Date: ____________

6.6 Solve Absolute Value Inequalities

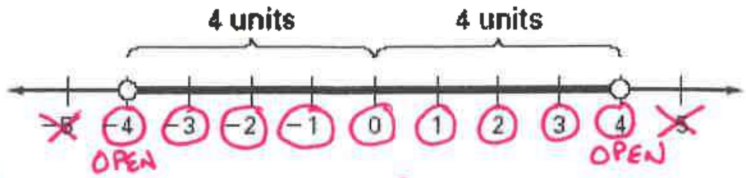
Goal: Solve absolute value inequalities.

Investigating Activity: Absolute Value Inequalities *(For use before Lesson 6.6)*

QUESTION: How can you use a number line to solve absolute-value inequalities in the form: $|x| < c$

1) **EXPLORE 1:** $|x| < 4$

- Determine which values of X are solutions for X= -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5



- Will you use open or closed circles? Why? OPEN CIRCLE WHEN <, >, ≠

- What compound inequality can describe the graph of the solution above?

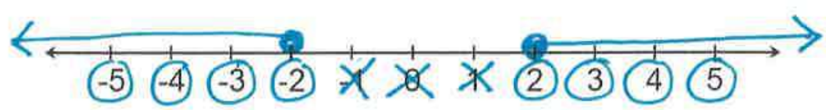
$\boxed{-4 < x < 4}$ OR

$\boxed{x > -4 \text{ AND } x < 4}$

QUESTION: How can you use a number line to solve absolute-value inequalities in the form: $|x| > c$

2) **EXPLORE 2:** $|x| \geq 2$

- Determine which values of X are solutions for X= -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5



- Will you use open or closed circles? Why? CLOSED CIRCLE : ≤, ≥, =

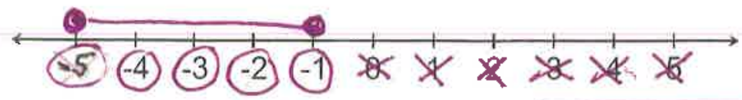
- What compound inequality can describe the graph of the solution above?

$\boxed{x \leq -2 \text{ OR } x \geq 2}$

3) **EXPLORE**: **TRY THESE**

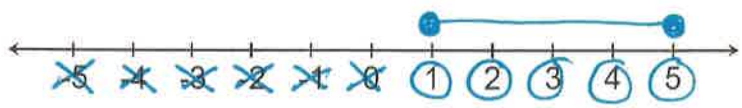
- For each absolute value inequality
  a) Determine which values of X are solutions for X= -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5
  b) Sketch the graph
  c) Write the compound inequality to describe the solution

1. $|x + 3| \le 2$



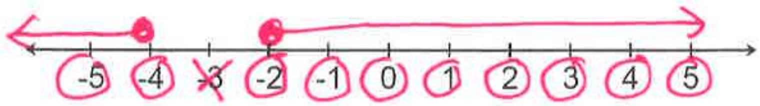
Compound inequality: $\boxed{-5 \le x \le -1}$

2. $|x - 3| \le 2$



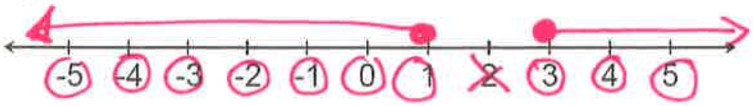
Compound inequality: $\boxed{1 \le x \le 5}$

3. $|x + 3| \ge 1$



Compound inequality: $\boxed{x \le -4 \text{ or } x \ge -2}$

4. $|x - 2| \ge 1$



Compound inequality: $\boxed{x \le 1 \text{ or } x \ge 3}$

4) **EXPLORE**: **DRAW CONCLUSIONS**

① ($\le$) WHEN AN ABSOLUTE VALUE INEQUALITY USES $\le$; IT IS AN "AND" COMPOUND INEQUALITY.

② ($\ge$) WHEN THE THE ABSOLUTE VALUE INEQUALITY USES $\ge$; IT IS AN "OR" COMPOUND INEQUALITY.

6.6 Notes – SOLVE ABSOLUTE VALUE INEQUALITIES

Rules to Solving Absolute Value Inequalities

- The inequality $|ax + b| < c$ where $c > 0$ is equivalent to the compound inequality $-c < ax+b < c$.
- The inequality $|ax + b| > c$ where $c > 0$ is equivalent to the compound inequality ____ or ____

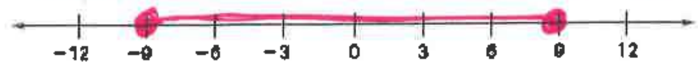
$ax+b < -c$ OR $ax+b > c$

Example 1 ***Solve an absolute value inequality***

Solve the inequality. Graph your solution.

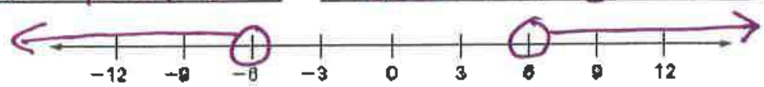
a. $|x| \le 9$

- The distance between x and 0 is less than or equal to 9. So, $\boxed{-9 \le x \le 9}$. The solutions are all real numbers Greater than EQUAL −9 and LESSTHAN EQUAL 9.



b. $|x| > 6$

- The distance between x and 0 is greater than 6. So, $\boxed{x > 6 \text{ or } x < -6}$. The solutions are all real numbers greater than 6 or less than −6.



Example 2 ***Solve "and" absolute value inequality***

Solve $|2x - 7| < 9$. Graph your solution.

Rewrite as compound inequality.

SOLVE

$-9 < 2x - 7 < 9$

$+7 \quad +7 \quad +7$

$\frac{-2}{2} < \frac{2x}{2} < \frac{16}{2}$

The solution is $-1 < x < 8$.

TIP: Check several solutions in the original inequality

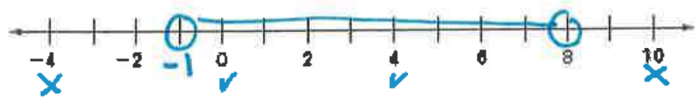
$C: x = -4 \quad |-15| < 9 \quad F$

$C: x = 0 \quad |-7| < 9 \quad T$

$C: x = 4 \quad |1| < 9 \quad T$

$C: x = 10 \quad |13| < 9 \quad F$

Graph:



Example 3 ***Solve "or" absolute value inequality***

Solve $|x + 8| - 4 \ge 2$. Graph your solution.

Isolate the absolute value expression.

Rewrite as compound inequality.

SOLVE

$+4 \quad +4$

$|x+8| \ge 6$

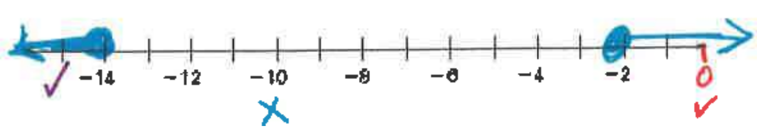
$x+8 \le -6$ OR $x+8 \ge 6$

$-8 \quad -8 \qquad -8 \quad -8$

The solution is $x \le -14$ OR $x \ge -2$.

TIP: Check several solutions in the original inequality

Graph:



$C: x = -15$

$|-7| - 4 \ge 2$

$3 \ge 2$ ✓

$C: x = -10$

$|-2| - 4 \ge 2$

$-2 \ge 2$ X

$C: x = 0 \quad 4 \ge 2$ ✓

Checkpoint -- Solve the inequality. Graph your solution.

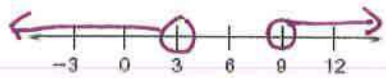
1) $\frac{3|x-6|}{3} > \frac{9}{3}$

$\rightarrow |x-6| > 3$

$x-6 < -3$ or $x-6 > 3$

$+6 \quad +6 \qquad\qquad +6 \quad +6$

$\boxed{x < 3 \text{ or } x > 9}$



2) $|2x-5| - 8 \leq -3$

$+8 \quad +8$

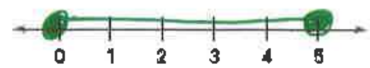
$\rightarrow |2x-5| \leq 5$

$-5 \leq 2x-5 \leq 5$

$+5 \qquad +5 \quad +5$

$\frac{0}{2} \leq \frac{2x}{2} \leq \frac{10}{2}$

$\boxed{0 \leq x \leq 5}$



3) $-5|6x-1| + 10 < 30$

$-10 \quad -10$

$\frac{-5|6x-1|}{-5} < \frac{20}{-5}$

$|6x-1| < -4$

ABSOLUTE VALUE CAN NOT BE NEGATIVE

X = NO SOLUTION

SOLVING INEQUALITIES

1) <u>One-Step and Multi-Step Inequalities</u>

- Follow the steps for solving an equation, **BUT** REVERSE the inequality symbol when mult. or divide the varicble by a negative. number.

2) <u>Compound Inequalities</u>

- If necessary, rewrite the inequality as two separate inequalities; solve each inequality separately. In the solution, you must include the words "and" or "or".

3) <u>Absolute Value Inequalities</u>

- If necessary, isolate the absolute value expression on one side of the inequality. Rewrite the absolute value inequality as a Compound inequality; then solve it.