Math 444HH
HW #53

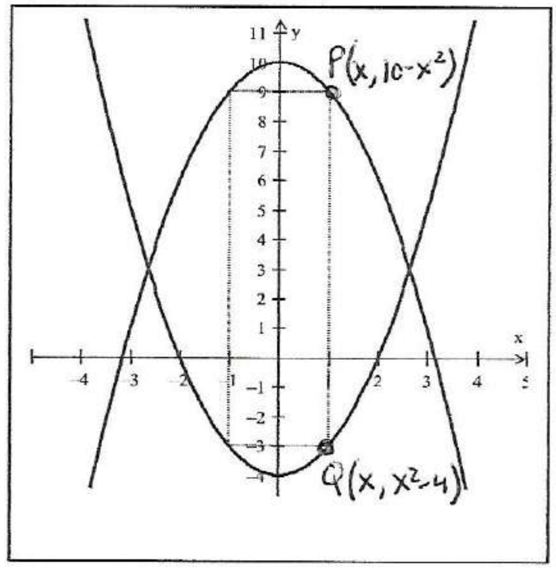
1) Using techniques of calculus (not your TI89), find the intervals of increase or decrease for the graph of $f(x)=x^3-12x+2$. State the coordinates of any relative extrema (rel max or rel min)

2) For $f(x)=\dfrac{1}{\sqrt{x}}$, find

(a) $f'(x)$ using two different definitions of the derivative.

(b) the equation of the line tangent to the curve at the point where $x=\dfrac{1}{9}$.

3) Find the dimensions that give the largest area for a rectangle that has two vertices on the graph of $y=10-x^2$ and two vertices on the graph of $y=x^2-4$. See the figure at the right. (Use your TI89, not techniques of calculus.)



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1) $f(x)=x^3-12x+2$

$f'(x)=3x^2-12$

$3x^2-12=0$

$x^2-4=0$

$x=\pm 2$

$3(x+2)$	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
f':	$+$	-2	$-$	2	$+$

f is INC on $(-\infty,-2]\cup[2,\infty)$ b/c $f'>0$. f is DEC on $[-2,2]$ b/c $f'<0$.

REL MAX: $(-2, 18)$ b/c f' changes from $(+)$ to $(-)$ at $x=-2$.

REL MIN: $(2,-14)$ b/c f' changes from $(-)$ to $(+)$ at $x=2$.

2) $f(x)=\dfrac{1}{\sqrt{x}}$

a) $$f'(x)=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}\cdot\frac{\frac{\sqrt{x}(\sqrt{x+h})}{1}}{\frac{\sqrt{x}\sqrt{x+h}}{1}}=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$=\lim_{h\to 0}\frac{x-x-h}{(h\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}=\lim_{h\to 0}\frac{\cancel{-h}\ \ -1}{(\cancel{h}\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}=\boxed{\frac{-1}{2x\sqrt{x}}}$$

a) $f'(x) = \lim_{h \to 0} \dfrac{\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x-h}}}{2h} \cdot \dfrac{\sqrt{x+h}\,\sqrt{x-h}}{\dfrac{1}{\sqrt{x+h}\,\sqrt{x-h}}} = \lim_{h \to 0} \dfrac{\sqrt{x-h} - \sqrt{x+h}}{2h\sqrt{x+h}\,\sqrt{x-h}}$

$= \lim_{h \to 0} \dfrac{\sqrt{x-h} - \sqrt{x+h}}{2h\sqrt{x+h}\,\sqrt{x-h}} \cdot \dfrac{\sqrt{x-h} + \sqrt{x+h}}{\sqrt{x-h} + \sqrt{x+h}} = \lim_{h \to 0} \dfrac{\cancel{x}-h-\cancel{x}-h}{2h\sqrt{x+h}\sqrt{x-h}\,(\sqrt{x-h}+\sqrt{x+h})}$

$= \lim_{h \to 0} \dfrac{\cancel{-2h}\ \ -1}{\cancel{2h}\sqrt{x+h}\sqrt{x-h}\,(\sqrt{x-h}+\sqrt{x+h})} = \boxed{\dfrac{-1}{2x\sqrt{x}}}$

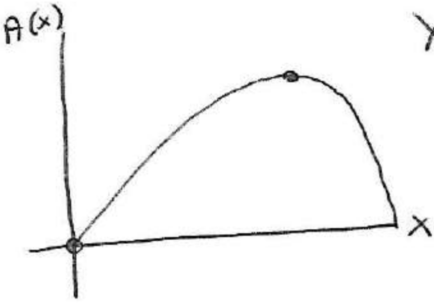
b) $f\left(\frac{1}{9}\right) = \dfrac{1}{\sqrt{\frac{1}{9}}} = 3 \qquad f'\left(\frac{1}{9}\right) = \dfrac{-1}{2\left(\frac{1}{9}\right)\sqrt{\frac{1}{9}}} = \dfrac{-1}{\frac{2}{27}} = \dfrac{-27}{2}$

$$\boxed{y - 3 = \frac{-27}{2}\left(x - \frac{1}{9}\right)}$$

3) $A = 2x\left(10 - x^2 - (x^2 - 4)\right)$

$A = 2x(-2x^2 + 14)$

$A = -4x^3 + 28x$



$x \approx 1.528$

$\boxed{3.056 \text{ by } 9.330}$

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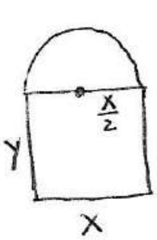


$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$A = xy + \frac{x^2}{8}\pi$

$P = 30$ ft.

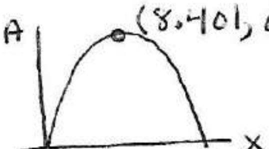
$\therefore\ x + 2y + \frac{1}{2}\left(2\pi \cdot \frac{x}{2}\right) = 30$

$\therefore\ y = 15 - \frac{x}{4}(2+\pi)$

$\therefore\ A = x\left(15 - \frac{x}{4}(2+\pi)\right) + \frac{\pi}{8}x^2$

$\boxed{A = -x^2\left(\frac{4+\pi}{8}\right) + 15x}$

b)



$\boxed{x \approx 8.401 \quad y \approx 4.201}$

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$\boxed{A = 100\,m^2}$ x
y

a) $L = 2x + 2y$

$100 = xy \rightarrow y = \frac{100}{x}$

$\therefore\ L = 2x + \frac{200}{x} = \boxed{\dfrac{2x^2 + 200}{x}}$

b)

$\boxed{\begin{array}{l} x \approx 10.000 \\ y \approx 10.000 \end{array}}$

XMIN = 0 YMIN = 0
XMAX = 25 YMAX = 100