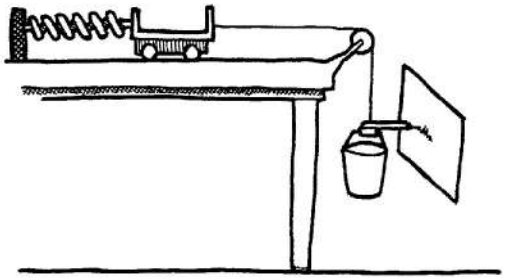
NAME _____ DATE _____

Scenario

Dominique's precalculus teacher asks her to make an exact graph of $y = \sin x$ *using a real-life process. She sets up equipment as shown in the diagram, where a very light cart on a flat table is connected on one end to a spring fixed to a wall and connected on the other end to a thread. The thread passes over an ideal pulley and supports a very light bucket. She attaches a pencil to the bucket; the marking end of the pencil is in light contact with a piece of paper. Dominique plans to have the bucket oscillate while the paper moves horizontally with constant velocity so that the pencil draws out a sinusoidal graph on the paper. With no mass in the cart or bucket and everything at rest, the pencil points directly to the top of the paper.*

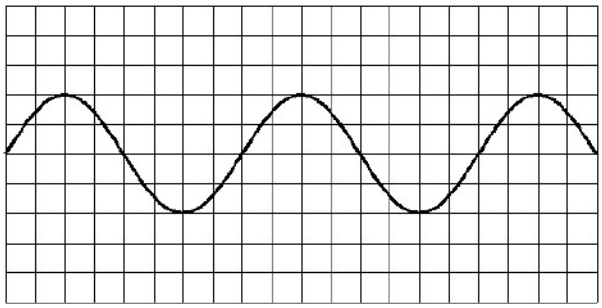


Argumentation

PART A: Dominique places a mass M in the cart and a mass m in the bucket. She finds that, with everything at rest, the pencil now points to the bottom of the paper, and when the pencil does oscillate, the period is shorter than desired. She wishes to make the pencil point to the center of the paper when everything is at rest and double the period of the pencil's oscillation. Explain in specific terms how Dominique can add or remove masses from the bucket and/or cart to accomplish this.

Quantitative Analysis

PART B: Dominique places 0.5 kg in the bucket and 9.5 kg in the cart and allows the setup to oscillate with an amplitude of 2 cm. As the paper moves horizontally with constant speed v past the oscillating pencil, the pencil draws the graph shown. Each square on the graph represents 1 cm. Recall that the pencil pointed to the highest line of the grid when neither the bucket nor the cart carried any mass, and everything was at rest. Calculate the speed v that the paper was moving and include units in your answer. Explain each part of your calculation with words.



Data Analysis

PART C: Suppose there was a small amount of friction between the pencil and paper, enough to affect the vertical motion of the bucket but not cause the bucket to swing back and forth horizontally. Answer the following questions assuming all other quantities are kept constant:

i. How, if at all, would this affect the calculated speed in Part B? Explain.

ii. How, if at all, would this affect the graph shown in Part B? Explain.