

AP Statistics – 6.3N Activity	Name:
Goal: Understand Normal Model as an Approximation to the Binomial Model	Date:

### I. Example: Teens and Debit Cards

In a survey of 506 teenagers ages 14-18, subjects were asked a variety of questions about personal finance. One question asked teens if they had a debit card. Suppose that exactly 10% of teens ages 14-18 have debit cards.

- (a) Show that the distribution of  $X$  is approximately binomial.

Let  $X$  = the number of teens in a random sample of size 506 that have a debit card.

B DEBIT OR NO DEBIT CARD

I Sampling without replacement. SINCE THERE ARE MILLIONS OF TEENS AND WE HAVE A SAMPLE OF 506, THE 10% CONDITION HAS BEEN MET.

N FIXED TRIALS  $n = 506$

S FIXED PROBABILITY  $p = 10\%$

This is a binomial distribution  $\rightarrow B(n=506, p=.1)$

- (b) Check the conditions for using a Normal approximation in this setting.

To use the Normal distribution you must check BOTH  $np$  and  $n(1-p)$  must be Greater or Equal to 10.

Check:

$$np \geq 10$$

$$506(.1) \geq 10$$

$$50.6 \geq 10 \checkmark$$

$$n(1-p) \geq 10$$

$$506(.9) \geq 10$$

$$455.4 \geq 10 \checkmark$$

- (c) Use a Normal distribution to estimate the probability that 40 or fewer teens in the sample have debit cards.

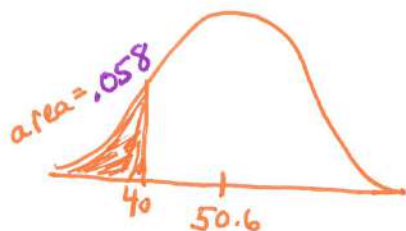
USING A NORMAL APPROXIMATION:

(a+b) Conditions met  $n = 506$   $p = .1$

ON Green Sheet

$$\begin{cases} \mu_x = np = 506(.1) = 50.6 \\ \sigma_x = \sqrt{np(1-p)} = \sqrt{506(.1)(.9)} = 6.75 \end{cases}$$

STATE DISTRIBUTION  $\rightarrow N(50.6, 6.75)$



$$P(X \leq 40) = .058$$

THERE IS APPROXIMATELY, A 6 % Chance that 40 OR FEWER TEENS will have a debit card.

EXACT PROBABILITY WITH BINOMIAL DISTRIB:

$$B(506, .1)$$

$$P(X \leq 40) = .063$$

Remember

- ① BINOMIAL DISTRIB GRAPH WITH  $p = .1$  IS SKewed RIGHT



THEREFORE IT WILL HAVE A LARGER AREA

- ② As  $n \uparrow$ , the distrib. approaches a Normal distrib.

## II. Notes –see definitions on page 395

The Normal Distribution can be used as an approximation for the binomial distribution

- If... the number of successes and failures are at least 10. ( $np \geq 10$  and  $n(1-p) \geq 10$ )
- In English that means – when the number of trials is large, this method can be used.

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

$$P(x \geq 4000) = P\left(z \geq \frac{x - \mu}{\sigma}\right)$$

Optional to calculate z-score,

but you **MUST CLEARLY** sketch normal graph.

***In the above model***, replace the inequality with less than, etc., whatever is appropriate for the problem you are solving. Additionally, replace the 4000 for your problem.

### **Steps:**

**Step 1: Define the Random Variable and check binomial conditions**

**Step 2: Check the normal conditions  $np \geq 10$  and  $n(1-p) \geq 10$**

**\*\* You must show BOTH calculations to indicate you verified the normal condition.**

**Step 3: Calculate the mean and standard deviation with the formulas above (green sheet); and state the normal model  $N(\mu, \sigma)$**

**Step 4: Sketch the normal graph (identify area for probability, mean and x-value); and calculate the probability of interest**

**Step 5: State your conclusion, in the context of the problem.**