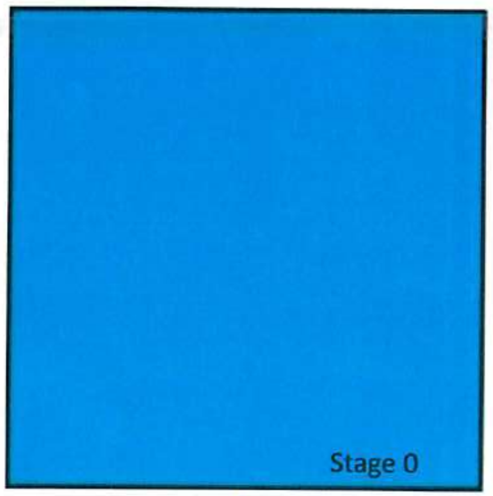
Math 454i
Palekar

H#
Fractals: Perimeter & Area

Name: ____________
Date: ____________

1. To generate the Sierpinski Carpet: Take a square with area A. Divide it into 9 equal-sized squares. Remove the middle one. Then take the remaining 8 squares. Divide each one into 9 equal squares. Remove the middle one from each group of 9.



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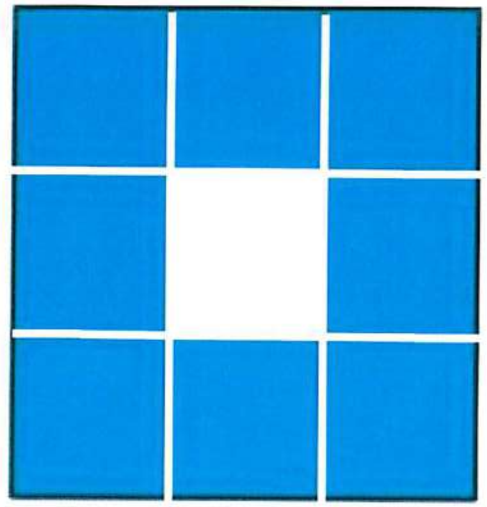
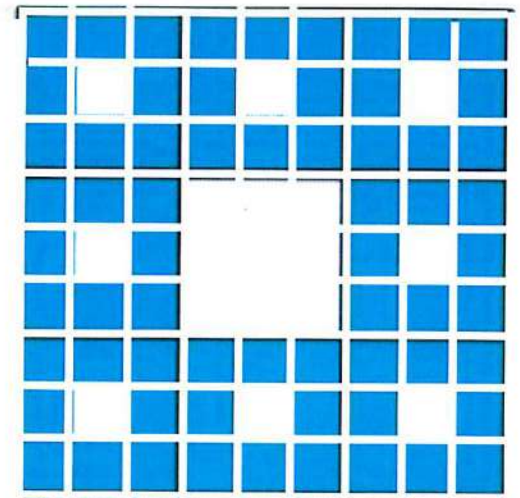


$1/3$



$\frac{1}{9}$

Stage	0	1	2	3	4	5	. . .	n
Number of shaded □s	1	8	64	512	4096	32,768		8^n
Length of one side	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$		$\left(\frac{1}{3}\right)^n$
Area of one □ out of whole	1	$\frac{1}{9}$	$\frac{1}{81}$	$\frac{1}{729}$	$\frac{1}{6561}$	$\frac{1}{59{,}049}$		$\left(\frac{1}{9}\right)^n$
Total Perimeter [total distance around all filled-in regions]	4	$\frac{32}{3} \approx$ 10.667	$\frac{256}{9} \approx$ 28.444	$\frac{2048}{27} \approx$ 75.852	$\frac{16{,}384}{81} \approx$ 202.272	$\frac{131{,}072}{243} \approx$ 539.391		$4\left(\frac{8}{3}\right)^n$
Total Shaded Area (fraction, then round to 3 dec places)	1	$\frac{8}{9} \approx$ 0.889	$\frac{64}{81} \approx$ 0.790	$\frac{512}{729} \approx$ 0.702	$\frac{4096}{6561} \approx$ 0.624	$\frac{32{,}768}{59{,}049} \approx$ 0.555		$\left(\frac{8}{9}\right)^n$

2. Consider $y = 3.5x(1-x)$. Take a value of $x = 0.2$ and follow the procedure discussed in class. Be sure to round off each calculation to ***three decimal places*** before doing the next calculation. Fill-in table below. The possible attractor(s) are 0.827, 0.501, 0.875, 0.383

Ti-89
$0.2 \rightarrow x$
$3.5x*(1-x) \rightarrow x$

0.2	0.85	0.846	0.84	0.833	0.828	0.827	0.827
0.56	0.446	0.456	0.47	0.486	0.497	0.501	0.501
0.862	0.865	0.868	0.872	0.874	0.875	0.875	0.875
0.415	0.409	0.4	0.391	0.385	0.383	0.383	0.383 ...

Multiple Choice. Show ALL work to receive credit. Formulas are listed below.

D 3.

Which of the following is the general term $T(n)$ of the sequence 1, 4, 27, 256, … ?

$1^1, 2^2, 3^3, 4^4, \ldots$

arith, geo? neither

A. n

B. n^2

C. 2^{2n-1}

D. n^n

B 4.

If the common difference of an arithmetic sequence is 4 and $T(6) = 15$, find the first term of the sequence.

A. −9

B. −5

C. $\frac{11}{5}$

D. 35

—, —, —, —, —, 15 (4)

$a_n = a_1 + (n-1)d$

$15 = a_1 + (6-1)$

$15 = a_1 + 20$

$-5 = a_1$

A 5.

Which of the following is not an arithmetic sequence?

A. $11, 2, -8, -19, \ldots$ (-9, -10, -11) not a common difference

B. 4, 7, 10, 13, …

C. 57, 51, 45, 39, …

D. −3, −5, −7, −9, …

A 6.

Which of the following are in geometric sequence?

I. $\frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}, \ldots$ $\quad \frac{1}{x}, \frac{2}{x^2}, \frac{4}{x^3}$ $\quad \frac{2}{x} \rightarrow \frac{2}{x}$

II. $x, x^2, x^3, \ldots$ $\quad r = x$

III. $\log x, \log x^2, \log x^3, \ldots$ $\quad \log x, 2\log x, 3\log x, \ldots$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Arithmetic Sequence	***Geometric Sequence***
$a_n = a + (n-1)d \qquad Sum = \frac{n}{2}(a_1 + a_n)$	$a_n = a \bullet r^{n-1} \qquad S_n = \frac{a_1(1-r^n)}{(1-r)},\ r \neq 1$ $S_\infty = \frac{a_1}{1-r},\ r \neq 1$

$$S_n = \frac{a_1(1-r^n)}{1-r},\ r \neq 1$$

D 7.

If x^2, $4x + 3$, 25 are in geometric sequence, find the values of x.

$x^2,\ 4x+3,\ 25$

$$\frac{x^2}{4x+3} = \frac{4x+3}{25}$$

$$25x^2 = 16x^2 + 24x + 9$$

$$9x^2 - 24x - 9 = 0$$

$$3(3x^2 - 8x - 3) = 0$$

$$3(3x+1)(x-3) = 0$$

$3x + 1 = 0$ or $x - 3 = 0$

$3x = -1$ $\quad$ $x = 3$

$x = -1/3$

A. $-\frac{1}{3}$

B. $\frac{1}{3}$ or -3

C. -3

D. $-\frac{1}{3}$ or 3

C 8.

Find the sum of the first 7 terms of the geometric series $0.25 + 0.75 + 2.25 + \ldots$

$\frac{1}{4}, \frac{3}{4},$ $r = 3$

A. 91

B. 182.25

C. 273.25

D. 546.25

$$S_7 = \frac{0.25(1-3^7)}{1-3}$$

$$= \frac{0.25}{-2}(1-3^7)$$

$$= 273.25$$

C 9.

The sum to infinity of a geometric series is 60. If its first term is 80, find the common ratio.

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. $-\frac{1}{2}$

$S_\infty = 60$

$80, \_, \_ \ldots$

$r = ?$

$$S_\infty = \frac{a_1}{1-r},\ r \neq 1$$

$$S_\infty = \frac{80}{1-r}$$

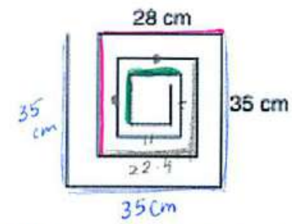
$$60 = \frac{80}{1-r}$$

$$60 - 60r = 80$$

$$-60r = 20$$

$$r = -\frac{1}{3}$$

D 10.



The figure shows the pattern on a birthday cake. The pattern has an outermost square of three sides with a length of 35 cm. The length of the side of each subsequent smaller square is 80% of that of the previous one. What is the maximum length of the pattern on the cake?

A. 175 cm

B. 210 cm

C. 350 cm

D. 385 cm

3(35)
2(28)
2(22.4)
2(17.92)
2(14.336)
2(11.4688)
2(9.17504

311.6

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{35}{1-.80}$$

$$= \frac{35}{.20}$$

$$= 175 \times 2 = 350$$

$+35$

385