

Math  
6<sup>th</sup> Grade

6 <sup>th</sup> Grade	
Domain	Cluster
Ratios and Proportional Relationships	<ul style="list-style-type: none"><li>• Understand ratio concepts and use ratio reasoning to solve problems</li></ul>
The Number System	<ul style="list-style-type: none"><li>• Apply and extend previous understandings of multiplication and division to divide fractions by fractions</li><li>• Compute fluently with multi-digit numbers and find common factors and multiples</li><li>• Apply and extend previous understandings of numbers to the system of rational numbers</li></ul>
Expressions and Equations	<ul style="list-style-type: none"><li>• Apply and extend previous understandings of arithmetic to algebraic expressions</li><li>• Reason about and solve one-variable equations and inequalities</li><li>• Represent and analyze quantitative relationships between dependent and independent variables</li></ul>
Geometry	<ul style="list-style-type: none"><li>• Solve real-world and mathematical problems involving area, surface area, and volume</li></ul>
Statistics and Probability	<ul style="list-style-type: none"><li>• Develop understanding of statistical variability</li><li>• Summarize and describe distributions</li></ul>

<b>Domain</b>	Ratios and Proportional Relationships				
<b>Cluster</b>	Understand Ratio Concepts and Use Ratio Reasoning to Solve Problems				
<b>Alignments</b>	CCSS – 6.RP.1; 6.RP.2; 6.RP.3a; 6.RP.3b; 6.RP.3c; 6.RP.3d KS – (MA) PS – DOK –				
<table border="1"> <thead> <tr> <th><u>Standards</u></th><th><u>Learning Targets</u></th></tr> </thead> <tbody> <tr> <td> <ol style="list-style-type: none"> <li>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes”</i></li> <li>Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math>, and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger”<sup>1</sup></i></li> <li>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations               <ol style="list-style-type: none"> <li>Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios</li> <li>Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i></li> </ol> </li> </ol> </td><td> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Use language to describe the relationship between two amounts</li> <li>Write a ratio to show the relationship between two amounts</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Understand and use language to describe a unit rate</li> </ul> </li> <li> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Make a table of equivalent ratios and find missing values in the table</li> <li>Plot the ratio values from a table on the coordinate plane</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Find the cost per unit of an item</li> <li>Find constant speed</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Find the percent of an amount by using a proportion</li> <li>Solve problems that involve finding the whole when given part of a whole and the percent</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Use proportions to convert measurement units</li> </ul> </li> </ol> </li></ol></td></tr> </tbody> </table>		<u>Standards</u>	<u>Learning Targets</u>	<ol style="list-style-type: none"> <li>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes”</i></li> <li>Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math>, and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger”<sup>1</sup></i></li> <li>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations               <ol style="list-style-type: none"> <li>Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. 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| <ul style="list-style-type: none"><li>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent</li><li>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities</li></ul> |  |
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**Instructional Strategies**

- Grocery ads/ labels for students to calculate unit rates
- Holt textbook PowerPoint
- Cooperative learning strategies
- Guided practice (dry erase boards)

**Assessments/Evaluations**

**Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

**Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

### Sample Assessment Questions

1.

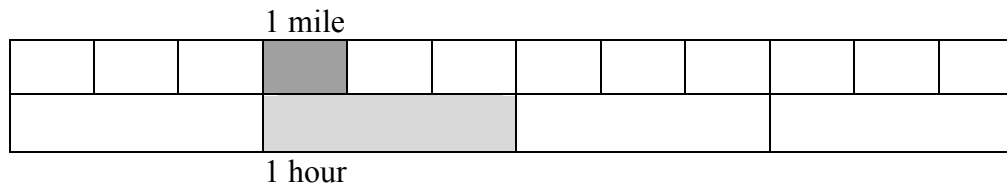
- A ratio is a comparison of two quantities which can be written as  $a$  to  $b$ ,  $a/b$ , or  $a:b$
- A comparison of 12 pennies to 6 dimes can be written as the ratio of 12:6 and can be regrouped into 6 pennies to 3 dimes (6:3) and 2 pennies to 1 dime (2:1)
- Students should be able to identify all these ratios and describe them using “For every... there are...”

[Example derived from Arizona Mathematics Standards]

2.

- On roller skates you can travel 12 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: You can travel 3 miles in 1 hour written as 3mi/1hr and it takes  $\frac{1}{3}$  of an hour to travel each mile written as  $\frac{1}{3}\text{hr./1mi}$



- Students can represent the relationship between 12 miles and 4 hours

[Example derived from Arizona Mathematics Standards]

3.

- a) Using the information in the table, find the number of yards in 24 feet

Feet	3	6	9	15	24
Yards	1	2	3	5	?

There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards must be 8 yards (3 yards and 5 yards)

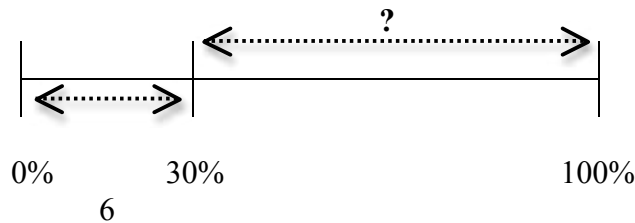
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards

b) Compare the number of black to white squares. If the ratio remains the same, how many black squares will you have if you have 60 white squares?



Black	3	30	15	45	?
White	2	20	10	30	60

c) If 6 is 30% of a value, what is that value? (Solution: 20)



d) A credit card company charges 15% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals \$350 for this month, how much interest would you have to pay if you didn't pay until the next month? Show the relationship on a graph and use the graph to predict the interest charges for a \$275 balance

Charges	\$1	\$50	\$100	\$200	\$350
Interest	\$0.15	\$7.50	\$15	\$30	?

[Examples derived from Arizona Mathematics Standards]

**Instructional Resources/Tools**

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website

**Literacy Connections**

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<b>Domain</b>	The Number System	
<b>Cluster</b>	Apply and Extend Previous Understandings of Multiplication and Division to Divide Fractions by Fractions	
<b>Alignments</b>	CCSS – 6.NS.1 KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <p>1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>. (In general, <math>(a/b) \div (c/d) = ad/bc</math>.) How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>3/4</math>-cup servings are in <math>2/3</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>3/4</math> mi and area <math>1/2</math> square mi?</i></p>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <p>1.</p> <ul style="list-style-type: none"> <li>• Divide fractions</li> <li>• Explain how an answer to a division of fractions problem is reasonable</li> <li>• Solve word problems involving division of fractions by using visual models and equations</li> </ul>
<p style="text-align: center;"><b><u>Instructional Strategies</u></b></p> <ul style="list-style-type: none"> <li>• Real world problems</li> <li>• Holt textbook PowerPoint</li> <li>• Cooperative learning strategies</li> <li>• Guided practice (dry erase boards)</li> <li>• Trail Run</li> <li>• “I Have, Who Has” game</li> <li>• 3 by 3 Square</li> </ul>		

### Assessments/Evaluations

#### **Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

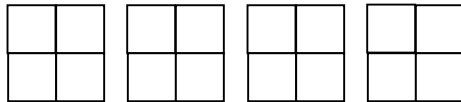
#### **Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

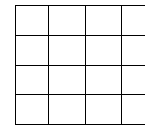
### Sample Assessment Questions

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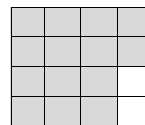
- $16 \div 4$  means → How many groups of 4 would make 16, or how many in each groups of 4 would make 16?



Or



- Thus  $7/2 \div 1/4$  can be solved the same way. How many objects in a group of  $1/4$  make  $7/2$ , or how many in each group when  $1/4$  fills  $7/2$ ?



[Example derived from Arizona Mathematics Standards]



**Instructional Resources/Tools**

- Holt Textbook Series
- Khan Academy
- Teacher created activities
- Illuminations website

**Literacy Connections**

-

<b>Domain</b>	The Number System	
<b>Cluster</b>	Compute Fluently with Multi-Digit Numbers and Find Common Factors and Multiples	
<b>Alignments</b>	CCSS – 6.NS.2; 6.NS.3; 6.NS.4 KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <p>2. Fluently divide multi-digit numbers using the standard algorithm</p> <p>3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation</p> <p>4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express <math>36 + 8</math> as <math>4(9 + 2)</math></p>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <p>2.</p> <ul style="list-style-type: none"> <li>Follow the steps to divide multi-digit numbers</li> </ul> <p>3.</p> <ul style="list-style-type: none"> <li>Follow the steps to: <ul style="list-style-type: none"> <li>add decimals</li> <li>subtract decimals</li> <li>multiply decimals</li> <li>divide decimals</li> </ul> </li> </ul> <p>4.</p> <ul style="list-style-type: none"> <li>Find the greatest common factor of two whole numbers</li> <li>Find the least common multiple of two whole numbers</li> <li>Use the distributive property to find the sum of two whole numbers</li> </ul>
<p style="text-align: center;"><b><u>Instructional Strategies</u></b></p> <ul style="list-style-type: none"> <li>Real world problems</li> <li>Holt textbook PowerPoint</li> <li>Cooperative learning strategies</li> <li>Guided practice (dry erase boards)</li> <li>Factor game</li> <li>Product game</li> <li>Venn Diagram</li> <li>Place Value chart</li> <li>Base ten blocks</li> </ul>		

**Assessments/Evaluations**

**Formative Assessment**

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**Summative Assessment**

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- Common assessment
- Unit projects

**Sample Assessment Questions**

2.

$\begin{array}{r} 2 \\ 32 \overline{)8456} \end{array}$	There are 200 thirty twos in 8456
$\begin{array}{r} 2 \\ 32 \overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	200 multiplied by 32 is 6400. 8456 minus 6400 is 2056
$\begin{array}{r} 26 \\ 32 \overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	There are 60 thirty twos in 2056
$\begin{array}{r} 26 \\ 32 \overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \end{array}$	60 multiplied by 32 is 1920. 2056 minus 1920 is 136
$\begin{array}{r} 264 \\ 32 \overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \\ 128 \end{array}$	There are 4 thirty twos in 136. 4 times 32 is 128
$\begin{array}{r} 264 \\ 32 \overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \end{array}$	The remainder is 8. There is not a full thirty-two in 8; there is only part of a thirty-two in 8. This can also be written as $8/32$ or $1/4$ . There is $1/4$ of a thirty-two in 8. $8456 = 264 * 32 + 8$

$\begin{array}{r} -128 \\ 8 \end{array}$	
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[Example derived from Arizona Mathematics Standards]

3.

- Max has 6 employees, and he gave each of them \$24.50. How much money did he give to his employees all together?

$$\begin{array}{r} 24.50 \\ \times \quad 6 \\ \hline 147.00 \end{array}$$

**Maxine gave \$147 to his employees altogether**

- Grace gave each of her employees \$42.50. If she gave a total of \$340, how many employees does Grace have?

$$\begin{array}{r} \quad \quad 8 \\ 42.50 \overline{)340} \end{array}$$

**Grace has 8 employees**

- Kate gave each of her 7 employees an equal amount of money. If she gave a total of \$227.50, how much did each employee get?

$$\begin{array}{r} \quad 32.50 \\ 7 \overline{)227.50} \end{array}$$

**Each employee received \$32.50**

4.

- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF? Solution:  $2 \times 2 \times 3 = 12$ . Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus  $2 \times 2 \times 3$  is the greatest common factor)

[Examples derived from Arizona Mathematics Standards]

**Instructional Resources/Tools**

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website

**Literacy Connections**

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<b>Domain</b>	The Number System	
<b>Cluster</b>	Apply and Extend Previous Understandings of Numbers to the System of Rational Numbers	
<b>Alignments</b>	CCSS – 6.NS.5; 6.NS.6a; 6.NS.6b; 6.NS.6c; 6.NS.7a; 6.NS.7b; 6.NS.7c; 6.NS.7d; 6.NS.8 KS – (MA) PS – DOK –	
<b><u>Standards</u></b>		<b><u>Learning Targets</u></b>
<p>5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation</p> <p>6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane</p>		<p>5.</p> <ul style="list-style-type: none"> <li>Recognize that positive and negative numbers describe amounts that have opposite directions on a number line and have opposite values</li> <li>Use positive and negative numbers to represent amounts in real world situations</li> </ul> <p>6a.</p> <ul style="list-style-type: none"> <li>Recognize the opposite signs of numbers are located on the opposite sides of 0 on the number line</li> <li>Recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite</li> </ul> <p>6b.</p> <ul style="list-style-type: none"> <li>Understand the quadrant a number is located in is based on the sign of the coordinates</li> <li>Recognize that when the signs of the coordinates are opposite, the location of the points are a reflection across one or both axes</li> </ul> <p>6c.</p> <ul style="list-style-type: none"> <li>Plot integers and other rational numbers on a vertical or horizontal number line</li> <li>Plot integers and other rational numbers on the coordinate plane</li> </ul>

<p>7. Understand ordering and absolute value of rational numbers</p> <ol style="list-style-type: none"> <li>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right</i></li> <li>Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math></i></li> <li>Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars</i></li> <li>Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than 30 dollars</i></li> </ol> <p>8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate</p>	<p>7a.</p> <ul style="list-style-type: none"> <li>Recognize how the position of two integers compare to one another on a number line</li> </ul> <p>7b.</p> <ul style="list-style-type: none"> <li>Write and explain the order of rational numbers in real-world situations</li> </ul> <p>7c.</p> <ul style="list-style-type: none"> <li>Understand that absolute value means the distance from zero on a number line</li> <li>Understand absolute value in real world situations</li> </ul> <p>7d.</p> <ul style="list-style-type: none"> <li>Recognize the meaning of an integer in a real world situation</li> <li>Solve real world problems by graphing points in all four quadrants on the coordinate plane</li> </ul> <p>8.</p> <ul style="list-style-type: none"> <li>Find the distance between points on the coordinate plane by using absolute value</li> </ul>
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### Instructional Strategies

- *Extreme Temperatures* big book
- Walk the number line
- Real world situations
- BrainPop on Absolute Value
- Create a coordinate plane on the floor and graph points
- Holt textbook PowerPoint
- Cooperative learning strategies
- Guided practice (dry erase boards)



### Assessments/Evaluations

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#### **Summative Assessment**

- Quizzes
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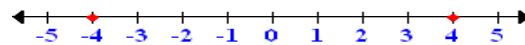
Examples:

5.

- Place 5 and -3 on a number line. What do you notice about the placement of each number?
- If the temperature outside is 9 degrees below 0, how do you represent that on a number line?

6.

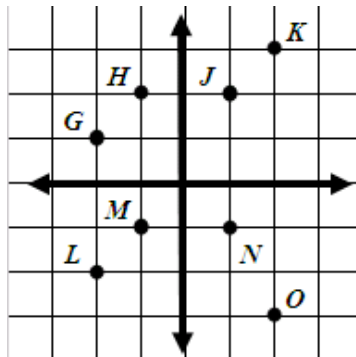
- a) Number lines can be used to show numbers and their opposites. Both 4 and -4 are 4 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids



- b) Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

(2, -3)    $(-\frac{1}{2}, -\frac{3}{4})$    (-0.35, 0.76)

c)



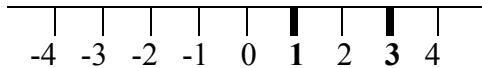
- Identify points G, J, L, and N
- Identify points (-1,2), (2,3), (-1,-1), and (2,-3)

Examples derived from Arizona Mathematics Standards

7.

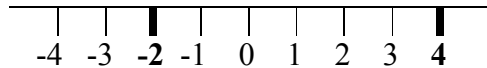
- a) In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers

- Case 1: Two positive numbers



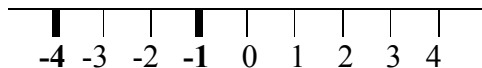
- $3 > 1$
- 3 is greater than 1

- Case 2: One positive and one negative number



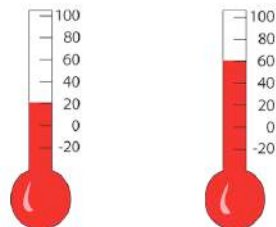
- $4 > -2$ ;  $-2 < 4$
- 4 is greater than -2
- -2 is less than 4

- Case 3: Two negative numbers



- $-1 > -4$ ;  $-4 < -1$
- -1 is greater than -4
- -4 is less than -1

- b) One of the thermometers shows 20°C and the other shows 60°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship



- c) Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.  
Example: Death Valley contains North America's lowest point of elevation and is located in California. It reaches down to 282 ft. Students could represent this value as less than 282 feet or a depth no greater than 282 feet below sea level
- d) If you have a balance of \$-30 in the bank what does that mean? → Answer: You are \$30 in debt

8.

- If the following points on a coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?

$(-3, 1), (1, 1), (-3, -1), (?, ?)$

- To determine the distance along the x-axis between the point  $(-3, 1)$  and  $(1, 1)$  a student must recognize that  $-3$  is 3 units to the left of 0 and  $1$  is 1 unit to the right of zero, so the two points are total of 4 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression,  $|-3| + |1|$

[Examples derived from Arizona Mathematics Standards]

### Instructional Resources/Tools

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illustrations website

**Literacy Connections**

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<b>Domain</b>	Expressions and Equations				
<b>Cluster</b>	Apply and Extend Previous Understandings of Arithmetic to Algebraic Expressions				
<b>Alignments</b>	CCSS – 6.EE.1; 6.EE.2a; 6.EE.2b; 6.EE.2c; 6.EE.3; 6.EE.4 KS – (MA) PS – DOK –				
<table border="1"> <thead> <tr> <th><u>Standards</u></th><th><u>Learning Targets</u></th></tr> </thead> <tbody> <tr> <td> <ol style="list-style-type: none"> <li>Write and evaluate numerical expressions involving whole-number exponents</li> <li>Write, read, and evaluate expressions in which letters stand for numbers <ol style="list-style-type: none"> <li>Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as <math>5 - y</math></i></li> <li>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression <math>2(8 + 7)</math> as a product of two factors; view <math>(8 + 7)</math> as both a single entity and a sum of two terms</i></li> <li>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas <math>V = s^3</math> and <math>A = 6s^2</math> to find the volume and surface area of a cube with sides of length <math>s = \frac{1}{2}</math></i></li> </ol> </li> <li><u>Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression <math>3(2 + x)</math> to produce the equivalent expression <math>6 + 3x</math>; apply the distributive property to the expression <math>24x + 18y</math> to produce the equivalent expression <math>6(4x + 3y)</math>; apply</i></u></li> </ol> </td><td> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Write an expression using exponents</li> <li>Find the value of an exponent</li> </ul> </li> <li> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Translate from words to an algebraic expression</li> <li>Identify the parts of an expression (sum, term, product, factor, quotient, and coefficient)</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>View one or more parts of an expression as one amount</li> <li>Solve problems using order of operations</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Find the value of an algebraic expression when given the value of the variable</li> <li>Solve real world problems when given formulas</li> </ul> </li> </ol> </li> <li> <ul style="list-style-type: none"> <li>Use the number properties to find equivalent expressions</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Recognize when two expressions are equivalent</li> </ul> </li> </ol> </td></tr> </tbody> </table>		<u>Standards</u>	<u>Learning Targets</u>	<ol style="list-style-type: none"> <li>Write and evaluate numerical expressions involving whole-number exponents</li> <li>Write, read, and evaluate expressions in which letters stand for numbers <ol style="list-style-type: none"> <li>Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as <math>5 - y</math></i></li> <li>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression <math>2(8 + 7)</math> as a product of two factors; view <math>(8 + 7)</math> as both a single entity and a sum of two terms</i></li> <li>Evaluate expressions at specific values of their variables. 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<p><u>properties of operations to <math>y + y + y</math> to produce the equivalent expression <math>3y</math></u></p> <p>4. <u>Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions <math>y + y + y</math> and <math>3y</math> are equivalent because they name the same number regardless of which number <math>y</math> stands for</u></p>	
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<p style="text-align: center;"><b><u>Assessments/Evaluations</u></b></p> <p><b>Formative Assessment</b></p> <ul style="list-style-type: none"> <li>• Observation</li> <li>• Pairs check</li> <li>• Individual practice</li> <li>• Target tickets</li> <li>• Quizzes</li> <li>• Projects</li> </ul> <p><b>Summative Assessment</b></p> <ul style="list-style-type: none"> <li>• Quizzes</li> <li>• Common assessment</li> <li>• Unit projects</li> </ul>	

### Sample Assessment Questions

1.

- Write the following as a numerical expressions using exponential notation
  - The area of a square with a side length of 6 m (Solution:  $6^2\text{m}^2$ )
  - The volume of a cube with a side length of 4 ft (Solution:  $4^3\text{ft}^3$ )
  - Ross has a pair of hamsters. The hamsters each have 2 babies. The babies grow up and have two babies of their own (Solution:  $2^3$  hamsters)

Evaluate:

- $5^3$  (Solution: 125)
- $4 + 3^3 \cdot 5$  (Solution: 155)
- $5^2 - 20 \div 4 + 15$  (Solution: 35)

2.

- a) It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number
  - $x + 12$  as “some number plus 12” as well as “x plus 12”
  - $s \cdot 3$  as “some number times 3” as well as “s times 3”
  - $m/6$  <sup>and</sup>  $m \div 6$  as “some number divided by 6” as well as “m divided by 6”
- b)
  - Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions
  - Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable
  - Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes
  - Consider the following expression:

$$x^2 + 3y + 2x + 4$$

The variables are x and y. There are 4 terms,  $x^2$ ,  $3y$ ,  $2x$ , and 4. There are 3 variable terms,  $x^2$ ,  $3y$ ,  $2x$ . They have coefficients of 1, 3, and 2 respectively. The coefficient of  $x^2$  is 1, since  $x^2 = 1x^2$ . The term  $3y$  represent 3 y's or  $3 \cdot y$ . There is one constant term, 4. The expression shows a sum of all four terms

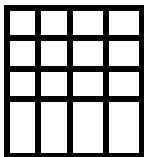


c) Examples:

- 5 more than 2 times a number (Solution:  $2x + 5$ )
- 2 times the sum of a number and 6 (Solution:  $2(x + 6)$ )
- 9 less than the product of 3 and a number (Solution:  $3x - 9$ )
- Twice the difference between a number and 7 (Solution:  $2(x - 7)$ )
- Evaluate  $4(r + 4) - 6r$ ; when  $r = \frac{1}{2}$
- The expression  $p + 0.06p$  can be used to find the total cost of an item with 6% sales tax, where  $p$  is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost \$24
- The perimeter of a parallelogram is found using the formula  $p = 2l + 2w$ . What is the perimeter of a rectangular picture frame with dimensions of 9 inches by 12 inches

3.

- Students use their understanding of multiplication to interpret  $4(3 + x)$ . *For example, 4 groups of  $(3 + x)$ .* They use a model to represent  $x$ , and make an array to show the meaning of  $4(3 + x)$ . They can explain why it makes sense that  $4(3 + x)$  is equal to  $12 + 4x$
- An array with 4 columns and  $x + 3$  in each column:



Students interpret  $y$  as referring to one  $y$ . Thus, they can reason that one  $y$  plus one  $y$  plus one  $y$  **must be**  $3y$ . They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that  $y + y + y = 3y$ :

$$y + y + y = y \times 1 + y \times 1 + y \times 1 = y \times (1 + 1 + 1) = y \times 3 = 3y$$

4.

- Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form

Example: Are the expressions equivalent? How do you know?

$$3m+7 \quad 3(m+1) \quad 2m + 7 + m$$

$$1 + 3m + 7 + m$$

Solution:

Expression	Simplifying the Expression	Explanation
$3m+7$	$3m+7$	Already in simplest form
$3(m+1)$	$3(m+1)$ $3m+3$	Distributive property
$2m + 7 + m$	$2m + 7 + m$ $2m + m + 7$ $(2m + m) + 7$ $3m + 7$	Combined like terms
$1 + 3m + 7 + m$	$1 + 3m + 7 + m$ $1 + 7 + 3m + m$ $(1 + 7) + (3m + m)$ $8 + 4m$ $4m + 8$	Combined like terms

[Examples derived from Arizona Mathematics Standards]

**Instructional Resources/Tools**

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website
- Learnzillion.com
- Smart Exchange

For Practice –

- Ixl.com
- Tenmarks.com
- Arizona Academic Content Standards
- Inside Mathematics –Problems of the Month-Leveled Word Problems – Under Common Core-Mathematical Content Standards – Tasks
- Illustrative mathematics:
  - Log Ride
  - Morning Walk
  - Fishing Adventures 1

**Literacy Connections**

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<b>Domain</b>	Expressions and Equations	
<b>Cluster</b>	Reason about and Solve One-Variable Equations and Inequalities	
<b>Alignments</b>	CCSS – 6.EE.5; 6.EE.6; 6.EE.7; 6.EE.8 KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <p>5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true</p> <p>6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set</p> <p>7. Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers</p> <p>8. Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams</p>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <p>5.</p> <ul style="list-style-type: none"> <li>Substitute numbers in an equation or inequality to determine what would make the statement true</li> </ul> <p>6.</p> <ul style="list-style-type: none"> <li>Use variables to represent numbers</li> <li>Write expressions to solve real world problems</li> </ul> <p>7.</p> <ul style="list-style-type: none"> <li>Write and solve an equation for a real world problem</li> </ul> <p>8.</p> <ul style="list-style-type: none"> <li>Write an inequality to represent a real world problem</li> <li>Recognize that inequalities can have an infinite amount of solutions</li> <li>Represent solutions of inequalities on a number line</li> </ul>
<p style="text-align: center;"><b><u>Instructional Strategies</u></b></p> <ul style="list-style-type: none"> <li>Guess My Rule partner game</li> <li>Set/not part of set activity</li> <li>Holt textbook PowerPoint</li> <li>Cooperative learning strategies</li> <li>Guided practice (dry erase boards)</li> <li>Anchor charts</li> </ul>		

### Assessments/Evaluations

#### **Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

#### **Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

### Sample Assessment Questions

5.

- Consider the following situation: Mackenzie had 32 papers in her desk. Her teacher gave her some more and now she has 100. How many papers did her teacher give her?

This situation can be represented by the equation  $32 + x = 100$  where  $x$  is the number of papers the teacher gives to Mackenzie. This equation can be stated as “some number was added to 32 and the result was 100”. Students ask themselves “What number was added to 32 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem

- Reasoning:  $32+60$  is 92.  $82+8$  is 100, so the number added to 32 to get 100 is 68
- Use knowledge of fact families to write related equations:  $x + 26 = 100$ ,  $100 - x = 32$ ,  $100 - 32 = x$ . Select the equation that helps you find  $n$  easily
- Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 32 from 100 to get the numerical value of  $x$
- Scale model: There are 32 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 68 blocks need to be added to the left side of the scale to make the scale balance

- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 32 and the unknown value together make 100

100	
32	x

6. Examples:

- Anna has three more than twice as many bracelets as Allison. Write an algebraic expression to represent the number of bracelets that Anna has. (Solution:  $2c + 3$  where  $c$  represents the number of bracelets that Allison has)
- A water park charges \$18 to enter and \$0.45 per ticket. Write an algebraic expression to represent the total amount spent. (Solution:  $18 + 0.45t$  where  $t$  represents the number of tickets purchased)
- Tom has a summer job doing yard work. He is paid \$12 per hour and a \$30 bonus when he completes the yard. He was paid \$105 for completing one yard. Write an equation to represent the amount of money he earned. (Solution:  $12h + 30 = 105$  where  $h$  is the number of hours worked)
- Describe a problem situation that can be solved using the equation  $3c + 4 = 16$ ; where  $p$  represents the price of an item
- Natalie earned \$7.00 mowing the lawn on Friday. She earned more money on Thursday. Write an expression that shows the amount of money Natalie has earned (Solution:  $\$7.00 + n$ )

7.

- Megan spent \$44.46 on three pairs of shoes. If each pair of shoes costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost

\$44.46		
S	S	S

Sample Solution: The bar model represents the equation  $3S = \$44.46$

- To solve the problem, I need to divide the total cost of \$44.46 between the three pairs of shoes. I know that it will be more than \$10 each because  $10 \times 3$  is only 30 but less than \$15 each because  $15 \times 3$  is 45. If I start with \$14 each, I am up to \$42. I have \$2.46 left. I then give each pair of jeans \$0.50. That's \$1.50 more dollars. I only have \$0.96 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.32. Each pair of jeans costs \$14.82 ( $14 + 0.50 + 0.32$ ). I double check that the jeans cost \$14.82 each because  $\$14.82 \times 3$  is \$44.46"

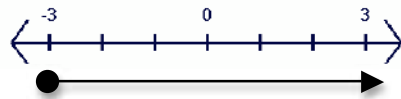
- Cole gets paid \$30 for selling his baseball cards. He spends \$9.99 on new headphones and \$7.25 on lunch. Write and solve an equation to show how much money Cole has left (Solution:  $30 = 9.99 + 7.25 + x$ ,  $x = \$12.76$ )

30		
9.99	7.25	Money left over (x)

8. Examples:

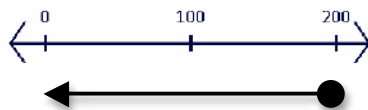
- Graph  $x \geq -3$

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- Sam spent more than \$45 at an arcade. Write an inequality to represent the amount of money Sam spent. What are some possible amounts of money Sam could have spent? Represent the situation on a number line
- The Lewis family spent less than \$200 on fast food last month. Write an inequality to represent this amount and graph this inequality on a number line

Solution:  $200 > x$



### Instructional Resources/Tools

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website
- Illustrativemathematics.com: Firefighter Allocation

**Literacy Connections**

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Domain	Expressions and Equations													
Cluster	Represent and Analyze Quantitative Relationships Between Dependent and Independent Variables													
Alignments	CCSS – 6.EE.9 KS – (MA) PS – DOK –													
<u>Standards</u>		<u>Learning Target</u>												
9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time</i>		9. <ul style="list-style-type: none"><li>• Use variables to write an equation that involves an independent and dependent variable in a real world problem</li><li>• Analyze the relationship between the independent and dependent variables using tables and graphs</li></ul>												
<u>Instructional Strategies</u>														
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<u>Assessments/Evaluations</u>														
9. Examples: <ul style="list-style-type: none"><li>• What is the relationship between the two variables? Write an expression that illustrates the relationship</li></ul>														
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td></tr></table>					x	1	2	3	4	y	1.5	3	4.5	6
x	1	2	3	4										
y	1.5	3	4.5	6										
<ul style="list-style-type: none"><li>• Use a coordinate plane to describe the change in <math>y</math> as <math>x</math> increases by 1</li></ul>														

- Susan started with \$1 in her savings. She plans to add \$4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account

- Language: Susan has \$1 in her savings account. She is going to save \$4 each week

- Equation:  $y = 4x + 1$

- Table:

x	y
0	1
1	5
2	9

- Graph

### Sample Assessment Questions

- 

### Instructional Resources/Tools

- 

### Literacy Connections

-

<b>Domain</b>	Geometry	
<b>Cluster</b>	Solve Real-World and Mathematical Problems Involving Area, Surface Area, and Volume	
<b>Alignments</b>	CCSS – 6.G.1; 6.G.2; 6.G.3; 6.G.4 KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <ol style="list-style-type: none"> <li>Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems</li> <li>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = lwh</math> and <math>V = bh</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems</li> <li>Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems</li> <li>Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems</li> </ol>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Find the area of: <ul style="list-style-type: none"> <li>triangles</li> <li>parallelograms, rhombus, trapezoids</li> <li>polygons by separating them into: <ul style="list-style-type: none"> <li>rectangles</li> <li>triangles</li> <li>other shapes</li> </ul> </li> <li>shapes in real world problems</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes</li> <li>Apply formulas to find volumes of right rectangular prisms with fractional edge lengths in real world problems</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Draw polygons in the coordinate plane to solve real world problems</li> </ul> </li> <li> <ul style="list-style-type: none"> <li>Recognize three-dimensional figures using nets made up of rectangles and triangles</li> <li>Use nets to find the surface area of three-dimensional figures</li> </ul> </li> </ul></li></ol>

**Instructional Strategies**

- Holt textbook PowerPoint
- Cooperative learning strategies
- Guided practice (dry erase boards)
- Using nets to create 3 dimensional polygons
- Real world problems
- Foldables
- Anchor chart
- BrainPop video
- Project

**Assessments/Evaluations**

**Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

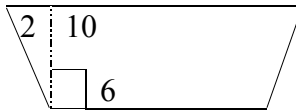
**Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

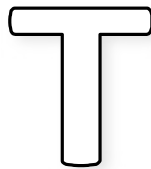
### Sample Assessment Questions

1. Examples:

- Find the area of a triangle with a base length of three units and a height of four units
- Find the area of the trapezoid shown below using the formulas for rectangles and triangles



- A rectangle measures 6 cm by 8 cm. If the lengths of each side double, what is the effect on the area?
- The area of the rectangular school garden is 32 square units. The length of the garden is 8 feet. What is the length of the fence needed to enclose the entire garden?
- The sixth grade class at Trinity School is building a giant wooden T for their school. The T will be 12 feet tall and 12 feet wide and the thickness of the block letter will be 3 feet
- How large will the T be if measured in square feet?
- The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 40 inches by 40 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?

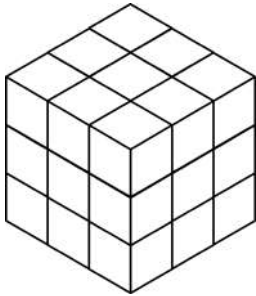


2.

- Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (<http://illuminations.nctm.org/ActivityDetail.aspx?ID=6>)

Examples:

- The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of  $\frac{1}{3}\text{ft}^3$



- A model shows a rectangular prism with dimensions  $\frac{3}{2}$  inches,  $\frac{5}{2}$  inches, and  $\frac{5}{2}$  inches. Each of the cubic units in the model is  $\frac{1}{8}\text{in}^3$
- Students work with the model to illustrate  $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2} = (3 \times 5 \times 5) \times \frac{1}{8}$ . Students reason that a small cube has volume  $\frac{1}{8}$  because 8 of them fit in a unit cube

3. Example:

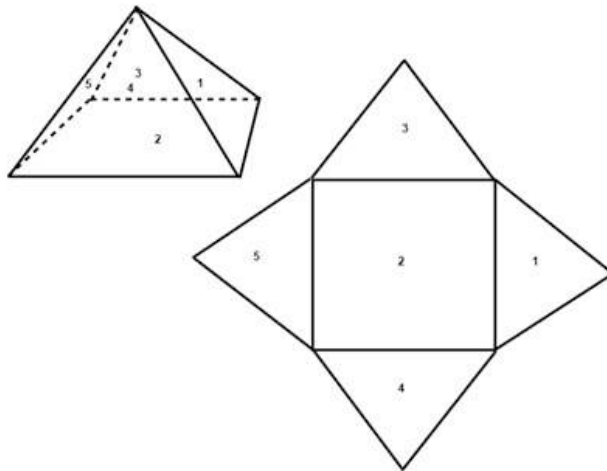
- On a map, the mall is located at  $(-3, 3)$ , the movie theater is located at  $(0,3)$ , and your house is located at  $(0,0)$ . Represent the locations as points on a coordinate grid with a unit of 1 mile
- What is the distance from the mall to the movie theater? The distance from the movie theater to your house? How do you know?
- Connecting the three locations creates what shape? The city council is planning to place a new apartment building in this area. How large is the area of the planned building?

4.

- Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (<http://illuminations.nctm.org/ActivityDetail.aspx?ID=205>)

Examples:

- Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?
- Create the net for a given prism or pyramid, and then use the net to calculate the surface area



[Examples derived from Arizona Mathematics Standards]

### Instructional Resources/Tools

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website

### Literacy Connections

•

<b>Domain</b>	Statistics and Probability	
<b>Cluster</b>	Develop Understanding of Statistical Variability	
<b>Alignments</b>	CCSS – 6.SP.1; 6.SP.2; 6.SP.3 KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <ol style="list-style-type: none"> <li>1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages</i></li> <li>2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape</li> <li>3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number</li> </ol>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <ol style="list-style-type: none"> <li>1. <ul style="list-style-type: none"> <li>• Recognize what is and is not a statistical question</li> </ul> </li> <li>2. <ul style="list-style-type: none"> <li>• Find the: <ul style="list-style-type: none"> <li>• mean</li> <li>• median</li> <li>• mode</li> </ul> of a set of data</li> <li>• Find the range and interquartile range of a set of data</li> </ul> </li> <li>3. <ul style="list-style-type: none"> <li>• Recognize that a measure of center for a set of data summarizes all of its values with a single number</li> </ul> </li> </ol>
<p style="text-align: center;"><b><u>Instructional Strategies</u></b></p> <ul style="list-style-type: none"> <li>• Data collection activities</li> <li>• Holt textbook PowerPoint</li> <li>• Cooperative learning strategies</li> <li>• Guided practice (dry erase boards)</li> <li>• Mean, Median, Mode song</li> <li>• Student line up</li> <li>• Real world problems</li> <li>• M&amp;M activity</li> </ul>		



### Assessments/Evaluations

#### **Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

#### **Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

### Sample Assessment Questions

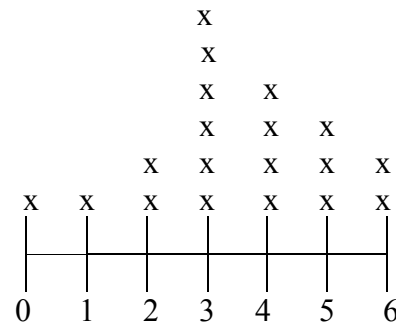
1.

- Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e., documents)
- Questions can result in a narrow or wide range of numerical values. For example, asking classmates “How old are the students in my class in years?” will result in less variability than asking “How old are the students in my class in months?”
- Students might want to know about the study habits of the students at their school. So rather than asking "Do you study?" they should ask about the amount of studying the students at their school do per week. A statistical question for this study could be: “How many hours per week on average do students at Washington Middle School study?”
- To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 2 hours per week, 3 hours per week, and so on. Be sure that students ask questions that have specific numerical answers

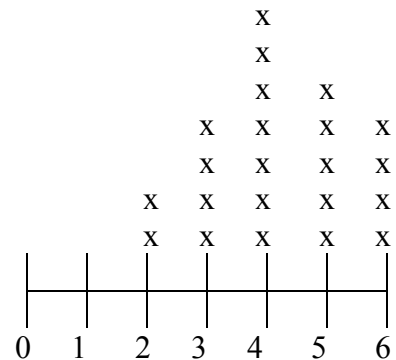
2.

- The two-dot plots show the 6-trait reading scores for a group of students on two different traits, understanding and evaluation. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the evaluation scores are generally higher than the understanding scores. One observation students might make is that the scores for understanding are clustered around a score of 2 whereas the scores for evaluation are clustered around a score of 4

6-Trait Reading Rubric  
Scores for Understanding



6-Trait Reading Rubric  
Scores for Evaluation

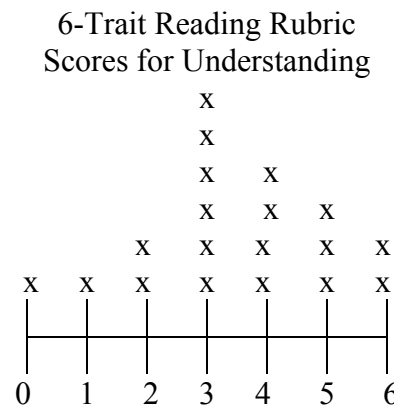


3.

- When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values

Example:

- Consider the data shown in the dot plot of the six trait scores for organization for a group of students
- How many students are represented in the data set?
- What are the mean, median, and mode of the data set? What do these values mean? How do they compare?
- What is the range of the data? What does this value mean?



[Examples derived from Arizona Mathematics Standards]

### Instructional Resources/Tools

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website

### Literacy Connections

•

<b>Domain</b>	Statistics and Probability	
<b>Cluster</b>	Summarize and Describe Distributions	
<b>Alignments</b>	CCSS – 6.SP.4; 6.SP.5a; 6.SP.5b; 6.SP.5c; 6.SP.5d KS – (MA) PS – DOK –	
<p style="text-align: center;"><b><u>Standards</u></b></p> <p>4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots</p> <p>5. Summarize numerical data sets in relation to their context, such as by:</p> <ol style="list-style-type: none"> <li>Reporting the number of observations</li> <li>Describing the nature of the attribute under investigation, including how it was measured and its units of measurement</li> <li>Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered</li> <li>Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered</li> </ol>		<p style="text-align: center;"><b><u>Learning Targets</u></b></p> <p>4.</p> <ul style="list-style-type: none"> <li>Display numerical data in plots on a number line, including: <ul style="list-style-type: none"> <li>dot plots</li> <li>histograms</li> <li>box plots</li> </ul> </li> </ul> <p>5a.</p> <ul style="list-style-type: none"> <li>Summarize numerical data by reporting the number of observations</li> </ul> <p>5b.</p> <ul style="list-style-type: none"> <li>I can describe my method of investigation, how it is measured, and the unit of measurement</li> </ul> <p>5c.</p> <ul style="list-style-type: none"> <li>I can find the median and all interquartile ranges</li> <li>I can find the mean and mean distribution</li> </ul> <p>5d.</p> <ul style="list-style-type: none"> <li>I can relate the center to the shape of the distribution</li> </ul>

### **Instructional Strategies**

- Cooperative learning strategies
- Guided practice (dry erase boards)
- Data collection activity
- Holt Course 2 PowerPoint

### **Assessments/Evaluations**

#### **Formative Assessment**

- Observation
- Pairs check
- Individual practice
- Target tickets
- Quizzes
- Projects

#### **Summative Assessment**

- Quizzes
- Common assessment
- Unit projects

### **Sample Assessment Questions**

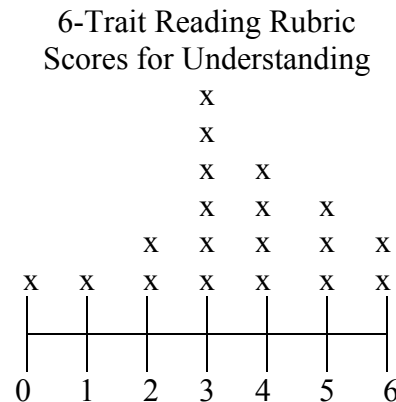
4.

- In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations
- Box Plot Tool - <http://illuminations.nctm.org/ActivityDetail.aspx?ID=77> Histogram Tool -- <http://illuminations.nctm.org/ActivityDetail.aspx?ID=78>
- Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers

- In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it
- Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summary of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side-by-side box plots on the same scale

Examples:

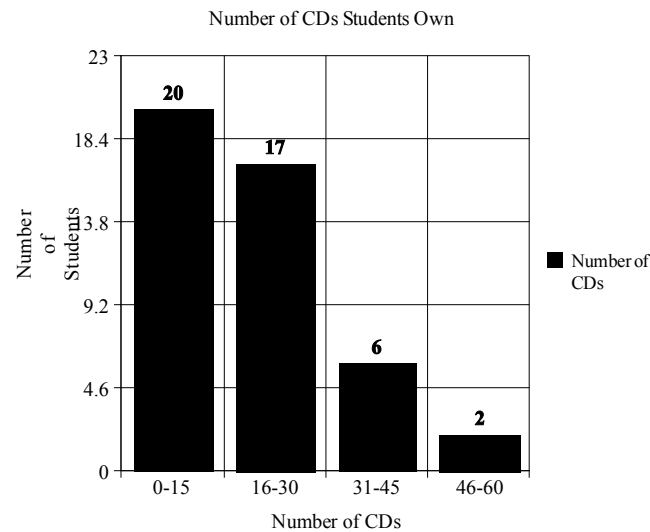
- Nineteen students completed a reading sample that was scored using the six traits rubric. The scores for the trait of understanding were 0, 1, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?



- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many CDs each student owns. A total of 45 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

14	32	27	25	12	5	16	11	29	35
11	27	14	33	18	41	3	22	49	12
16	24	18	34	7	9	0	14	20	15
19	13	14	29	25	15	7	15	28	16
37	6	27	4	51					

- A histogram using 5 ranges (0-9, 10-19, ...100-109) to organize the data is displayed below



- Mrs. Hermann asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the chalkboard. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133
134	136	137	137	139	139	139	140
141	142	142	142	143	143	144	145
147	149	150	138				

### Five number summary

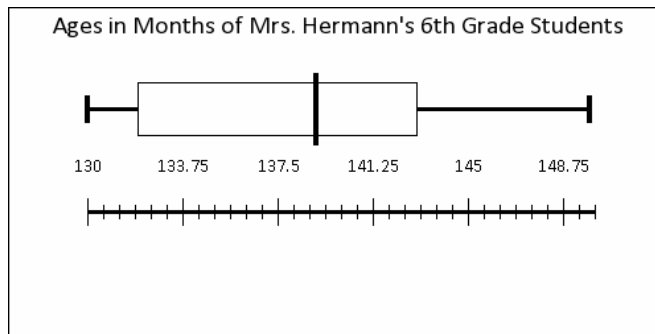
Minimum – 130 months

Quartile 1 (Q1) –  $(132 + 133) \div 2 = 132.5$  months

Median (Q2) – 139 months

Quartile 3 (Q3) –  $(142 + 143) \div 2 = 142.5$  months

Maximum – 150 months



This box plot shows that:

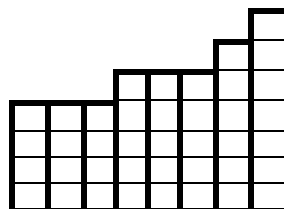
- 1/4 of the students in the class are from 130 to 132.5 months old
- 1/4 of the students in the class are from 142.5 months to 150 months old
- 1/2 of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

[Examples derived from Arizona Mathematics Standards]

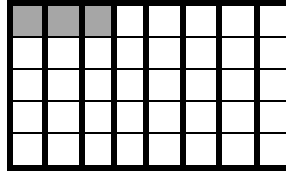


5.

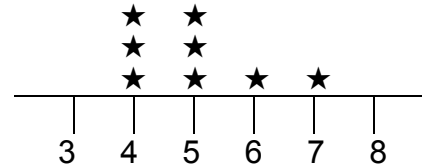
- a) Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation
- b) The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set
- c) Understanding the Mean: The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean
- For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points that are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes. Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Megan, Ross, Grace, Kate, Natalie, Claire, Cole, and Jayme there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data set could be represented with stacking cubes



- Students can model the mean by “leveling” the stacks or distributing the blocks so the stacks are “fair”. Students are seeking to answer the question “If all of the students had the same number of letters in their name, how many letters would each person have?”
- One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5

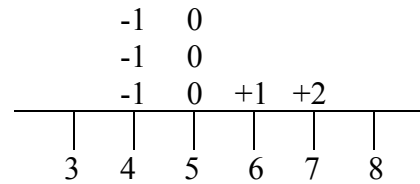


- If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have
- Understanding Mean Absolute Deviation: The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets
- In the previous data set, the names drawn were Megan, Ross, Grace, Kate, Natalie, Claire, Cole, and Jayme. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5

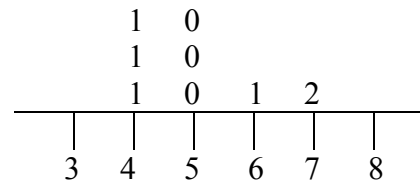


- To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference

**Deviations from the mean**



**Absolute Deviations**



Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Ross	4	-1	1
Kate	4	-1	1
Cole	4	-1	1
Megan	5	0	0
Grace	5	0	0
Jayme	5	0	0
Claire	6	+1	1
Natalie	7	+2	2
<b>Total</b>	40	0	6

- The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be  $6 \div 8$  or  $\frac{3}{4}$  or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set
- Consider a different data set also containing 8 names. If the names were Max, Tom, Sam, Lui, Sabrina, Chelsea, Anthony, and Laurena. Summarize the data set and its variability. How does this compare to the first data set?

The mean of the data set is still 5.  $(3+3+3+3+7+7+7+7)/8=40/8=5$

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Max	3	-2	2
Tom	3	-2	2
Sam	3	-2	2
Lui	3	-2	2
Sabrina	7	+2	2
Chelsea	7	+2	2
Anthony	7	+2	2
Laurena	7	+2	2
<b>Total</b>	40	0	16

The mean deviation of this data set is  $16 \div 8$  or 2. Although the mean is the same, there is much more variability in this data set

- Understanding Medians and Quartiles: Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles ( $Q3 - Q1$ ). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed
- Consider the first data set again. Recall that the names drawn were Megan, Ross, Grace, Kate, Natalie, Claire, Cole, and Jayme. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest

54547645 → 44455567

- The middle value in the ordered data set is the median. If there is an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4<sup>th</sup> and 5<sup>th</sup> values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values, which is an even number of values. Q1 would be the average of the 2<sup>nd</sup> and 3<sup>rd</sup> value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6<sup>th</sup> and 7<sup>th</sup> value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data.

4   4<sup>Q1</sup>4   5<sup>M</sup>5   5<sup>Q3</sup>6   7

Q1=4 Median=5 Q3=5.5

### Instructional Resources/Tools

- Holt textbook series
- Khan Academy
- Teacher created activities
- Illuminations website

### Literacy Connections

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**These topics are currently taught in the 6<sup>th</sup> grade.  
The CCSS does not have these listed for sixth grade.**

- Patterns
- Exponents
- Add, Subtract, Multiply fractions
- Equivalent Fractions, Decimals, and Percents
- Identify Polygons (types of angles, triangles, quadrilateral)
- Perimeter and Area of Rectangles
- Compare and Order Decimals
- Reading Decimals
- Place Value

- Prime and Composite numbers
- Convert within a system of measurement
- Circle graphs
- Stem and Leaf plots
- Angle measurement
- Transformations and Symmetry
- 3D shapes
- Elapsed Time
- Probability (sample space)