Find and position pairs of rational numbers on a coordinate plane.

**Clarification**

The standard builds upon students' previous knowledge of number lines and the coordinate plane. In 6<sup>th</sup> grade, students plot rational numbers on number lines and coordinate planes.

**The Number Line**

Using a number line, students demonstrate that they interpret a number as having both a distance from 0 (magnitude) and a direction (positive or negative). Students should be exposed to number lines, which include negative numbers, that are both horizontal and vertical, and build understanding from real world examples. Students interpret the negative sign as being the "opposite of." This reflects the magnitude of the number across 0. Students know that each iteration of the negative sign reflects the magnitude of the number across 0.

**The Coordinate Plane**

In previous grades, students were limited to coordinates in the first quadrant. In 6<sup>th</sup> grade, students are expected to identify the quadrant in which an ordered pair is located and to plot an ordered pair comprised of two rational numbers based on their understanding of horizontal and vertical number lines.

Students are expected to know that points  $(a, b)$  and  $(-a, b)$  are reflections of each other because they are:

- located on the same horizontal line
- equidistant from the  $y$ -axis but on opposite sides.

Students are expected to know that points  $(a, b)$  and  $(a, -b)$  are reflections of each other because they are:

- located on the same vertical line
- equidistant from the  $x$ -axis but on opposite sides.

**Checking for Understanding**

Place the following on a number line.

- a)  $3\frac{2}{3}$
- b)  $-3\frac{2}{3}$
- c)  $-(-3\frac{2}{3})$
- d)  $-(-(-3\frac{2}{3}))$

Use the previous work to answer the following question. What is the absolute value of each number? How is this possible?

---

Without graphing coordinates, how can you determine in which quadrant each point would be located? In which quadrant is each point located?

- a)  $(5\frac{1}{2}, -6\frac{3}{4})$
- b)  $(-5\frac{1}{2}, -6\frac{3}{4})$
- c)  $(5.5, 6.75)$
- d)  $(-5.5, 6.75)$

Graph the coordinates from the previous question. How do the locations of the points relate to each other and the axes?

---

The point  $(2.1, -3.5)$  is reflected over the  $x$ -axis, what is the coordinate of that point of reflection?

Return to: [Standards](#)

**Apply and extend previous understandings of numbers to the system of rational numbers.**

**NC.6.NS.7** Understand ordering of rational numbers.

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- Write, interpret, and explain statements of order for rational numbers in real-world contexts.

**Clarification**

Students are expected to compare and order rational numbers, which in 6<sup>th</sup> grade can be defined as integers, fractions, and decimals. Students interpret an inequality by describing its position on a number line.

**For example:** Describe the relationship between the numbers in the following inequality:  $-\frac{1}{3} > -.35$

Sample solution: Negative one third is greater than negative 35 hundredths. This means that on a number line negative one third would be to the right of negative 35 hundredths.

Given a set of rational numbers in a real-world context, students place the numbers in a particular order, explain their reasoning, and interpret meaning based on the context.

**Checking for Understanding**

Place the following points on a number line: 5,  $-3.5$ ,  $\frac{9}{2}$ ,  $-3$ ,  $-2\frac{1}{2}$   
Using the points and number line from the previous question, compare the following using  $>$ ,  $<$ , or  $=$  to describe the relationship between the value of each number.

a)  $5 \underline{\hspace{1cm}} \frac{9}{2}$

b)  $-3 \underline{\hspace{1cm}} -2\frac{1}{2}$

c)  $-3 \underline{\hspace{1cm}} -3.5$

d)  $\frac{9}{2} \underline{\hspace{1cm}} -3$

Fill in the blank: If a number is located to the left of another number on the number line, that number is \_\_\_\_\_ the other number.

The balance in Sue's checkbook was  $-\$12.55$ . The balance in John's checkbook was  $-\$10.45$ . Write an inequality to show the relationship between these amounts. Who owes more? How do you know?

Return to: [Standards](#)

**Apply and extend previous understandings of numbers to the system of rational numbers.**

**NC.6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

**Clarification**

Students use the coordinate plane as a tool to solve problems. In previous grades, students were limited to working in the first quadrant. In 6<sup>th</sup> grade, students will be expected to solve problems using coordinates that are in different quadrants or on the axes. Students are expected to find the distance between points on the same horizontal or same vertical line.

**Checking for Understanding**

What is the distance between  $(-5, 2)$  and  $(-9, 2)$ ?

Rectangle  $RSTU$  has vertices at  $R(-4, 3)$ ,  $S(-4, -2)$ ,  $T(5, -2)$  and  $U(5, 3)$ . Plot the rectangle on a coordinate plane and find the perimeter of the figure.

The perimeter of a square is 22 units. One of the vertices of the square is located on the origin of a coordinate plane. One of the vertices is located in the 3<sup>rd</sup> quadrant. What are the possible coordinates of the vertices of the square?

Return to: [Standards](#)

**Apply and extend previous understandings of numbers to the system of rational numbers.**

**NC.6.NS.9** Apply and extend previous understandings of addition and subtraction.

- Describe situations in which opposite quantities combine to make 0.
- Understand  $p + q$  as the number located a distance  $q$  from  $p$ , in the positive or negative direction depending on the sign of  $q$ . Show that a number and its additive inverse create a zero pair.
- Understand subtraction of integers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two integers on the number line is the absolute value of their difference.
- Use models to add and subtract integers from -20 to 20 and describe real-world contexts using sums and differences.

**Clarification**

The standard addresses adding and subtracting integers. Students add and subtract integers between -20 and 20, using models. Rules are not expected at this grade level. When derived from a real-world problem, students describe the sum or difference in context. These problems may require multiple steps. For example, evaluate  $6 + (-4) + (3) - 7$ .

**Making Zero Pairs**

Students are expected to create examples in which a number and the opposite of that number combine to make zero. Students describe these numbers as an additive inverse of each other and recognize that together they make a zero pair.

**Adding and Subtracting Integers**

Students are expected to interpret integers as having both a distance and a direction. Students demonstrate this understanding using a number line to:

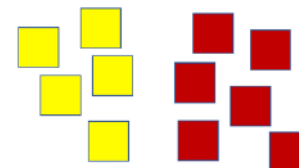
- Add integers
  - Students interpret the sum as the combination of distances with their corresponding direction.
  - Students explain how additive inverses create a zero pair.
- Subtract integers
  - Students interpret the absolute value of the difference as the distance between numbers.
  - Students explain why they can rewrite subtraction as addition and use this property as needed.

While students are required to understand addition and subtraction of integers using number lines, students may use and interpret other models to find sums and differences or to demonstrate an understanding of the concepts of this standard. Students may start using physical models, such as algebra tiles and integer chips. By the end of the year, students should move to visual models, such as number lines.

**Checking for Understanding**

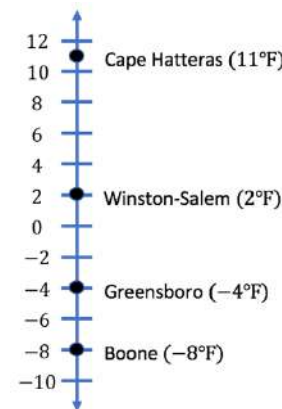
Answer the following questions. A student laid out these squares to represent a positive and a negative number. Each yellow square represents a positive one while each red square represents a negative one.

- What number is represented by the yellow squares?
- What number is represented by the red squares?
- How many zero pairs are represented by the yellow and red squares? How do you know?
- If the squares represented an addition problem, write an expression to represent the problem, and what would be the sum?



The number line shows the record low temperatures for these North Carolina cities in the month of February.

- How much warmer was the record low in Cape Hatteras than the record low in Boone?
- How much cooler was the record low in Boone than the record low in Greensboro?
- How much warmer was the record low in Winston-Salem than in Greensboro?
- How much cooler was the record low in Greensboro than Winston-Salem?
- A student got the same answer for questions c) and d). The students shared in a discussion, "I thought that when I was counting down the number line, I would get a negative answer, but I got a positive answer no matter which way I counted." Explain to the student why all of these answers were positive.



Rewrite the following into equivalent expressions and then evaluate each expression.

- $5 + (-3)$
- $8 - 17$
- $-7 + (-15)$
- $-4 - 12$

## Expressions and Equations

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

**NC.6.EE.1** Write and evaluate numerical expressions, with and without grouping symbols, involving whole-number exponents.

### Clarification

The standard places a focus on understanding the evaluation, meaning, notation, and vocabulary of whole-number exponents. The base of an expression with an exponent may be any positive rational number. Students write numerical expressions from verbal and visual representations that can involve grouping symbols and whole-number exponents. Students evaluate numerical expressions, using mathematical reasoning to develop a proper sequence of steps.

Examples of student reasoning:

- multiplication is done before addition because multiplication is repeated addition
- addition and subtraction are done in the order they are written because subtraction can be written as addition (See NC.6.NS.9)

In 5<sup>th</sup> grade, only grouping symbols used were parentheses. In 6<sup>th</sup> grade, grouping symbols may include: parentheses, brackets, braces and multiple sets of parentheses.

Students understand that parts of an expression have understood grouping symbols, such as numerators or denominators of fractions.

### Checking for Understanding

In the following pictures, write an expression that represents the total number of squares in the picture and then find the total number of squares.

a)



b)



For the question above, describe how you found the total with a partner. What was the most efficient way to find the total?

---

Consider the following expressions:  $6 + 3^2$  and  $(6 + 3)^2$

Evaluate each expression and explain why they have different answers.

---

The expression  $4(4^2)(8 \cdot 2)$  and  $4^5$  are equivalent.

Show that the two expressions are equivalent. Describe the steps that can be applied to  $4(4^2)(8 \cdot 2)$  to create the equivalent expression  $4^5$ .

Taken from: SBAC Mathematics Practice Test Scoring Guide Grade 6 p. 34

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What is the value of each expression?

e)  $0.2^3$

f)  $[4(2 + 3)]^2 - 5(30 - 10)$

g)  $7^2 - 24 \div 3 + 26$

h)  $2(1 + (5 - 4)^3)$

i)  $\frac{1+3(1+2)}{5^2}$

j)  $\frac{11}{25} - \left(\frac{2}{6-1}\right)^2$

Return to: [Standards](#)

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

**NC.6.EE.2** Write, read, and evaluate algebraic expressions.

- Write expressions that record operations with numbers and with letters standing for numbers.
- Identify parts of an expression using mathematical terms and view one or more of those parts as a single entity.
- Evaluate expressions at specific values of their variables using expressions that arise from formulas used in real-world problems.

**Clarification**

**Write expressions that record operations with numbers and with letters standing for numbers.**

Students:

- translate numerical and algebraic expressions from verbal representations and
- translate given numerical and algebraic expressions using words.

**Identify parts of an expression using mathematical terms and view one or more of those parts as a single entity.**

Students are expected to identify parts of expression, using terms like: constant, coefficient, variable, base, exponent, quantity, sum, difference, product, factor, quotient, and term.

Students understand that terms such as quantity, sum, difference, product, and quotient identify multiple parts of an expression that can be treated as a single entity and often have understood grouping symbols.

**Evaluate expressions at specific values of their variables using expressions that arise from formulas used in real-world problems.**

Students evaluate algebraic expressions derived from real-world problems.

In order to understand a formula, students formalize the definition of an equation as expressions with equivalent values. This means that if the value of one expression is found through evaluation, the value of the other expression is also known.

Students will be able to evaluate formulas in which the algebraic expressions are limited to two variables in one expression that is set equal to another variable, for example,  $V = s^3$  and  $P = 2l + 2w$ .

**Checking for Understanding**

Write the following as an algebraic expression:

- 7 less than 3 times a number
- 3 times the sum of a number and 5
- Twice the cube of  $z$
- The quotient of the sum of  $x$  plus 4 and 2

Write the following in words:

- $r + 21$
- $6n^2$
- $\frac{6}{b-1}$
- $\frac{1}{2}(k + 3)^2$

For parts f) though i), identify at least one example of the following: constant, coefficient, variable, base, exponent, quantity, sum, difference, product, factor, quotient, and term.

The formula to find the volume of a cube can be written as  $V = s^3$ . The length of the side of a square of one cube is 5 in., and the length of the side of a square of another cube is 2 inches longer.

- What do you expect to be the difference in the volumes of the cubes?
- Write an expression to represent the volume of each cube.
- Evaluate the expressions.
- Was your prediction close? Why was that the case?

You and a friend are traveling to Canada. Watching the weather forecast, you see the projected high temperature for the day is  $28^\circ\text{C}$ . You look up the formula to convert Celsius to Fahrenheit and see that it is  $F = \frac{9}{5}C + 32$ . Based on the result of the formula, what clothes should you pack?

Return to: [Standards](#)

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

**NC.6.EE.3** Apply the properties of operations to generate equivalent expressions without exponents.

**Clarification**

Students use the properties of operations to rewrite expressions into equivalent forms. The properties of operation include the commutative, associative, identity and distributive properties, and combining like terms.

**For example:** Produce an equivalent expression for  $3(2 + x)$ .

*Solution:* Using the definition of multiplication, students should see the expression as 3 groups of  $(2 + x)$ .

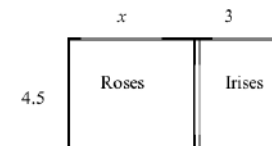
As seen in the visual, this produces 3 groups of 2 and 3 groups of  $x$ . Written as multiplication, this is  $3 \cdot 2 + 3 \cdot x$  which is  $6 + 3x$ .



In this grade level, students are not expected to distribute a variable to an expression, factor a variable from algebraic expression, or rewrite algebraic expressions that contain exponents.

**Checking for Understanding**

Students are planting a flower bed for science class. The flower bed can be 4.5 ft wide and will be divided into 2 sections for roses and irises. The iris section will be 3 ft long and it has not yet been determined how long the rose section will be.



- A student in the class claims that the area of the flower bed could be written as  $4.5(x + 3)$ . Write an equivalent form of this expression.
- How do you know the two expressions are equivalent?

Write an equivalent expression for  $3(x + 4) + 2(x - 2)$  that has only two terms.

Use the distributive property to write an equivalent expression for  $30x + 18y$ .

Use properties of operations to write at least 3 different expressions equivalent to the each of the following:  $6(2x + 3)$  and  $3x + 6 + 9$

Return to: [Standards](#)

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

**NC.6.EE.4** Identify when two expressions are equivalent and justify with mathematical reasoning.

**Clarification**

Students show that two algebraic expressions are equivalent, explaining their steps using mathematical reasoning and mathematical properties. In 6<sup>th</sup> grade, the focus on showing equivalency of algebraic expressions relies on substituting the same number for the variable(s) in both expressions and showing that the resulting values of the expressions are equivalent.

Students notice that this equivalency is not dependent on the number substituted in for the variable and should work for all numbers. It is also important for students to check with multiple numbers as some numbers (especially 0, 1, and 2) can lead to a false equivalency claim.

Students may also use the properties of operations to rewrite one expression into another to show equivalency (see NC.6.EE.3).

**Checking for Understanding**

Some high school students were disagreeing about their Math 1 homework. One students claimed that  $(2x - 3)^2$  was equivalent to  $4x^2 + 9$ . The other students said, no,  $(2x - 3)^2$  is equivalent to  $4x^2 - 12x + 9$ . Help out these high school students! Who is correct? Explain how you know.

Return to: [Standards](#)

**Reason about and solve one-variable equations.****NC.6.EE.5** Use substitution to determine whether a given number in a specified set makes an equation true.**Clarification**

Students use substitution to determine when given numbers make an equation true and identify this number as a solution of the equation. Students define a true equation as having the same numerical value on both sides of the equal sign. Students understand that typically almost all values substituted into an equation will make it false. Students are not expected to know or use set notation.

**Checking for Understanding**

Four high school students are working on a Math 1 problem to find the solution to  $(2x - 3)^2 = 121$ . Each student got a different answer. The four answers were 5, 6, 7, and 9.

- Which of these numbers make the equation true?
- How do you know?

Return to: [Standards](#)**Reason about and solve one-variable equations.****NC.6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem**Clarification**

Students interpret a context to write an expression that contains a variable. This standard deals with writing algebraic expressions beyond direct mathematical translations (see NC.6.EE.2) and understanding what was written.

Students state, with precision, the meaning of a variable and describe when a variable, in an expression or equation, represents:

- a single number, often when the expression can be written as an equation

**For example:** A school is using 12 passenger vans to transport students on a field trip. With 36 students going on the field trip, how many vans will be needed?

- all numbers, such as in an expression

**For example:** It is 12 degrees warmer than yesterday. Write an expression to represent the temperature today.

- a range of numbers, which in 6<sup>th</sup> grade is limited by interpretation from context, such as only whole numbers.

**For example:** A school is using 12 passenger vans to transport students on a field trip. Write an expression that represent the largest number of students that can be transported in  $v$  vans.

**Checking for Understanding**

Write an expression to represent the following:

- Susan's age in three years, when  $a$  represents her present age.
- The number of wheels,  $w$ , on any number of bicycles.
- The value of any number of quarters,  $q$ .

Write an expression that represents the following:

- The skating rink charges \$100 to reserve the place and then \$5.50 per person. Write an expression to represent the cost for any number of people.
- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.

A school is using 12 passenger vans to transport students on a field trip. Write an expression represent the number of vans needed for  $s$  students.

Describe a situation that can be represented by the expression  $2c + 3$ .

Return to: [Standards](#)

**Reason about and solve one-variable equations.****NC.6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form:

- $x + p = q$  in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers; and,
- $p \cdot x = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers

**Clarification**

Students write and solve one-step equations.

In 6<sup>th</sup> grade, students write equations by first writing an algebraic expression (see NC.6.EE.6) and then setting it equal to the known value of the expression.

Many students may find the solution to these basic problems using an arithmetic process.

**For example:** A school is using 12 passenger vans to transport students on a field trip. With 36 students going on the field trip, how many vans will be needed?

In this question, many students may solve this problem using operations directly, in this case,  $v = 36 \div 12$ . This means that they start with the end, 36, and work backwards through the problem.

The expectation of the standard is that students will learn to write an equation to represent this problem,  $12 \cdot v = 36$ . This means starting at beginning and working forward through the problem to the known value of the expression, using variables to represent unknown quantities. Students see the relationship between the equation and the arithmetic process. This leads students to seeing the relationship of inverse operations and the beginning of an algebraic approach to solving equations. As problems become more complex, the algebraic approach becomes the more efficient method to find solutions.

*Note: While both  $v = 36 \div 12$  and  $12 \cdot v = 36$  are equations, to meet the expectation of this standard, students who write the initial equation with the variable by itself, should be asked to represent the situation with an equation like the forms mentioned in the standard.*

Students know that the process for finding the number(s) that makes an equation true, the solution, using mathematical reasoning is called solving. Students use the skills learned in NC.6.EE.5 to verify they have found the solution.

*While subtraction and division can be used when selecting problems for this standard, problems involving negative numbers, negative variables, a variable in the denominator, and complex fractions are beyond the expectation of this standard.*

**Checking for Understanding**

Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

---

Select all equations that have  $n = 6$  as a solution.

- |                     |                    |
|---------------------|--------------------|
| A. $2 + n = 6$      | B. $n + 6 = 12$    |
| C. $4 \cdot n = 24$ | D. $n \cdot 3 = 2$ |

---

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

---

Robert has  $X$  books. Marie has twice as many books as Robert has. Together they have 18 books. Which of the following equations can be used to find the number of books that Robert has?

- |                   |                     |
|-------------------|---------------------|
| A. $x + 2 = 18$   | B. $x + x + 2 = 18$ |
| C. $x + 2x = 18$  | D. $2x = 18$        |
| E. $2x + 2x = 18$ |                     |

NAEP – Released Item (2011) [Question ID: 2011-8M12 #5 M153101](#)

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Solve the following equations:

- |                        |                                         |
|------------------------|-----------------------------------------|
| a) $12 = 8 + y$        | b) $f - \frac{2}{3} = \frac{1}{4}$      |
| c) $2.3 + r = 7.1$     | d) $1\frac{2}{5} = k \cdot \frac{1}{6}$ |
| e) $\frac{w}{4} = 3.3$ | f) $9.5 - 2.8 + a = 20.2$               |

Return to: [Standards](#)

**Reason about one variable inequalities.**

**NC.6.EE.8** Reason about inequalities by:

- Using substitution to determine whether a given number in a specified set makes an inequality true.
- Writing an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem.
- Recognizing that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions.
- Representing solutions of inequalities on number line diagrams.

**Clarification**

Students interpret inequalities and use them to describe situations. Since students can already determine equality, this standard includes the use of  $\geq$ ,  $\leq$ ,  $>$  or  $<$  throughout. The use of  $\leq$  and  $\geq$  are new to students.

**Using substitution to determine whether a given number in a specified set makes an inequality true.**

Students identify the solutions of inequalities of the form  $x \geq c$  or  $x \leq c$  when  $c$  is either positive or negative. Students also determine if  $c$  is included in the solutions.

**Writing an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem.**

Students use reasoning to determine the appropriate inequality to use in a given situation. For example, a person must be at least 16 years old to obtain a driver's license.

**Recognizing that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions.**

Students recognize the relationship between an infinite number of solutions and the need to shade number lines to represent these infinite solutions.

**Representing solutions of inequalities on number line diagrams.**

Students represent inequalities on a number line, using appropriate symbols. Students are also expected to write an inequality from its graphical representation.

*In 6<sup>th</sup> grade, students are not expected to solve inequalities or to write compound inequalities.*

**Checking for Understanding**

Consider the following numbers:  $-3.25$ ,  $-2$ ,  $3.5$ ,  $4$ ,  $4\frac{2}{3}$ ,  $5$

Which of these numbers are a possible solution to the following inequalities?

- a)  $b \leq 3\frac{1}{2}$
- b)  $-2 > m$
- c)  $h \geq -3.2$
- d)  $4 < p$

Write an inequality to represent each situation.

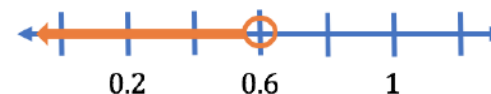
- a) The Flores family spent less than \$400.00 last month on groceries.
- b) The class must raise at least \$100 to go on the field trip.
- c) In order to get an attendance award, a student can have at most 2 absences for the year.

A student wrote  $1.75d$  to represent the cost of sports drinks with  $d$  representing the number of drinks. Write an inequality that describes the be values  $d$  could be. Explain.

Create a number line that represents the following inequalities.

- a)  $b \leq 3\frac{1}{2}$
- b)  $-2 > m$
- c)  $h \geq -3.2$
- d)  $4 < p$

Write the inequality represented in the graph.



Return to: [Standards](#)

**Represent and analyze quantitative relationships between dependent and independent variables.**

**NC.6.EE.9** Represent and analyze quantitative relationships by:

- Using variables to represent two quantities in a real-world or mathematical context that change in relationship to one another.
- Analyze the relationship between quantities in different representations (context, equations, tables, and graphs).

**Clarification**

Students describe and analyze how one variable changes in relation to the other.

**Using variables to represent two quantities in a real-world or mathematical context that change in relationship to one another.**

Students analyze the relationship between variables in a given situation and represent that situation as a two-variable equation. Students may be given a partially completed table or graph along with the context of the situation.

In 6<sup>th</sup> grade, students write an expression from a context and set that expression equal to a variable that represents the value of the created expression (output). This variable will be in a dependent relationship to the variable in the created algebraic expression. From this, students can describe the mathematical relationship between the variables.

**For example:** The cost to get into a high school basketball game is \$5 for each ticket. Write an equation to represent this situation and describe the relationship between the variables.

Solution: Sample answer:  $c = 5 \cdot t$

Possible description: As the number of tickets increases by 1, the cost increases 5 dollars.  $c$  is in a dependent relationship to  $t$ .

**Analyze the relationship between quantities in different representations (context, equations, tables, and graphs).**

Students create tabular and graphical representations of equations. In 6<sup>th</sup> grade, students are expected to create the graphical representation from the corresponding tabular representation. Student can use a tabular or graphical representation to analyze the relationship between variables. Students can then relate their finding back to the equation and the context.

Students understand that:

- A table is an organized list of solutions to its corresponding equation.
- A graph is a visual representation of the solutions to its corresponding equation.

From the context, equation, or graph, students determine which variable is in a dependent relationship. Students recognize that on a coordinate plane, the variable that is in a dependent relationship is graphed on the  $y$ -axis. From the example above, since cost,  $c$ , is in a dependent relationship with tickets,  $t$ , cost would be graphed on the  $y$ -axis and tickets would be graphed on the  $x$ -axis.

In 6<sup>th</sup> grade, the relationships analyzed should focus on proportional and linear relationships. Students are not required to use the terms proportional or linear at this level. The restrictions placed on ratios tables apply to this standard. The initial values given should be whole numbers.

**Checking for Understanding**

Chris and his friends are going to the ice cream shop that is having a sale on milkshakes. Each milkshake cost \$2.

- Write an equation that represents the total cost of buying any number of milkshakes for Chris and his friends.
- Create a table and a graph to represent this situation.
- Describe how the variables relate to each other and explain how you see these relationships in the equation, table and graph.

The student government is hosting a breakfast for charity and they need to know how many people will attend so they can make plans. The four members of the student government will attend the breakfast along with the number of people who bought a ticket.

- Write an equation that represents the total attending the breakfast.
- Create a table and a graph to represent this situation.
- Describe how the variables relate to each other and explain how you see these relationships in the equation, table and graph.

Looking at the two tables created in the previous examples. One table contains equivalent ratios, the other does not. How could you have determined this from the equations written?

A traveling basketball team is selling candy bars to raise money for new uniforms. There are 36 candy bars in each box and each candy bar cost \$1.75 each. The team sold 20 boxes.

- Complete the table that shows how much money was collected for each box.
- Create an equation to represent the amount of money collected based on the number of boxes sold.
- Graph the equation from the ordered pairs in the table.

| Boxes Sold | Money Collected |
|------------|-----------------|
| 1          | \$63            |
| 2          | \$126           |
| 3          |                 |
| 4          |                 |
| 5          |                 |
| 6          | \$378           |
| 7          |                 |
| 8          |                 |
| 9          |                 |
| 10         |                 |
| 11         |                 |
| 12         |                 |
| 13         |                 |
| 14         | \$882           |
| 15         |                 |
| 16         |                 |
| 17         |                 |
| 18         |                 |
| 19         |                 |
| 20         |                 |

Return to: [Standards](#)

## Geometry

### Solve real-world and mathematical problems involving area, surface area, and volume.

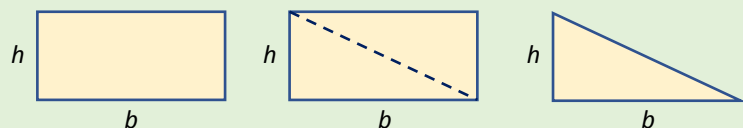
**NC.6.G.1** Create geometric models to solve real-world and mathematical problems to:

- Find the area of triangles by composing into rectangles and decomposing into right triangles.
- Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles.

#### Clarification

This standard builds on student understanding of area as the number of squares needed to cover a plane figure and an understanding of composite shapes from triangles and rectangles. Students have found the area of rectangles in the elementary grades. They have also worked with composite shapes of triangles and rectangles.

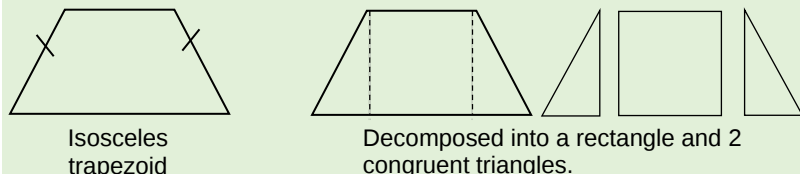
Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Students will use this understanding in future grades to find the area of polygons.



The area of the triangle is  $\frac{1}{2}$  the area of the rectangle. The area of a rectangle can be found by multiplying base  $\times$  height; therefore, the area of the triangle is  $A_{\Delta} = \frac{1}{2}bh = \frac{(b \times h)}{2}$ .

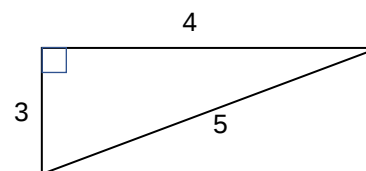
Additionally, students decompose shapes into rectangles and triangles to determine the area.

**For example**, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.

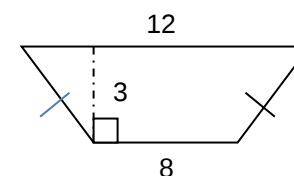


#### Checking for Understanding

Find the area of a right triangle below.



Show how you can find the area of the isosceles trapezoid shown below by decomposing into triangles and rectangles and using their area formulas to find the total area of the figure.



A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area? Describe visually and or verbally how you arrived at your answer.

Return to: [Standards](#)

**Solve real-world and mathematical problems involving area, surface area, and volume.**

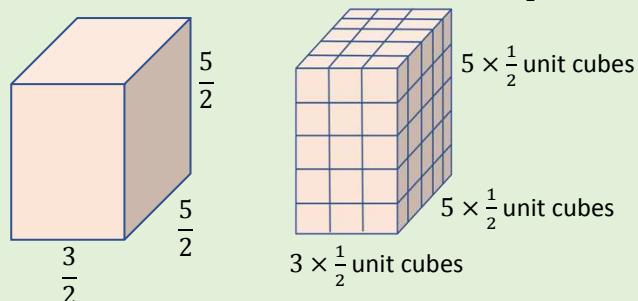
**NC.6.G.2** Apply and extend previous understandings of the volume of a right rectangular prism to find the volume of right rectangular prisms with fractional edge lengths. Apply this understanding to the context of solving real-world and mathematical problems.

**Clarification**

This standard builds on previous understanding of volume of right rectangular prisms. Students previously worked with volume of right rectangular prisms with whole number edges by packing unit cubes into the figure to build understanding of the volume formula for rectangular prisms (NC.5.MD.4 and NC.5.MD.5).

Students will use what they know about fractions, specifically unit fractions, to decompose the visual image into cubes with unit fractional edge lengths connecting to multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

**For example,** the model shows a rectangular prism with dimensions  $\frac{3}{2}$ ,  $\frac{5}{2}$ , and  $\frac{5}{2}$  inches. Each of the cubic units in the model is  $\frac{1}{2}$  inch on each side.



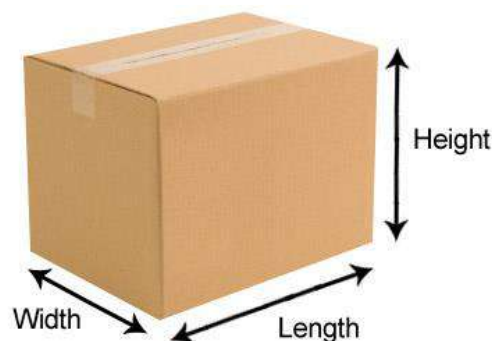
Students work with the model to illustrate  $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2} = (3 \times 5 \times 5) \times \frac{1}{8}$ . Students reason that a small cube has volume of  $\frac{1}{8} \text{ in}^3$  because 8 of them fit in a unit cube. Students recognize that there are 75 small cubes in the prism and if 8 of them make a unit cube, then there are 9 unit cubes and 3 small cubes in the prism, so  $9\frac{3}{8}$  is a way to express the volume in cubic units.

**Checking for Understanding**

A right rectangular prism has edges of  $1\frac{1}{4}$ ", 1" and  $1\frac{1}{2}$ ". How many cubes with side lengths of  $\frac{1}{4}$ " would be needed to fill the prism? What is the volume of the prism? (Note: The small unit cube is  $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$ ).

Each smaller cube within the  $3 \times 3$  cube below has a side length of  $\frac{3}{4}$  in. What is the volume of the  $3 \times 3$  cube? The toy manufacturer is looking for a box to package the cubes in for shipping. Which company, A or B, will allow them to send more cubes? Explain your response. How many cubes can be packaged?

|                  | Width | Length | Height |
|------------------|-------|--------|--------|
| <b>Company A</b> | 18"   | 12"    | 28"    |
| <b>Company B</b> | 16"   | 24"    | 20"    |



Return to: [Standards](#)

**Solve real-world and mathematical problems involving area, surface area, and volume.**

**NC.6.G.3** Use the coordinate plane to solve real-world and mathematical problems by:

- Drawing polygons in the coordinate plane given coordinates for the vertices.
- Using coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.

**Clarification**

This standard connects shapes in the Geometry standards to the coordinate system. Students have plotted points on the coordinate plane. The coordinate grid creates a good visual for examining properties of polygonal figures.

Students will draw polygons in the coordinate plane and calculate distances for vertical and horizontal lines in the plane to solve a variety of problems.

**For example,** parallelogram  $ABCD$

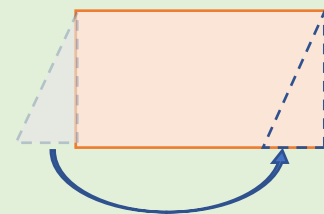
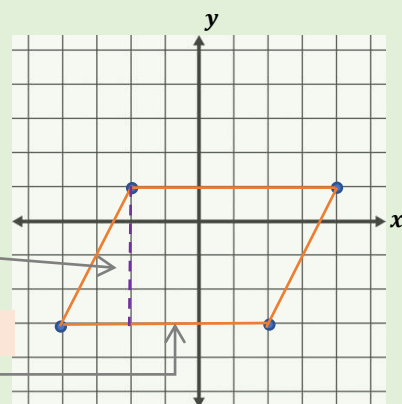
has vertices

$A(-2, 1)$ ,  $B(-4, -3)$ ,  $C(2, -3)$ , and  $D(4, 1)$ . Students can plot the points in the coordinate grid and use the formula for the area of a parallelogram ( $A = bh$ ) to find the area of the figure.

So,  $A = 6 \times 4 = 24 \text{ units}^2$

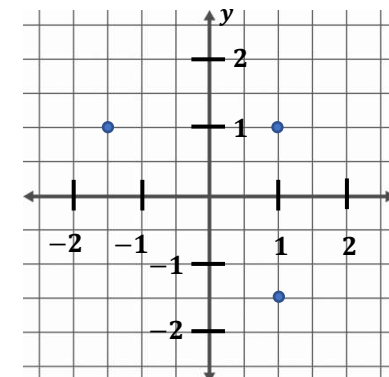
Students can also use what they know about composite figures to verify the area for the parallelogram.

This standard can be used to assist in the development of the understanding for determining the area formula for a triangle and other special quadrilaterals.



**Checking for Understanding**

If the points on the coordinate plane are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



**Extension:** Can the shape be identified as something other than a rectangle? Why or why not?

On a map, the library is located at  $(-2, 2)$ , the city hall building is located at  $(0, 2)$ , and the high school is located at  $(0, 0)$ . Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Return to: [Standards](#)

**Solve real-world and mathematical problems involving area, surface area, and volume.**

**NC.6.G.4** Represent right prisms and right pyramids using nets made up of rectangles and triangles, and use nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems.

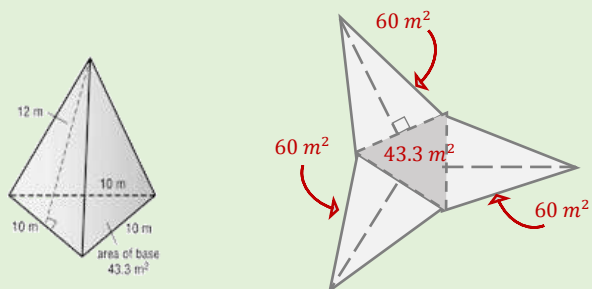
**Clarification**

This standard helps students to develop their visualization skills by examining **polyhedron** (a 3-dimensional shape with multiple faces made of polygons) using nets. A **net** is a two-dimensional representation of a three-dimensional figure. Students use nets to represent right prisms and pyramids, figures composed of rectangles and triangles. Nets help students to examine attributes of prisms and pyramids. They can determine the shapes of the lateral edges of the figure, the base, and attributes of lines and angles within the figure.

Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add them together to find the surface area of the figure.

Students visualize and describe the types of faces needed to create a three-dimensional figure. They also make and test conjectures to determine what shapes create specific three-dimensional figure.

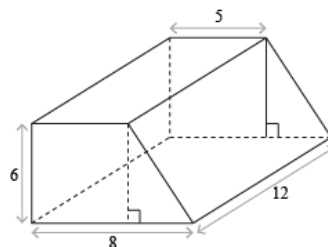
**For example**, the following is a triangular prism and its corresponding net. The area of the base is  $43.3 \text{ m}^2$ . The base length of each side and the height of each side is given enabling students to determine the area of each side as  $A = \frac{1}{2} 10(12) = 60 \text{ m}^2$ . So, the surface area (SA) is  $SA = 43.3 + 3(60) = 223 \text{ m}^2$ .



**Checking for Understanding**

Describe the shapes of the faces needed to construct a rectangular pyramid. Draw the net of the figure and label the sides and the bases.

Create the net for a given prism or pyramid, and then use the net to calculate the surface area.



Return to: [Standards](#)

## Statistics and Probability

### **Develop understanding of statistical variability.**

**NC.6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

#### **Clarification**

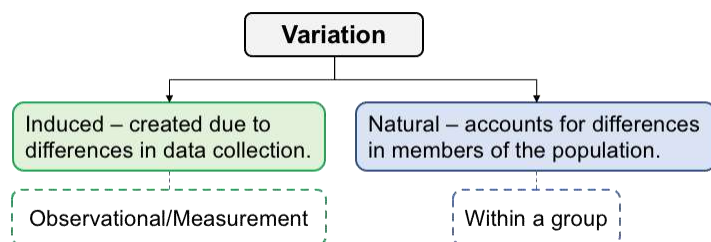
The intention of this standard is the distinction between a statistical question and a non-statistical question and to begin the discussion of variability and its role in statistical problem solving. Beginning with a working definition of statistics...

*Statistics is a collection of procedures and principles for gaining and analyzing information in order to help people make decisions when faced with uncertainty (Utts, 2005).*

Students know that a statistical question is one that collects information addressing the differences (variability) in a population. Students can also differentiate a statistical question from a research question. Research questions are answered from statistical analysis of data; therefore, research questions form the basis for the information that will be collected. They are based on a hypothesized outcome that is supported or refuted through analysis of the data.

**For example**, the question, “How tall am I?” is NOT a statistical question because there is only one response; however, the question, “How tall are the students in my class?” is a statistical question it anticipates [natural] variability in student heights.

Students understand that variability means that outcomes of data collection may be different and that statistics is a tool to explain the variability. Students understand that variation in data can be natural (due to differences in the population) or induced (by data collection). As students encounter statistics throughout middle school and high school, they will address variability at higher levels of complexity.



In 6<sup>th</sup> grade, students will focus on natural variability within a group. They will also examine how variability is induced through data collection and measurement (NC.6.SP.5).

*Students are NOT expected to list or name the types of variability, but they should be familiar with variability as a natural part of the statistical problem-solving process.*

#### **Checking for Understanding**

Which of the following represent a statistical question? Select all that apply. Explain.

1. What teaching styles are used by teachers for managing the behavior of students in this middle school?
2. How many hours per week, on average do students in our school exercise outside of school activities?
3. Who has the longest name in our class?
4. What was the temperature this morning at 6 am at Raleigh-Durham International Airport?
5. Does getting fewer than eight hours of sleep the night before a standardized exam make 6<sup>th</sup> graders more likely to do poorly on their EOGs at Middletown Middle School?
6. How many aunts and uncles do you have?
7. What proportion of students at your school eat cold pizza?
8. What is the relationship between playing AAU basketball in elementary school and making the varsity basketball team in high school?

Collect data and create statistical questions.

- Explain the difference between a statistical question and a non-statistical question.
- Create a class survey collecting both categorical and numerical data. Create at least 3 statistical questions that can be addressed by the data collected.

**Develop understanding of statistical variability.**

**NC.6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

**Clarification**

This standard supports mathematics as a tool to quantify and describe numerical data that has been generated from a statistical question. In 6<sup>th</sup> grade, students will ONLY calculate measures of center (mean and median) of a distribution.

Students will use graphical displays to describe the spread and shape of the data based on visual characteristics of the representations; therefore, it is important that students have graphed distributions to fully meet this standard.

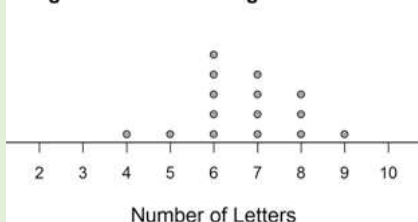
Key observations include: data clusters (mode), overall width (spread) of the values, data values that stand out from the rest (outliers), etc. Students do not need to *formally* determine the range of the data; however, they should be able to discuss the specific values in which the data falls in context.

**For example**, the graph shows the length of American League Baseball Team Names. Possible observations include:

- The range of all name lengths.
- The length of most names (mode).
- The mean and/or median name lengths.

The idea here is for students to make their own observations based on the context and the graph.

**Length of American League Team Names**

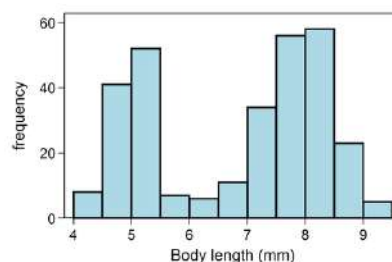


Students are expected to describe a data set given in various forms, this includes raw data or graphical displays of data. If students are presented with a graphical display of data, they are expected to know what information can/cannot be determined from the display (NC.6.SP.4).

**Checking for Understanding**

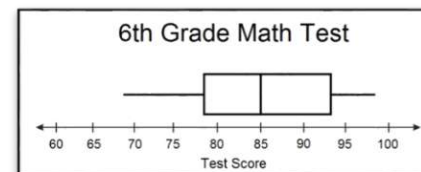
Ms. Flowers' 6<sup>th</sup> grade science class were studying insects and wanted to know the average length of a red work ant. As a class, they measured the length of red work ants and displayed the data in the following histogram.

- Describe the distribution.
- What information does the graph give us about the group of red work ants in the sample?



Ms. Williams wanted information about how well her students were performing on their mid-semester exam. She created a box plot using their test scores.

Using the graph, determine what information she determine about her class performance on the test.



Return to: [Standards](#)

### Develop understanding of statistical variability.

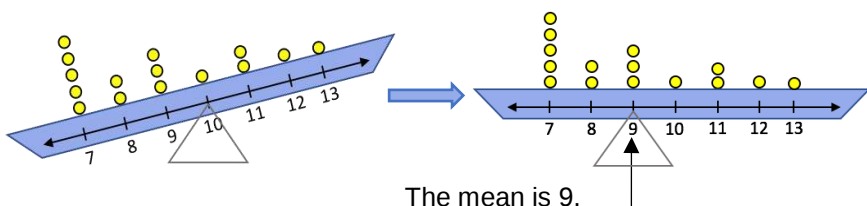
**NC.6.SP.3** Understand that both a measure of center and a description of variability should be considered when describing a numerical data set.

- Determine the measure of center of a data set and understand that it is a single number that summarizes all the values of that data set.
  - Understand that a mean is a measure of center that represents a balance point or fair share of a data set and can be influenced by the presence of extreme values within the data set.
  - Understand the median as a measure of center that is the numerical middle of an ordered data set.
- Understand that describing the variability of a data set is needed to distinguish between data sets in the same scale, by comparing graphical representations of different data sets in the same scale that have similar measures of center, but different spreads.

### Clarification

This standard defines the **descriptive statistics** that students will use to summarize data distributions at this grade level. Students will use mean and median to describe measures of center in the middle grades. Students are not expected to calculate measures of variability; however, they should examine graphical displays of data to compare data sets with the same center to help them understand the importance of examining variability between different data sets when analyzing data.

**Center:** This standard focuses on building conceptual understanding of the mean of a data set. Students develop understandings of the mean by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point).



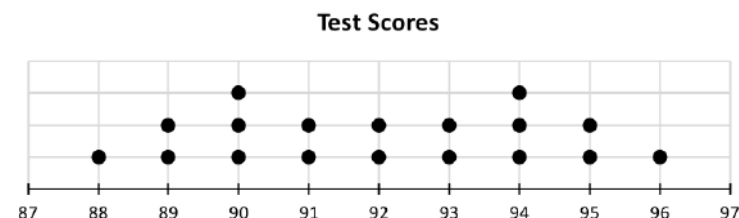
Students also recognize that the median is the actual data value (for an odd data set) that falls in the middle of the data set when the data is in order, noting that they have to calculate the mean of the middle two data values for an even data set.

Emphasis should be placed on the differences in the mean and median in terms of how they are determined and to what extent their values influence the measurement of the statistic (mean or median).

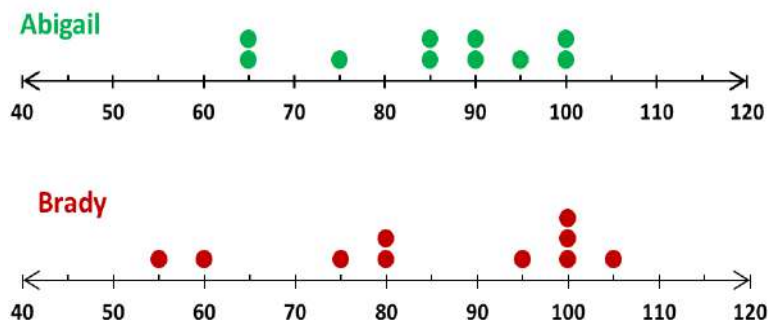
**Variability:** Students understand that measures of center, alone, are insufficient summaries for statistical data. Variability is as important as measures of center when analyzing and describing data. Students are not required to calculate measures of variability in 6<sup>th</sup> grade; however, they should recognize that when data sets have the same mean that variability can be used to distinguish the data sets.

### Checking for Understanding

The diagram below shows the Test Scores for a 6<sup>th</sup> grade mathematics class. *Without performing any calculations, use the graph below to show how you know that the mean and median test scores are both 92.*



The graphs below show the test scores of two students. *Who is the more consistent student? Why is it inappropriate to use the mean ONLY to help us decide? Explain your reasoning.*



Return to: [Standards](#)

**Summarize and describe distributions.****NC.6.SP.4** Display numerical data in plots on a number line.

- Use dot plots, histograms, and box plots to represent data.
- Compare the attributes of different representations of the same data.

**Clarification**

Students are expected to use dot plots, histograms and box plots to display numerical data. They are also expected to compare different types of representations for a given data set noting the advantages and disadvantages for using particular representations. Visual and numerical comparisons should be included. Students should also understand how measures of center and variability are represented by graphical displays and which displays reveal specific information (actual data values, number of data, mean, median, minimum, maximum, shape – symmetrical or skewed, etc.) relating to the data.

Students have created bar graphs and line plots in previous grade levels. Line plots are very similar to dot plots in that they represent the count of specific data values along a consistent scale on a horizontal axis. Histograms, not to be confused with bar graphs, and box plots are new content in the 6<sup>th</sup> grade standards.

Students can use a variety of methods and tools to create graphical displays, including but not limited to, by hand, applets, computer programs, or calculators.

Sample Applets:

- Box Plot Tool – <http://illuminations.nctm.org/ActivityDetail.aspx?ID=77>
- Histogram Tool – <http://illuminations.nctm.org/ActivityDetail.aspx?ID=78>

*Students do need to know the quartiles to create a box-plot. However, they are NOT expected to use the interquartile range (IQR) to quantify and interpret variability.*

**Checking for Understanding**

**A class of grade 6 students were collecting data for a class math project. They decided they would survey the other two 6<sup>th</sup> grade classes to determine how many DVDs each student owns. A total of 38 students were surveyed. The data are shown in the table below in no specific order.**

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 11 | 21 | 5  | 12 | 10 | 31 | 19 | 13 | 23 | 33 |
| 10 | 11 | 25 | 14 | 34 | 15 | 14 | 29 | 8  | 5  |
| 22 | 26 | 23 | 12 | 27 | 4  | 25 | 15 | 7  |    |
| 2  | 19 | 12 | 39 | 17 | 16 | 15 | 28 | 16 |    |

- 1) Create two different data displays to organize the data.
- 2) Describe the shape, center and spread of the distribution.
- 3) What attributes of the data can you easily see in each of the chosen displays? Which values can you approximate?

Return to: [Standards](#)

### Summarize and describe distributions.

**NC.6.SP.5** Summarize numerical data sets in relation to their context.

- Describe the collected data by:
  - Reporting the number of observations in dot plots and histograms.
  - Communicating the nature of the attribute under investigation, how it was measured, and the units of measurement.
- Analyze center and variability by:
  - Giving quantitative measures of center, describing variability, and any overall pattern, and noting any striking deviations.
  - Justifying the appropriate choice of measures of center using the shape of the data distribution.

### Clarification

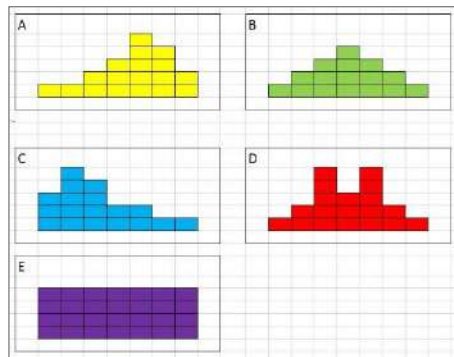
As students further develop their understanding of variability, they describe and analyze numerical data based on various representations of data. Students can identify the attributes of data represented in dot plots and histograms to quantitatively and qualitatively summarize distributions (NC.6.SP.4).

Students will use the shape of the data distribution to determine the appropriate measure of center to quantitatively describe the distribution. Students understand that symmetrical data displays reflect an “ideal” situation where the measures of center (mean and median) have very similar values. Furthermore, students understand that a skewed distribution is reflected by a visible and calculable difference between the mean and median. Additionally, students understand the effect of extreme values on the mean and median and can explain their choice of which measure more appropriately describes the center of a data distribution:

- Mean is appropriate to use with symmetrical distributions
- Median is appropriate for non-symmetrical distributions and/or distributions with extreme data values.

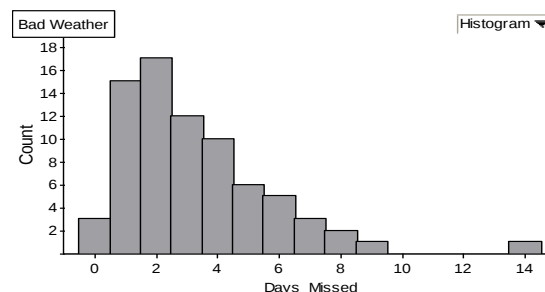
Students need to be familiar with various types of shapes, including but not limited to unimodal, bimodal/multimodal, uniform and skewed distributions.

- Skewed left
- Symmetrical, unimodal
- Skewed right
- Bimodal/multimodal
- Uniform



### Checking for Understanding

During the winter, schools are often closed due to severe weather. Sometimes schools have to make up for the missed days. A local school district wanted to look at historical data on the number of days missed due to inclement weather. The following graph shows the frequency distribution of the number of days missed due to snowstorms per year.



- Statistical problem solving is a process that accounts for variation in data. What type of variation (induced or natural) is present in the data set? Explain.
- Approximately how many years of data were collected and analyzed? Use the graph to show how you came up with your answer.
- Describe the shape of the data. Does there appear to be any data values that are outside of the normal pattern of the data? If so, what are they and what do they mean?
- What are the mean and the median of the data? What do they tell us about the data? Which measure is a better descriptor as the measure of center? Why?

Return to: [Standards](#)