Math 454 Partner Quiz Name: *Moi*
Palekar ***Related Rates*** $\frac{}{36}$ Date: ______
Q2 Dec 2016-7

Show all work to receive credit.

1) A point is moving on the graph of $\frac{d}{dt}\left[5x^3 + 6y^3 = xy\right]$. When the point is at $\left(\frac{1}{11}, \frac{1}{11}\right)$ (x, y), its y-coordinate is increasing at a speed of 5 units per second. What is the speed of the x-coordinate at that time and in which direction is the x-coordinate moving?

$\frac{dy}{dt} = 5$ un/sec

$\left.\frac{dx}{dt}\right|_{\left(\frac{1}{11}, \frac{1}{11}\right)} = ?$

$15x^2 \frac{dx}{dt} + 18y^2 \frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$ [4] deriv.

$\frac{15}{121}\frac{dx}{dt} + \frac{18}{121}(5) = \frac{1}{11}(5) + \frac{1}{11}\frac{dx}{dt}$ [1] plug in

$\left(\frac{15}{121} - \frac{1 \cdot 11}{11 \cdot 11}\right)\frac{dx}{dt} = \frac{-90}{121} + \frac{5 \cdot 11}{11 \cdot 11}$

$\frac{4}{121}\frac{dx}{dt} = \frac{-35}{121}$

$\frac{dx}{dt} = \frac{-35}{121}\left(\frac{121}{4}\right) = \frac{-35}{4}$ un/sec. [2] [1] moving left [1]

-8.75 un/sec

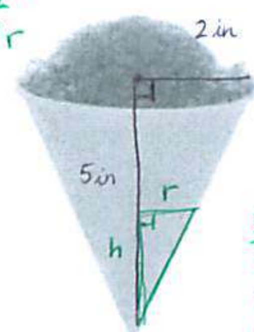
2) A cherry flavored raspa sno-cone is leaking from its paper cone at a rate of 2 cubic inches per minute. The paper cone's top radius is 2 inches and is 5 inches tall. When the depth of melted cherry raspa mixture is 3 inches,

when $h = 3in$

$h = \frac{5}{2}r$

$3 = \frac{5}{2}r$

$\frac{6}{5} = r$



(1) prop

$\frac{5}{2} = \frac{h}{r}$

$2h = 5r$

$h = \frac{5}{2}r$

a) How fast is the radius of the raspa changing? $\left.\frac{dr}{dt}\right|_{h=3in} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2\left(\frac{5}{2}r\right)$ [1] subst

$\frac{d}{dt}\left[V = \frac{5}{6}\pi r^3\right]$

$\frac{dV}{dt} = \frac{5\pi}{6}\left[3r^2\frac{dr}{dt}\right] = \frac{5\pi}{2}r^2\frac{dr}{dt}$

$-2 = \frac{5\pi}{2}\left(\frac{6}{5}\right)^2\frac{dr}{dt}$ [2] deriv [1] radius

$-2 = \frac{18\pi}{5}\frac{dr}{dt}$

$\boxed{\frac{-5}{9\pi}\ \text{in/min}} = \frac{-10}{18\pi} = \left(\frac{5}{18\pi}\right)(-2) = \frac{dr}{dt}$

[1] The radius of the raspa is decreasing at a rate of 0.18 in per min [1]

b) How fast is the raspabertty melt leaking onto Gooby's clothes? $\frac{dC}{dt} = 2\ \frac{in^3}{min}$ [1]

$\frac{dV}{dt} = -2\ \frac{in^3}{min}$

c) If Gooby has a small cylindrical cup with a 2 inch diameter beneath the leaking raspa, at this moment, how fast is the height of the raspa juice in the "catch cup" changing?

$\left.\frac{dh}{dt}\right|_{r=1} = ?$

$dV_{cylin} = 2\ \frac{in^3}{min}$

$V = \pi r^2 h$

$\frac{d}{dt}\left[V = \pi h\right]$ [1] radius constant

$\frac{dV}{dt} = \pi\frac{dh}{dt}$ [1] deriv

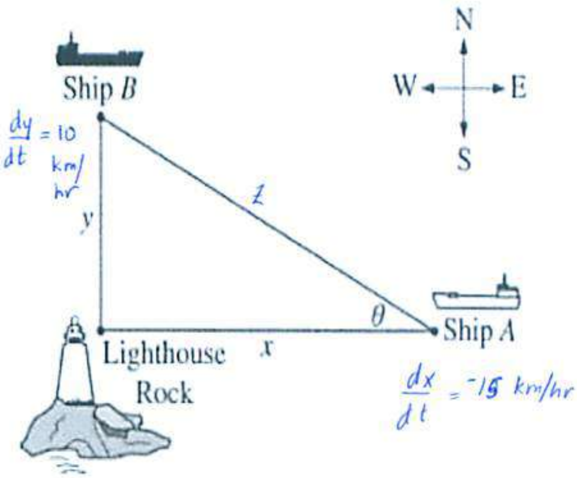
$2 = \pi\frac{dh}{dt}$

$\frac{2}{\pi} = \frac{dh}{dt}$ [1]

The height of the raspa juice in the cylindrical catch cup is increasing at 0.64 inches per minute. [1]

20

3) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$ [1] equ

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$ [1] deriv

a) $\boxed{3^2 + 4^2 = z^2 \quad 5 = z}$ [1]

$4(-15) + 3(10) = (5)\frac{dz}{dt}$

[1]

$-30 = 5\frac{dz}{dt}$

$-6 = \frac{dz}{dt}$ [1]
km/hr

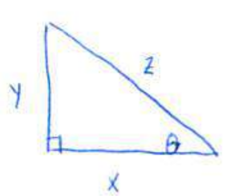
The distance btw the ships is decreasing at 6 km/hr.
[1]

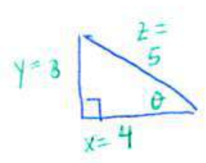
(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

$\left.\frac{dz}{dt}\right|_{\substack{x=4\text{ km}\\ y=3\text{ km}}} = ?$

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

$d\theta$ $\left.\frac{d\theta}{dt}\right|_{\substack{x=4\text{ km}\\ y=3\text{ km}}} = ?$





$\frac{d}{dt}\left[\tan\theta = \frac{y}{x}\right]$ [2] trig equ.

$\sec^2\theta\,\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$ [4] deriv quot rule

$\left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = \frac{(4)(10) - (3)(-15)}{16}$ plug in [1]

$\frac{d\theta}{dt} = \frac{85}{16}\left(\frac{16}{25}\right)$

$\frac{d\theta}{dt} = \frac{85}{25} = \frac{17}{5}$ [1]

[1]
The angle is changing at a rate of 3.4 radians per hour.

15

Math 454 | Partner Quiz | Name: ____________
Palekar | ***Related Rates*** | Date: ____________
Q2 | | Dec 2016-7

Show all work to receive credit.

1) A point is moving on the graph of $5x^3 + 6y^3 = xy$. When the point is at $\left(\frac{1}{11}, \frac{1}{11}\right)$, its y-coordinate is increasing at a speed of 5 units per second. What is the speed of the x-coordinate at that time and in which direction is the x-coordinate moving?

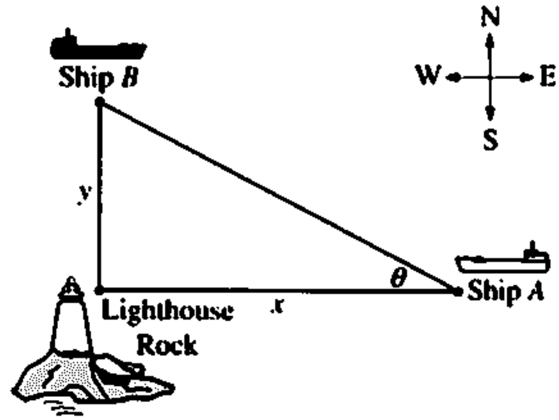
2) A cherry flavored raspa sno-cone is leaking from its paper cone at a rate of 2 cubic inches per minute. The paper cone's top radius is 2 inches and is 5 inches tall. When the depth of melted cherry raspa mixture is 3 inches,

a) How fast is the radius of the raspa changing?

b) How fast is the raspabertty melt leaking onto Gooby's clothes? ______

c) If Gooby has a small cylindrical cup with a 2 inch diameter beneath the leaking raspa, at this moment, how fast is the height of the raspa juice in the "catch cup" changing?

3) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.