| Scientific Calculator Only |

Name_____(1 pt.)

I. Answer each question in this section. All answers should include supporting evidence. Think carefully and good luck!

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1. Consider the parabola defined by $x = -\dfrac{1}{4}y^2 + 2y - 1$.

(a) Find the coordinates of the focus of this parabola. (6 pts.)

(b) What is an equation of the directrix of this parabola? (2 pts.)

(c) What is the length of the focal chord (latus rectum) of this parabola? (2 pts.)

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2. Find the cube roots of $-\sqrt{3} - i$ in $\mathbb{C}$. Your final answers must be in $rcis\theta$ form. (6 pts.)

3. Solve: $\cos\theta < \cos 2\theta$ for $\theta \in [0, 2\pi)$. (6 pts.)

4. (a) Solve $|x+4| \leq |x-6|$. Express your answer using interval notation. (3 pts.)

(b) Express in simplest form:

$$2\cos^3 \theta + \frac{2\sin^2 \theta}{\sec \theta} \quad \text{(4 pts.)}$$

5. If $\dfrac{6}{5}$ is a root of $10x^5 + kx^4 + 3x^2 + b = 0$ where all the coefficients are integers and $k > 0$, find two possible values of b. Justify your conclusion. (5 pts.)

6. If $3 + 2i$ is a root of $2x^5 - 7x^4 - 7x^3 + 83x^2 - 39x = 0$.

State the x-intercepts of the graph of: $y = 2x^5 - 7x^4 - 7x^3 + 83x^2 - 39x$. (8 pts.)

7. Using a complex number approach, find the image of $(5,-2)$ after a rotation of $150°$. Your answer should be expressed as an ordered pair using simplest radical form as necessary. (5 pts.)

8. Find an equation of the parabola whose focus is the center of the circle $x^2 + y^2 - 10x + 1 = 0$ and whose directrix is $y = -4$. (6 pts.)

9. Suppose $-\sqrt{2} - i\sqrt{2}$ and $2i$ are roots are both nth roots of some complex number A. (9 pts.)

(a) Can $2 - i$ be a root? Explain.

(b) Can $\sqrt{3} + i$ be a root? Explain.

(c) Can $n = 8$? Explain.

10. If $z_1 = -\dfrac{\sqrt{2}}{2} + i\dfrac{\sqrt{2}}{2}$ and $\dfrac{(6cis125°)(3cism°)}{18cis160°} = \overline{z_1}$, find the value m, such that $m \in [0°, 360°)$. (6 pts.)

II. Multiple Choice. (2 pts. each)

_____ 1. The focus of the parabola $y = -\dfrac{1}{2}x^2$ is

(a) $\left(0, \dfrac{1}{2}\right)$

(b) $\left(0, -\dfrac{1}{2}\right)$

(c) $\left(0, \dfrac{1}{8}\right)$

(d) $\left(0, -\dfrac{1}{8}\right)$

(e) $(0,0)$

_____ 2. If $z_1 \in \mathbb{C}$, $Arg\left(-\dfrac{|z_1|}{8}\right) =$

(a) $0°$

(b) $90°$

(c) $180°$

(d) $270°$

(e) undefined

_____ 3. $\csc(2Arc\tan x) =$

(a) $\dfrac{1+x^2}{2x}$

(b) $\dfrac{\sqrt{1+x^2}}{2x}$

(c) $\dfrac{2x}{1+x^2}$

(d) $\dfrac{1}{1+x^2}$

(e) $\sqrt{1+x^2}$