AP Physics - Light Wrap Up

Here beith the equations for the light/optics deal. There are several of them, but not nearly enough.

$$v = f\lambda$$

Here we have the equation for the velocity of a wave as a function of frequency and wavelength.

$$n = \frac{c}{v}$$

This is the equation for the index of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This is Snell's law. Use the thing to find the angle of incidence or refraction.

$$\sin\theta_C = \frac{n_1}{n_2}$$

This is the equation for the critical angle – you know, the whole total internal reflection deal.

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

This is the equation for a thin lens. It relates image distance (s_i), object distance (s_o) and focal length.

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

Magnification anyone? This calculates the magnification provided by a lens as a function of image height, object height, image distance, and object distance.

$$f = \frac{R}{2}$$

This equation calculates the focal length of a converging (concave) spherical mirror as a function of the radius of curvature.

$d \sin\theta = m\lambda$

This equation is used for interference problems with double slits, single slits, and diffraction gratings. It equates the path difference ($d \sin\theta$) to the wavelength of the light. The ***m*** term has to do with order number of the fringe. ***m*** is 0, 1, 2, 3, &tc.

$$x_m \approx \frac{m\lambda L}{d}$$

This equation is used to find the spacing between fringes for the double slit, single slit, and diffraction grating deal.

Here's the stuff we're supposed to be able to do.

B. Physical Optics

1. You should understand the interference and diffraction of waves so you can:

a. Apply the principles of interference to coherent sources oscillating in phase in order to:

(1) Describe conditions under which the waves reaching an observation point from two or more sources will all interfere constructively or under which the waves from two sources will interfere destructively.

Can you do this? Do you know why waves interfere constructively and destructively? The Physics Kahuna hopes you do. Basically its just the old law of superposition. The waves add algebraically. $d \sin\theta = m\lambda$ ***can be used to determine the spacings and &tc.***

(2) Determine locations of interference maxima or minima for two sources or determine the frequencies or wavelengths that can lead to constructive or destructive interference at a certain point.

Basically two sources produce waves that spread out in circles (if we're talking about two dimensions) or spheres (for three dimensions). Where the crests meet we get a maxima – the waves are in phase and add up. Where a crest meets a trough we get destructive interference and a minima – the out of phase waves cancel each other out.

(3) Relate amplitude and intensity produced by two or more sources that interfere constructively to the amplitude and intensity produced by a single source.

For a single source, the incident waves arrive one after the other. Think of a crest followed by a trough. For waves from two or more sources there are bunches of waves that arrive either in phase, out of phase, or somewhere in between. If they arrive in phase we get constructive interference and a large amplitude is produced (much larger than what we saw for a single source). If they arrive out of phase, we get destructive interference. So for multiple sources we get minima and maxima.

b. Apply the principles of interference and diffraction to waves that pass through a single or double slit or through a diffraction grating so they can:

(1) Sketch or identify the intensity patterns that result when monochromatic waves pass through a single slit and fall on a distant screen, and describe how this pattern will change if slit width or the wavelength of the waves is changed.

Not difficult. Consult the handout to see what a single slit diffraction pattern ought to look like. As for changing the slit width, look at the equation. $x_m \approx \frac{m\lambda L}{d}$ ***d in the equation is the slit width. As it gets larger the distance between the dark fringes decreases, as the slit width gets smaller, the dark fringe spacing would increase.***

(2) Calculate for a single-slit pattern, the angles or the positions on a distant screen where the intensity is zero.

Easy as pie. Simply use the equation: $x_m \approx \frac{m\lambda L}{d}$. ***This equation calculates the various positions where the intensity is zero.***

(3) Sketch or identify the intensity pattern that results when monochromatic waves pass through a double slit, and identify which features of the pattern result from single-slit diffraction and which from two-slit interference.

Consult the handout to learn how to do this.

(4) Calculate, for a two-slit interference pattern, the angles or the positions on a distant screen at which intensity maxima or minima occur.

Two equations help you do this: $d\sin\theta = m\lambda$ ***and*** $x_m \approx \frac{m\lambda L}{d}$. ***Simply apply the equations.***

(5) Describe or identify the interference pattern formed by a grating of many equally spaced narrow slits, calculate the location of intensity maxima, and explain qualitatively why a multiple-slit grating is better than a two-slit grating for making accurate determinations of wavelength.

This is the grating deal where you use $d\sin\theta = m\lambda$ ***and*** $x_m \approx \frac{m\lambda L}{d}$. ***Once again, simply apply the equations.***

c. Apply the principles of interference to light reflected by thin films so they can:

(1) State under what conditions a phase reversal occurs when light is reflected from the interface between two media of different indices of refraction.

Okay, the phase reversal occurs when light is reflected by a medium that has a higher index of refraction than the medium the light was going though originally. When the new medium has a smaller index of refraction, no phase change takes place.

(1) Determine whether rays of monochromatic light reflected from two such interfaces will interfere constructively or destructively, and thereby account for Newton's rings and similar phenomena, and explain how glass may be coated to minimize the reflection of visible light.

Just apply the rules for light reflection above with the explanation in the handout on Newton's rings. For the coating thing, you need nonreflective coatings which means you want destructive interference. This means that the minimum thickness for the film is $2t = \frac{\lambda}{2}$***. The index of refraction for the film must be such that we get a phase change on the top and at the film/glass interface.***

2. You should understand dispersion and the electromagnetic spectrum so you can:

a. Relate a variation of index of refraction with frequency to a variation in refraction.

Dispersion is the deal where short wavelength light has a bigger index of refraction than does light with a longer wavelength in a transparent media. So blue light is bent more than red light. Examples are the prism and rainbows.

b. Know the names associated with electromagnetic radiation and be able to arrange in order of increasing wavelength the following: visible light of various colors, ultraviolet light, infrared light, radio waves, x-rays, and gamma rays.

This is a simple bit of memorization for a bright AP Physics student such as yourself.

3. You should understand the transverse nature of light waves so they can explain qualitatively why light can exhibit polarization.

See the notes. The Physics Kahuna did a fine job of explaining the whole thing in the handout. Please consult it.

C. Geometrical Optics

1. You should understand the principles of reflection and refraction so you can:

a. Determine how the speed and wavelength of light change when light passes from one medium to another.

This is the old index of refraction deal.

b. Show on a diagram the directions of reflected and refracted rays.

The Physics Kahuna showed you how to make these drawings. Consult your notes or the handout.

c. Use Snell's laws to relate the directions of the incident ray and the reflected ray, and the indices of refraction of the media.

This basically is using Snell's law to find the various angles of incidence and refraction, et al.

d. Identify conditions under which total internal reflection will occur.

The conditions is when you get the critical angle.

2. You should understand image formation by plane or spherical mirrors so you can:

a. Relate the focal point of a spherical mirror to its center of curvature.

Focal length is half the radius of the mirror. The center of curvature is positioned a distance of one radius from the center of the mirror.

b. Given a diagram of a mirror with the focal point shown, locate by ray tracing the image of a real object and determine whether the image is real or virtual, upright or inverted, enlarged or reduced in size.

We did a bunch of these using the various rules for the different rays reflecting off the mirror.

3. You should understand image formation by converging or diverging lenses so you can:

a. Determine whether the focal length of a lens is increased or decreased as a result of a change in the curvature of its surface or in the index of refraction of the material of which the lens is made or the medium in which it is immersed.

If the curvature changes, the focal length changes. In general, the greater the curvature the shorter the focal length. As the index of refraction is changed; the greater the index of refraction the smaller will be the focal length (bigger angle of refraction so smaller focal length, right?)

b. Determine by ray tracing the location of real object located inside or outside of the focal point of the lens, and state whether the resulting image is upright or inverted, real or virtual.

We did a bunch of these using the various rules for the different rays being refracted by the lens.

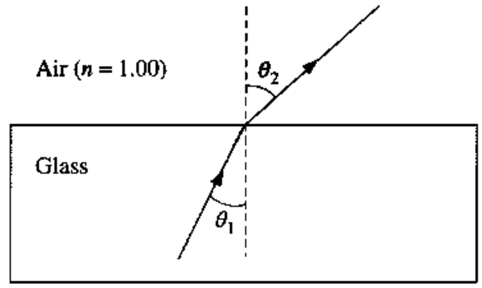
c. Use the thin lens equation to relate the object distance, image distance, and focal length for a lens, and determine the image size in terms of the object size.

Just use the equation. We did several problems.

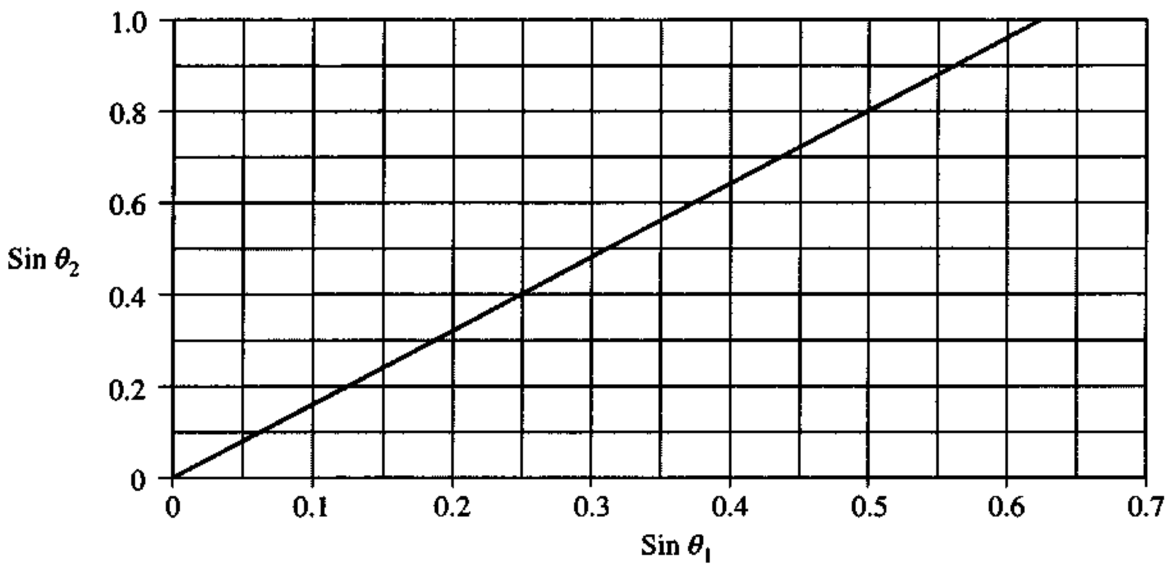
d. Analyze simple situations in which the image formed by one lens serves as the object for another lens.

The idea here is that the first lens forms an image which is then the object for the next lens, which can then form another, final image. We did some of these problems and your handout shows you an example.

From 2001:



- In an experiment a beam of red light of wavelength 675 nm in air passes from glass into air, as shown above. The incident and refracted angles are θ_1 and θ_2, respectively. In the experiment, angle θ_2 is measured for various angles of incidence θ_1, and the sines of the angles are used to obtain the line shown in the following graph.



(a) Assuming an index of refraction of 1.00 for air, use the graph to determine a value for the index of refraction of the glass for the red light. Explain how you obtained this value.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

The slope is n_1/n_2 since n_2 is 1, the index of refraction for the glass, n_1 is the slope.

$$slope = \frac{\Delta\sin\theta_2}{\Delta\sin\theta_1} = \frac{0.8-0}{0.5-0} = \boxed{1.6}$$

(b) For this red light, determine the following.

i. The frequency in air

$$c = f\lambda \quad f = \frac{c}{\lambda} = 3\,x\,10^8\,\frac{m}{s}\left(\frac{1}{675\,x\,10^{-9}\,m}\right) = 4.44\,x\,10^{14}\,Hz$$

ii. The speed in glass

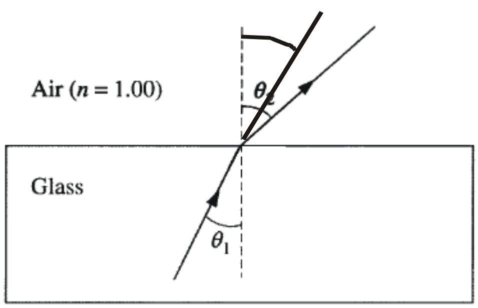
$$n = \frac{c}{v} \quad v = \frac{c}{n} \quad = 3\,x\,10^8\,\frac{m}{s}\left(\frac{1}{1.6}\right) \quad = \quad \boxed{1.88\,x\,10^8\,\frac{m}{s}}$$

iii. The wavelength in glass

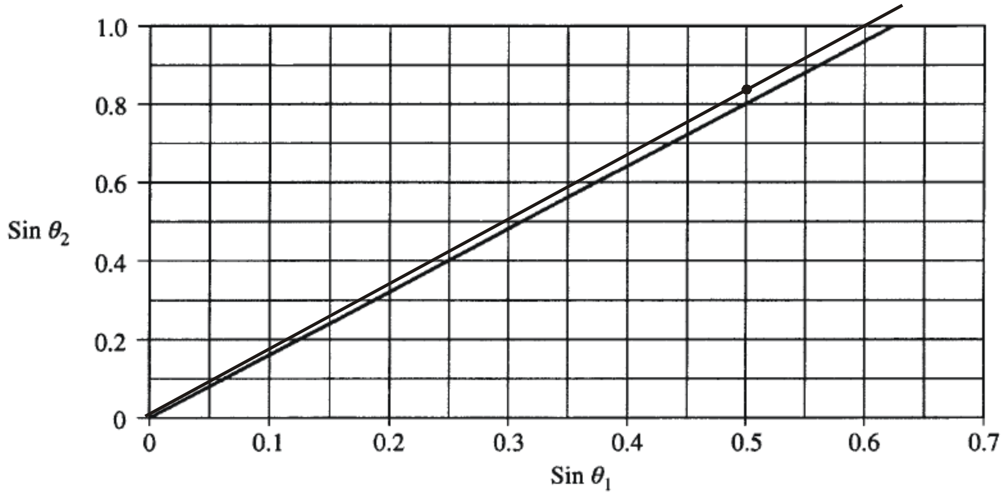
$$v = f\lambda \quad \lambda = \frac{v}{f} \quad = 1.88\,x\,10^8\,\frac{m}{\cancel{s}}\left(\frac{1}{4.44\,x\,10^{14}\,\frac{1}{\cancel{s}}}\right) \quad = \quad \boxed{423\,nm}$$

(c) The index of refraction of this glass is 1.66 for violet light, which has wavelength 425 nm in air.

i. Given the same incident angle θ_1, show on the ray diagram on the previous page how the refracted ray for the violet light would vary from the refracted ray already drawn for the red light.



ii. Sketch the graph of sin θ_2 versus sin θ_1 for the violet light on the figure on the previous page that shows the same graph already drawn for the red light.



Pick a point on the graph, say where sin θ_1 = 0.5, we can use Snell's law to find the value for $\sin\theta_2$:

$$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.66(0.5)}{1} = 0.83$$

We plot the point (0.5, 0.83) and connect it to the origin. This gives us the new curve. Note that the slope is slightly different..

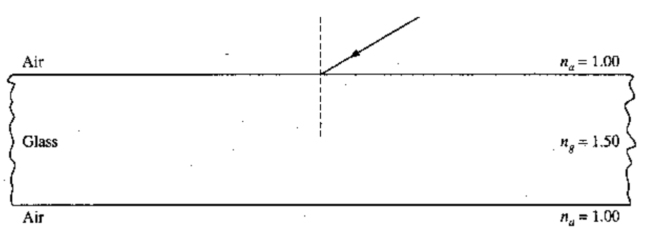
(d) Determine the critical angle of incidence θ_C, for the violet light in the glass in order for total internal reflection to occur.

$$\sin\theta_C = \frac{n_2}{n_1} = \frac{1}{1.66} \qquad \boxed{\theta_c = 37.0^o}$$

From 2000:

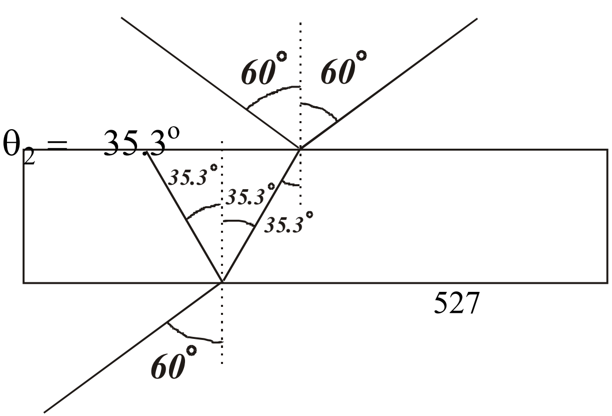
- A sheet of glass has an index of refraction n_g = 1.50. Assume that the index of refraction for air is n_a = 1.00.

(a) Monochromatic light is incident on the glass sheet, as shown in the figure below, at an angle of incidence of 60^0. On the figure, sketch the path the light takes the first time it strikes each of the two parallel surfaces. Calculate and label the size of each angle (in degrees) on the figure, including angles of incidence, reflection, and refraction at each of the two parallel surfaces shown.

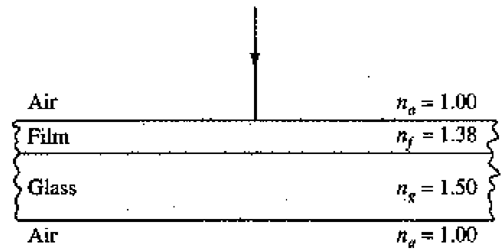


$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2} = \frac{(1)\sin 60^0}{1.50} = 0.5774 \quad \theta_2 = 35.3^o$$



(b) Next a thin film of material is to be tested on the glass sheet for use in making reflective coatings. The film has an index of refraction n_f = 1.38. White light is incident normal to the surface of the film as shown below. It is observed that at a point Where the light is incident on the film, light reflected from the surface appears green (λ = 525 nm).



i. What is the frequency of the green light in air?

$$v = f\lambda \qquad f = \frac{v}{\lambda} = \frac{3.0\,x\,10^{8}\,\cancel{m}/s}{525\,x\,10^{-9}\,\cancel{m}} = \boxed{5.71\,x\,10^{14}\,Hz}$$

ii. What is the frequency of the green light in the film?

The frequency is the same in any medium $\boxed{5.71\,x\,10^{14}\,Hz}$

iii. What is the wavelength of the green light in the film?

$$n = \frac{c}{v} \qquad c = nv \qquad n_1 v_1 = n_2 v_2 \qquad v = f\lambda$$

$$n_1\left(\cancel{f}\lambda_1\right) = n_2\left(\cancel{f}\lambda_2\right) \qquad n_1\lambda_1 = n_2\lambda_2 \qquad \lambda_2 = \frac{n_1\lambda_1}{n_2} = \frac{525\,nm}{1.38} = \boxed{380\,nm}$$

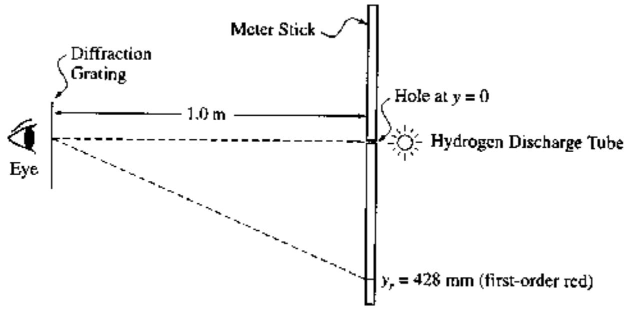
iv. Calculate the minimum thickness of film that would produce this green reflection.

The minimum thickness happens when the light path going straight down through the film and back out is equal to the wavelength of the light in the film (this will get you a path difference of one wavelength). So two time the thickness must equal the wavelength.

$$2t = \lambda_f \qquad t = \frac{\lambda_f}{2} = \frac{380\,nm}{2} = \boxed{190\,nm}$$

From 1998:

- A transmission diffraction grating with 600 *lines/mm* is used to study the line spectrum of the light produced by a hydrogen discharge tube with the setup shown below. The grating is 1.0 *m* from the source (a hole at the center of the meter stick). An observer sees the first-order red line at a distance $y_r = 428\ mm$ from the hole.



Note: Figure is drawn to scale.

a. Calculate the wavelength of the red line in the hydrogen spectrum.

$$600\,\frac{lines}{mm}$$

Number of mm/line

$$= \frac{1}{600\,\dfrac{line}{mm}} = 0.001667\,\frac{mm}{line} = 1.667\,x\,10^{-3}\,\frac{\cancel{mm}}{line}\left(\frac{10^{-3}\,m}{1\,\cancel{mm}}\right) = 1.667\,x\,10^{-6}\,m$$

$$\theta = \tan^{-1}\left(\frac{428\,x\,10^{-3}\,\cancel{m}}{1.0\,\cancel{m}}\right) = 23.17^{o}$$

$$d\sin\theta = m\lambda$$

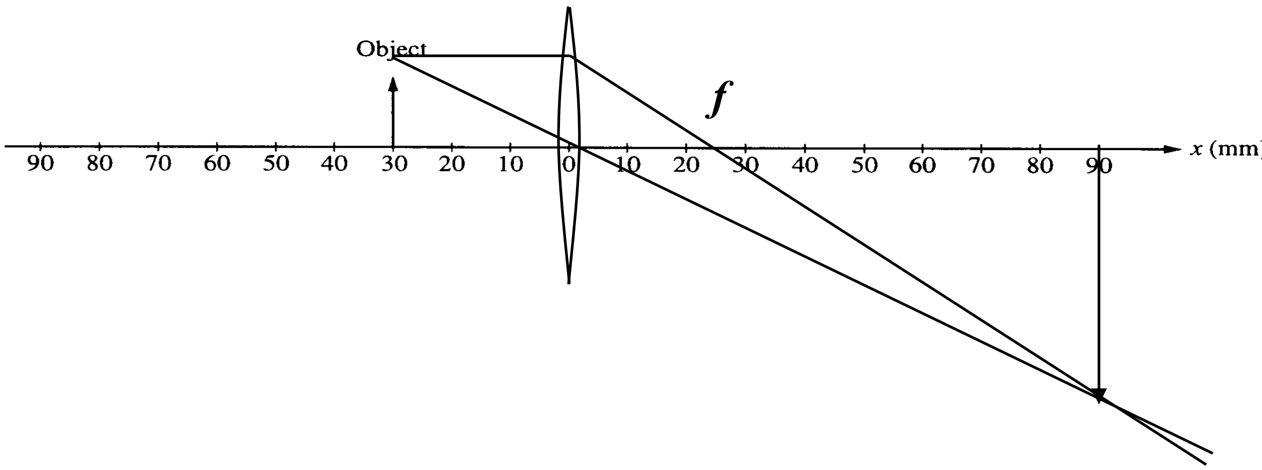
$$\lambda = \frac{\left(1.667\,x\,10^{-6}\,m\right)\sin 23.17^{o}}{1} = 0.656\,x\,10^{-6}\,m = 656\,x\,10^{-9}\,m = \boxed{656\,nm}$$

From 1997:

- An object is placed 30 *mm* in front of a lens. An image of the object is located 90 *mm* behind the lens.
  a. Is the lens converging or diverging? Explain your reasoning.
  Converging. Only converging lenses can project real images behind the lens. Diverging lens always produce a virtual image on the same side as the object.
  b. What is the focal length of the lens?

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f} \qquad \frac{1}{90\ mm} + \frac{1}{30\ mm} = \frac{1}{f} \qquad f = \boxed{22.5\ mm}$$

c. On the axis below, draw the lens at position $\boldsymbol{x} = 0$. Draw at least two rays and locate the image to show the situation described below.



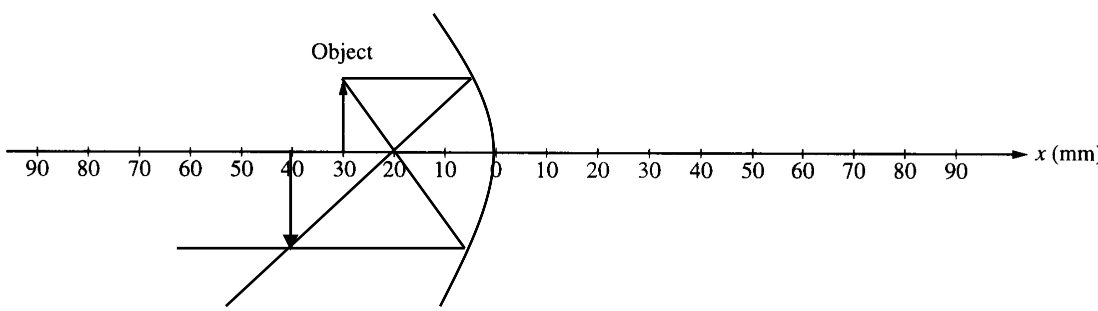
d. Based on your diagram in (c), describe the image by answering the following questions in the blank spaces provided.

i. Is the image real or virtual? ***Real***

ii. Is the image smaller than, larger than, or same size as the object? ***Larger***

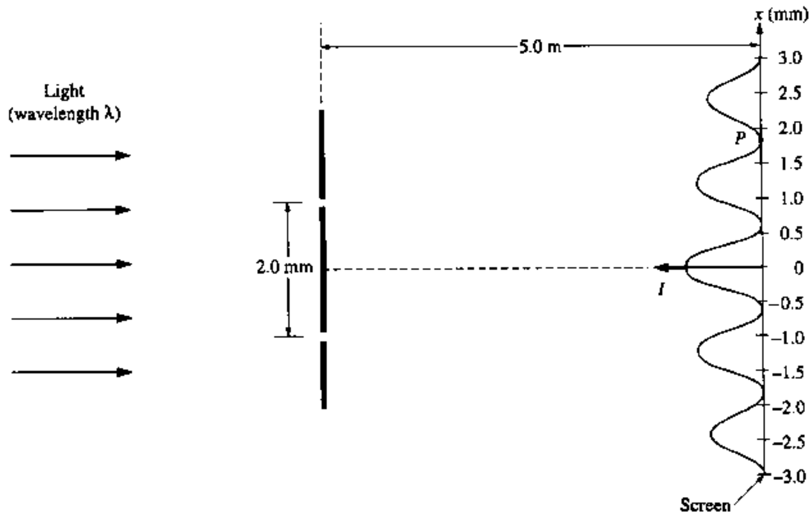
iii. Is the image inverted or upright compared to the object? ***Inverted***

e. The lens is replaced by a concave mirror of focal length 20 *mm*. On the axis below, draw the mirror at position $\boldsymbol{x} = 0$ so that a real image is formed. Draw at least two rays and locate the image to show this situation



From 1996:

- Coherent monochromatic light of wavelength **λ** in air is incident on two narrow slits, the centers of which are 2.0 *mm* apart, as shown. The interference pattern observed on a screen 5.0 *m* away is represented in the figure by the graph of light intensity ***I*** as a function of position ***x*** on the screen.



Note: Figure not drawn to scale.

a. What property of light does this interference experiment demonstrate?

Diffraction, Wave Property of Light, or Constructive & Destructive interference

b. At point ***P*** in the diagram, there is a minimum in the interference pattern. Determine the path difference between the light arriving at this point from the two slits.

m = 1 (first bright fringe is at m = 0) so the dark fringe must happen at m + ½, the path difference is $d\sin\theta$ ***. So we can use the equation given for the thing.***

$$d\sin\theta = m\lambda \quad d\sin\theta = \left(m + \frac{1}{2}\right)\lambda = \left(1 + \frac{1}{2}\right)\lambda = \boxed{\frac{3}{2}\lambda}$$

c. Determine the wavelength, **λ**, of the light.

$$x_m \approx \frac{m\lambda L}{d} = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad x = \left(\frac{3}{2}\right)\frac{\lambda L}{d} \qquad \lambda = \frac{dx}{mL}\left(\frac{2}{3}\right)$$

$$\lambda = \frac{\left(2\,x\,10^{-3}\,m\right)\left(1.8\,x\,10^{-3}\,\cancel{m}\right)}{1\left(5.0\,\cancel{m}\right)}\left(\frac{2}{3}\right) = 0.48\,x\,10^{-6}\,m = 4.8\,x\,10^{-7}\,m = \boxed{480\,nm}$$

d. Briefly and qualitatively describe how the interference pattern would change under each of the following separate modifications and explain your reasoning.

i. The experiment is performed in water, which has an index of refraction greater than 1.

$$x_m \approx \frac{m\lambda L}{d}$$ ***For m = 0 Wavelength is less under water. So*** x_m ***is less, therefore the interference pattern will be compressed toward the center.***

ii. One of the slits is covered.

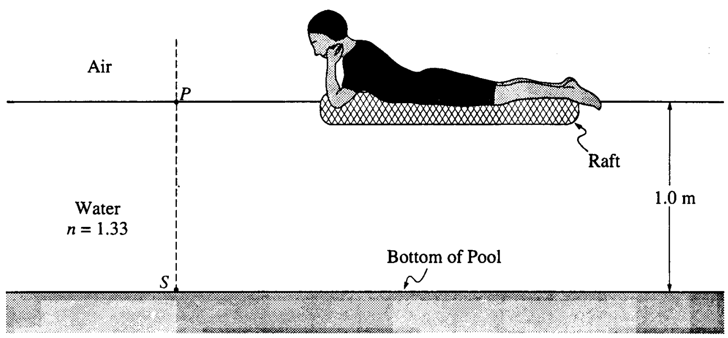
New pattern is a single slit pattern. The pattern will spread with a larger central maximum.

iii. The slits are moved farther apart.

$$x_m = \frac{m\lambda L}{d}$$ ***as d increases x decreases. Interference pattern will be compressed toward the center.***

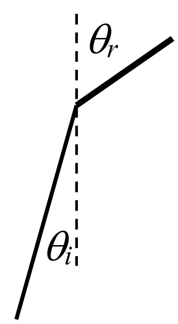
From 1994:

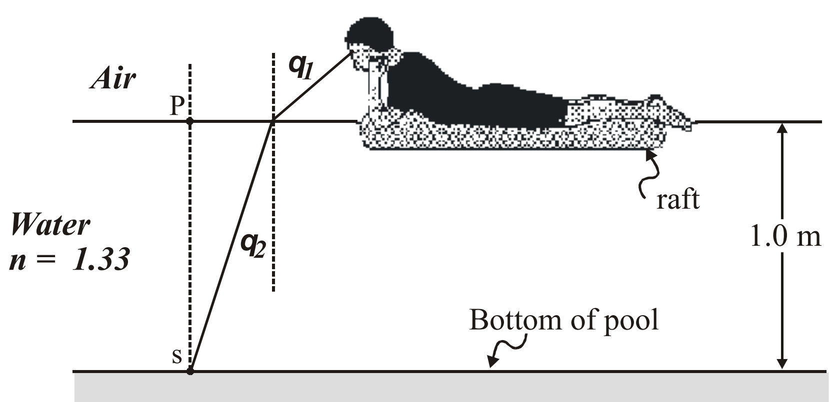
- A point source ***S*** of monochromatic light is located on the bottom of a swimming pool filled with water to a depth of 1.0 meter, as shown below. The index of refraction of water is 1.33 for this light. Point ***P*** is located on the surface of the water directly above the light source. A person floats motionless on a raft so that the surface of the water is undisturbed.



a. Determine the velocity of the source's light in water.

$n = \frac{c}{v}$ $\quad n = \frac{c}{v} \quad v = \frac{c}{n} \quad v = \dfrac{3.00\,x\,10^8\,\frac{m}{s}}{1.33} = \boxed{2.26 \times 10^8\,\frac{m}{s}}$

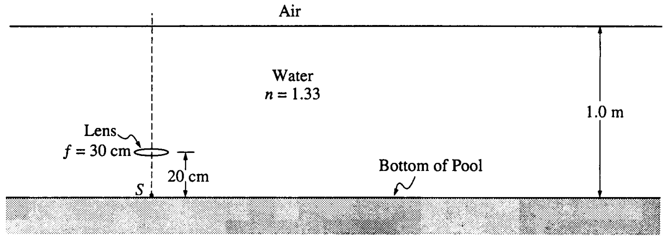




b. On the diagram above, draw the approximate path of a ray of light from the source ***S*** to the eye of the person. It is not necessary to calculate any angles.

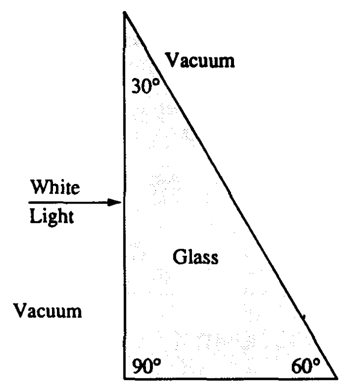
c. Determine the critical angle for the airwater interface.

$$\sin\theta_c = \frac{n_1}{n_2} = \left(\frac{1}{1.33}\right) \quad \boxed{\theta_c = 48.8^{\circ}}$$



Suppose that a converging lens with focal length of 30 centimeters in water is placed 20 centimeters above the light source, as shown in the diagram above. An image of the light source is formed by the lens.

	Wavelength in Vacuum	Index of Refraction of Glass
Red Light	700 nm	1.5
Blue Light	480 nm	1.6



Calculate the position of the image with respect to the bottom of the pool.

$$\frac{1}{s_i}+\frac{1}{s_o}=\frac{1}{f} \qquad \frac{1}{s_i}=\frac{1}{f}-\frac{1}{s_o} \qquad \frac{1}{s_i}=\frac{1}{20\,cm}-\frac{1}{30\,cm} \qquad s_i = \boxed{-60\,cm}$$

Image is 60 cm below the lens so it is 40 cm below the pool bottom.

d. If, instead, the pool were filled with a material with a different index of refraction, describe the effect, if any, on the image and its position in each of the following cases.

i. The index of refraction of the material is equal to that of the lens.

No refraction occurs. No image is formed.

ii. The index of refraction of the material is greater than that of water but less than that of the lens.

An image forms, but it is smaller and closer to S than in part d. This is because the angle of refraction is smaller – the light is bent at a smaller angle.

From 1993:

- The glass prism shown below has an index of refraction that depends on the wavelength of the light that enters it. The index of refraction is 1.50 for red light of wavelength 700 nanometers (700×10^9 meter) in vacuum and 1.60 for blue light of wavelength 480 nanometers in vacuum. A beam of white light is incident from the left, perpendicular to the first surface, as shown in the figure, and is dispersed by the prism into its spectral components.

a. Determine the speed of the blue light in the glass.

$$n = \frac{c}{v} \quad v = \frac{c}{n} \quad = \frac{3\,x\,10^8\,\frac{m}{s}}{1.60} \quad = 1.9\,x\,10^8\,\frac{m}{s} \quad = \boxed{0.625\,c}$$

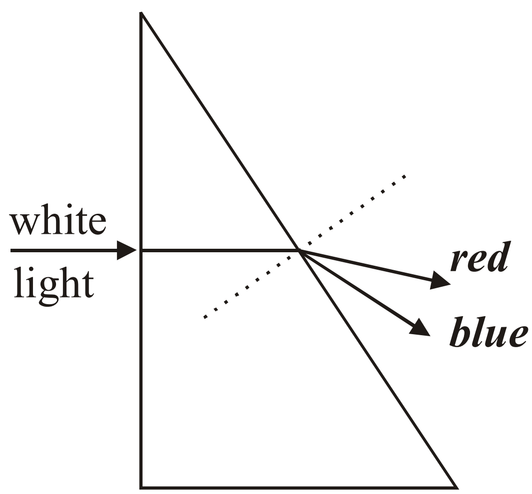
b. Determine the wavelength of the red light in the glass.

$$n = \frac{c}{v} \quad c = nv \qquad n_1 v_1 = n_2 v_2 \quad v = f\lambda$$

$$n_1\left(\cancel{f}\lambda_1\right) = n_2\left(\cancel{f}\lambda_2\right) \quad n_1\lambda_1 = n_2\lambda_2 \qquad \lambda_2 = \frac{n_1\lambda_1}{n_2} \quad = \frac{700\,nm}{1.60} \quad = \boxed{438\,nm}$$

c. Determine the frequency of the red light in the glass.

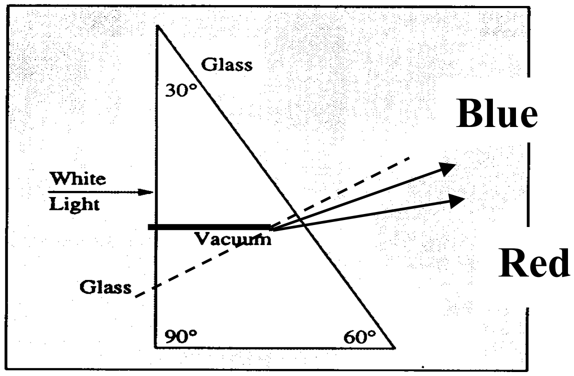
$$v = \lambda f \qquad f = \frac{c}{\lambda} \quad = \frac{3\,x\,10^8\,m/s}{700\,x\,10^{-9}\,m} \quad = \boxed{4.3\,x\,10^{14}\,Hz}$$



d. On the figure above, sketch the approximate paths of both the red and the blue rays as they pass through the glass and back out into the vacuum. Ignore any reflected light. It is not necessary to calculate any angles, but do clearly show the change in direction of the rays, if any, at each surface and be sure to distinguish carefully any differences between the paths of the red and the blue beams.

The red light has a smaller index of refraction, so it is bent less. Has a smaller angle of refraction. The blue light has a larger angle of refraction.

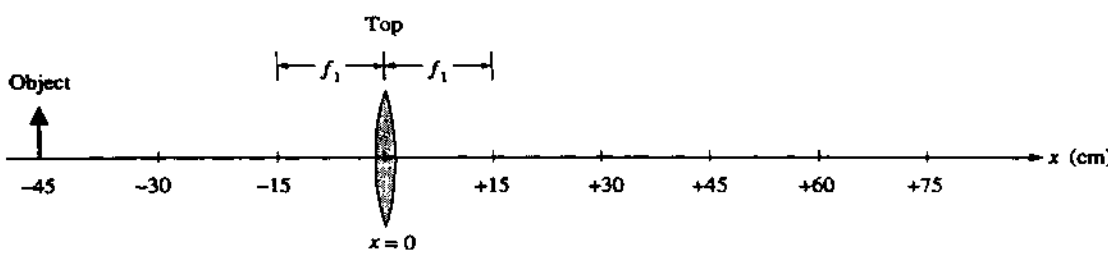
e. The figure below represents a wedgeshaped hollow space in a large piece of the type of glass described above. On this figure, sketch the approximate path of the red and the blue rays as they pass through the hollow prism and back into the glass. Again, ignore any reflected light, clearly show changes in direction, if any, where refraction



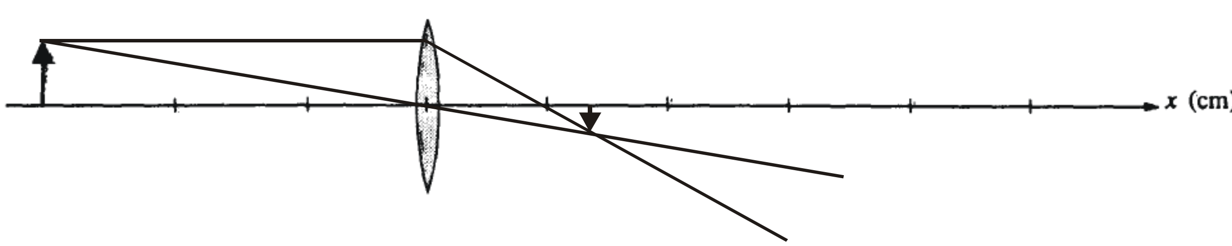
occurs, and carefully distinguish any differences in the two paths.

From 1992:

6. A thin double convex lens of focal length $f_1 = +15$ centimeters is located at the origin of the $\boldsymbol{x}$-axis, as shown. An object of height 8 centimeters is placed 45 centimeters to the left of the lens.



a. On the figure below, draw a ray diagram to show the formation of the image by the lens. Clearly show principal rays.



b. Calculate (do not measure) each of the following.

i. The position of the image formed by the lens.

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \qquad \frac{1}{s_o}=\frac{1}{f}-\frac{1}{s_i} \qquad \frac{1}{s_i}=\frac{1}{15\ cm}-\frac{1}{45\ cm} \qquad s_i=\boxed{22.5\ cm}$$

ii. The size of the image formed by the lens.

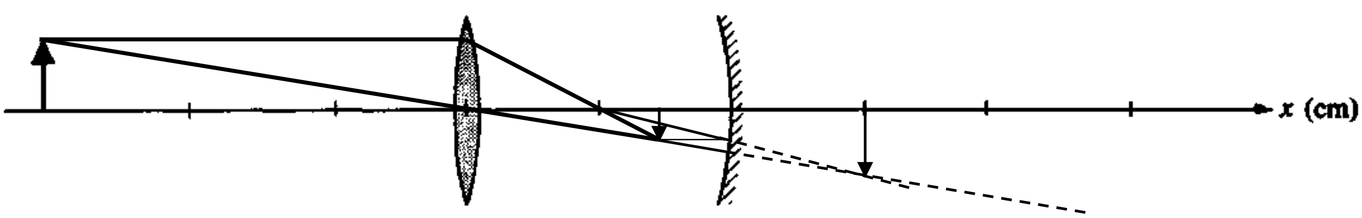
$$\frac{h_i}{h_o}=-\frac{s_i}{s_o} \qquad h_i=-h_o\frac{s_i}{s_o} \qquad =-(8\ cm)\frac{22.5\cancel{cm}}{45\cancel{cm}} \qquad \boxed{h_i=4\ cm}$$

c. Describe briefly what would happen to the image formed by the lens if the top half of the lens were blocked so that no light could pass through.

Every part of the lens forms the image, so the complete image would still be formed, with half the lens covered, less light is refracted, so the image would be dimmer. About half as bright.

A concave mirror with focal length $\boldsymbol{f_2}$ = + 15 centimeters is placed at $\boldsymbol{x}$ = + 30 centimeters.

d. On the figure below, indicate the position of the image formed by the lens, and draw a ray diagram to show the formation of the image by the mirror. Clearly show principal rays.



The Road Not Taken

TWO roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.