AP Statistics – 6.2B	Name:
Goal: Combining Random Variables	Date:

I. **Example #1:** **CYU (page 370):**

- Always state what your random variables are and distributions given:

X = # OF CARS SOLD DURING 1ST HOUR ON A RANDOMLY SELECTED FRIDAY

Y = # OF CARS LEASED "

T = TOTAL # OF CARS SOLD OR LEASED

B = MANAGERS BONUS = 500 X + 300 Y

1. Find and interpret μ_t

$\mu_T = \mu_X + \mu_Y = 1.1 + 0.7 = 1.8$

ON AVERAGE, THIS DEALARSHIP SELLS OR LEASES 1.8 CARS IN THE FIRST HOUR OF BUSINESS ON FRIDAYS

2. Calculate σ_t showing work clearly

$\sigma_T^2 = \sigma_X^2 + \sigma_Y^2 = (.943)^2 + (.64)^2 = 1.299$

$\sigma_T = \sqrt{1.299} = \boxed{1.14}$

3. Calculate μ_b and σ_b showing work clearly

Bonus: \$500/sale \$300/LEASED

$\mu_B = 500\,\mu_X + 300\,\mu_Y = 500(1.1) + 300(.7) = \boxed{\mu_B = \$760}$

$\sigma_B^2 = (500\,\sigma_X)^2 + (300\,\sigma_Y)^2 = [500 \cdot .943]^2 + [300 \cdot .64]^2 = 259,176.25$

$\boxed{\sigma_B = \$509.09}$

II. **Example #2:** **CYU (page 372):**

- **Define random variables.** X, Y and B were Defined in Example 1

D = X - Y

1. Find and interpret μ_d.

$\mu_D = \mu_X - \mu_Y = 1.1 + 0.7 = 1.8$

ON AVERAGE, THIS DEALERSHIP SELLS 0.4 CARS MORE THAN IT LEASES DURING THE FIRST HOUR OF BUSINESS ON FRIDAYS.

2. Calculate σ_d showing work clearly. How does this compare with Example #2

$\sigma_D^2 = \sigma_X^2 + \sigma_Y^2 = (.943)^2 + (.64)^2 = 1.299$

$\sigma_D = \sqrt{1.299} = \boxed{1.14}$ The variances are the same!

3. Calculate μ_b and σ_b showing work clearly.

$\mu_B = 500\,\mu_X - 300\,\mu_Y = 500(1.1) - 300(.7) = \$340$

$\sigma_B = \sqrt{(500 \cdot .943)^2 + (300 \cdot .64)^2} = \boxed{\$509.09}$ * NOTICE THIS IS THE SAME AS THE ABOVE 3.

iii. Your Notes

Rules for Combining Random Variable (p365 and 367)

Independent Random Variable (p365)

IV. Example #3

Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of:

a) $2Y + 20$
b) $3X$
c) $0.25X + Y$
d) $X - 5Y$
e) $X_1 + X_2 + X_3$

	Mean	SD
X	80	12
Y	12	3

[A] LINEAR TRANSFORMATION OF Y

$E(2Y+20) = 2\mu_Y + 20 = 2(12)+20 = \boxed{\mu = 44}$

$SD(2Y+20) = 2\sigma_Y = 2(3) = \boxed{\sigma = 6}$

constant does NOT change variability

[B] LINEAR TRANSFORMATION OF X

$E(3X) = 3 \cdot \mu_X = 3(80) = \boxed{\mu = 240}$

$SD(3X) = 3 \cdot \sigma_X = 3(12) = \boxed{\sigma = 36}$

C – E include both linear transformations and combining RV's

[C] $E(.25X+Y) = .25\mu_X + \mu_Y = .25(80)+12 = \boxed{\mu = 32}$

$VAR(.25X+Y) = (.25\sigma_X)^2 + \sigma_Y^2 = (.25 \cdot 12)^2 + 3^2 = 18$ $\boxed{\sigma = 4.24}$

[D] $E(X-5Y) = \mu_X - 5\mu_Y = 80 - 5(12) = \boxed{\mu = 20}$

$VAR(X-5Y) = \sigma_X^2 + (5\sigma_Y)^2 = 12^2 + (5 \cdot 3)^2 = 369$ $\boxed{\sigma = 19.21}$

[E] $E(X_1+X_2+X_3) = \mu_X + \mu_X + \mu_X = 80+80+80$ $\boxed{\mu = 240}$

$VAR(X_1+X_2+X_3) = \sigma_X^2 + \sigma_X^2 + \sigma_X^2 = 12^2+12^2+12^2 = 432$

$\boxed{\sigma = 20.78}$

V. **Example ~~3~~ 4 *"Apples"***

Suppose that the weights of a certain variety of apples have weights that are approximately Normally distributed with a mean of 9 ounces and a standard deviation of 1.5 ounces. If bags of apples are filled by randomly selecting 12 apples, what is the probability that the sum of the weights of the 12 apples is less than 100 ounces?

LET A = A RANDOMLY SELECTED APPLE. WITH $N(9, 1.5)$

T = TOTAL WEIGHT $A_1 + A_2 + A_3 \cdots A_{12}$ *

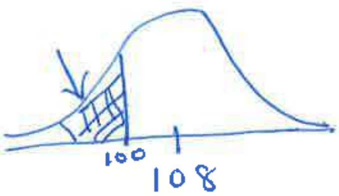
* THE APPLES ARE RANDOMLY SELECTED SO WE HAVE 12 INDEPENDENT VARIABLES

$\boxed{\text{FIND } P(T < 100)}$ $N(108, 5.2)$

$= P\left(Z < \frac{100-108}{5.2}\right)$

$= P(Z < -1.54) = .0618$

normalcdf(-E99, -1.54, 0, 1)



$\mu_T = \mu_{A_1} + \mu_{A_2} \cdots + \mu_{A_{12}}$

$= 9+9+9+9+9+9+9+9+9+9+9+9$

$\mu_T = 108\,oz$

$\sigma_T^2 = \sigma_{A_1}^2 + \cdots + \sigma_{A_{12}}^2$

$= 1.5^2 + \cdots + 1.5^2 = 27\,oz$

$\sigma_T = 5.2\,oz$

Conclude: There is about a 6.2% chance that the 12 randomly selected apples will have a total weight of less than 100 ounces

VI. **Example ~~3~~ 5 *"Speed Dating"***

Suppose that the height M of male speed daters follows a Normal distribution with a mean of 70 inches and a standard deviation of 3.5 inches and the height F of female speed daters follows a Normal distribution with a mean of 65 inches and a standard deviation of 3 inches. What is the probability that a randomly selected male speed dater is taller than the randomly selected female speed dater with whom he is paired?

M = height of male speed dater — $N(70, 3.5)$

F = height of female speed dater — $N(65, 3)$

Assume M and F are independent

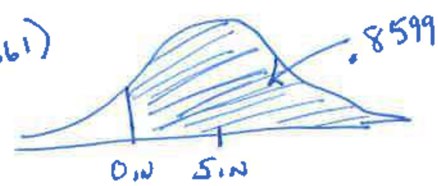
$P(M > F) = P(M - F > 0)$

LET: $D = M - F$

$\mu_D = \mu_M - \mu_F = 70 - 65 = 5\,in$

$\sigma_D^2 = \sigma_M^2 + \sigma_F^2 = \sqrt{3.5^2 + 3^2} = 4.61\text{ inches}$

$N(5, 4.61)$



$Z = \frac{0-5}{4.61} = -1.08$

$P(Z \geq -1.08) = .8599$

normalcdf(-1.08, E99, 0, 1)

CONCLUDE: THERE IS ABOUT AN 86% CHANCE THAT A RANDOMLY MALE SPEED DATER WILL BE TALLER THAN THE FEMALE HE IS RANDOMLY PAIRED WITH.

OR IN ABOUT 86% OF SPEED DATING COUPLES, THE MALE WILL BE TALLER THAN THE FEMALE.