Given as Critique Final 2015

FORMULA SHEET – THIS SHEET MAY BE REMOVED FROM TEST

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of Triangle

$K = \frac{1}{2}ab\sin C$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Standard Deviation

$$\sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}}$$

Z-Score

$z = \frac{x - \overline{x}}{s}$,

S=standard deviation

X=mean

X=Data Point

Arithmetic

$t_n = t_1 + d(n-1)$ $\qquad S_n = \frac{n(a_1 + a_n)}{2}$

Geometric

$t_n = t_1(r^{n-1})$ $\qquad S_n = \frac{a_1(1 - r^n)}{1 - r}$ $\qquad S_\infty = \frac{a_1}{1 - r}$

FORMULA SHEET – THIS SHEET MAY BE REMOVED FROM TEST

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of Triangle

$K = \frac{1}{2}ab\sin C$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Standard Deviation

$$\sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}}$$

Z-Score

$z = \frac{x - \overline{x}}{s},$

S=standard deviation

X=mean

X=Data Point

Arithmetic

$t_n = t_1 + d(n-1)$ $\qquad$ $S_n = \frac{n(a_1 + a_n)}{2}$

Geometric

$t_n = t_1(r^{n-1})$ $\qquad$ $S_n = \frac{a_1(1-r^n)}{1-r}$ $\qquad$ $S_\infty = \frac{a_1}{1-r}$

A scientific calculator may be used for this test. Graphing calculators are not permitted.

Part 1: Answer **all** of the following multiple choice questions. Circle the correct answer. [2 points each]

1. For all values of x for which the expression is defined, $\sec x \cdot \csc x \cdot \cos x$ is equivalent to a. $\tan x$ b. $\sin x$ c. $\dfrac{1}{\sin x}$ d. $\dfrac{1}{\cos x}$	2. For five days in January, Buffalo, NY, recorded the following daily high temperatures: $5^\circ, 7^\circ, 6^\circ, 5^\circ$ and 7° Which statement is true for this group of data? a. mean = median b. mean = mode c. median = mode d. mean < median
3. Jeremy hikes 7 miles due east and then 3 miles due north. How far, to the *nearest tenth* of a mile, is he from his starting point? a. 7.7 b. 7.6 c. 6.3 d. 6.4	4. Given the function $y = 3\cos(2x)$. What is the minimum value y can equal? a. π b. 2 c. 3 4. -3
5. Which of the following sequences is a geometric sequence? a. 1, 1, 2, 3, 5, 8, … b. 8, 6, 4, 2, … c. 1, 4, 9, 16, … d. 2, 6, 18, 54, …	6. Which of the following is the recursive formula for the sequence given by 2, 5, 26, … a. $a_1 = 2$, $a_n = (n-1)^2 + 1$ b. $a_1 = 2$, $a_n = (a_{n-1})^2 + 1$ c. $a_1 = 2$, $a_n = n^2$ d. $a_1 = 2$, $a_n = n^2 + 1$

A scientific calculator may be used for this test. Graphing calculators are not permitted.

Part 1: Answer **all** of the following multiple choice questions. Circle the correct answer. [2 points each]

1. For all values of x for which the expression is defined, $\sec x \cdot \csc x \cdot \cos x$ is equivalent to

   a. $\tan x$ b. $\sin x$

   c. $\frac{1}{\sin x}$ d. $\frac{1}{\cos x}$

   $\frac{1}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x$

2. For five days in January, Buffalo, NY, recorded the following daily high temperatures:

   $5°, 7°, 6°, 5°$ and $7°$

   $5, 5, \boxed{6}, 7, 7$

   Which statement is true for this group of data?

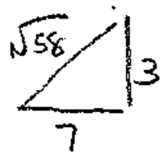
   $\bar{x} = \frac{5+7+6+5+7}{5} = 6$

   a. mean = median b. mean = mode

   c. median = mode d. mean < median

3. Jeremy hikes 7 miles due east and then 3 miles due north. How far, to the *nearest tenth* of a mile, is he from his starting point?

   a. 7.7 b. 7.6

   c. 6.3 d. 6.4

   $\sqrt{58} = 7.616 \sim 7.6$

4. Given the function $y = 3\cos(2x)$. What is the minimum value y can equal?

   $y = 3\cos(2x)$

   a. π b. 2

   max = 3

   c. 3 d. -3

   min = -3

5. Which of the following sequences is a geometric sequence?

   a. 1, 1, 2, 3, 5, 8, … b. 8, 6, 4, 2, …

   $\frac{1}{1} \neq \frac{2}{1}$ $\frac{6}{8} \neq \frac{4}{6}$

   c. 1, 4, 9, 16, … d. 2, 6, 18, 54, …

   $\frac{4}{1} \neq \frac{9}{4}$ $\frac{6}{2} = \frac{18}{6}$

6. Which of the following is the recursive formula for the sequence given by 2, 5, 26, …

   a. $a_1 = 2$, $a_n = (n-1)^2 + 1$

   b. $a_1 = 2$, $a_n = (a_{n-1})^2 + 1$

   c. $a_1 = 2$, $a_n = n^2$

   d. $a_1 = 2$, $a_n = n^2 + 1$

Part 2: OMIT ONE QUESTION. Clearly indicate which question you are omitting by writing "OMIT" in the space provided for that question. Show all of your work in order to receive full credit. [4 points each]

7. Find the first four terms of the sequence given by $a_n = n^2 - 4n$

8. Evaluate the following sum.

$$\sum_{k=1}^{4}(-1)^{n+1}\cdot(2k-1)$$

9. In triangle ABC, $m\angle A = 13°$, $m\angle B = 9°$, $c = 55\,\text{cm}$. Find b. Round to the nearest whole number.

10. Find all solutions in the interval $0° \le \theta < 360°$. Round your answers to the nearest degree.

$$\cos^2\theta + 2\cos\theta - 3 = 0$$

Part 2: OMIT ONE QUESTION. Clearly indicate which question you are omitting by writing "OMIT" in the space provided for that question. Show all of your work in order to receive full credit. [4 points each]

7. Find the first four terms of the sequence given by $a_n = n^2 - 4n$

$a_1 = 1 - 4(1)$ | $a_1 = -3$

$a_2 = 2^2 - 4(2)$ | $a_2 = -4$

$a_3 = 3^2 - 4(3)$ | $a_3 = -3$

$a_4 = 4^2 - 4(4)$ | $a_4 = 0$

8. Evaluate the following sum.

$$\sum_{k=1}^{4}(-1)^{n+1}\cdot(2k-1)$$

$k=1: (-1)^{1+1} \cdot [2(1)-1] = 1$

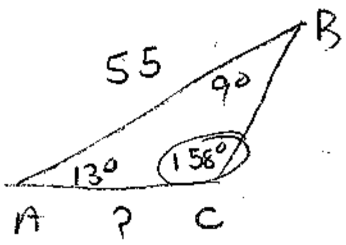
$k=2: (-1)^{2+1} \cdot [2(2)-1] = -3$

$k=3: (-1)^{3+1} \; [2(3)-1] = 5$

$k=4 \; (-1)^{4+1} \; [2(4)-1] = -7$

$$\sum_{k=1}^{4}(-1)^{k+1}(2k-1) = \boxed{-4}$$

9. In triangle ABC, $m\angle A = 13°$, $m\angle B = 9°$, $c = 55$ cm. Find b. Round to the nearest whole number.



$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 13°} = \frac{55}{\sin 158°}$$

$b \sin 158° = 55 \cdot \sin 13°$

$$b = \frac{55 \cdot \sin 13°}{\sin 158°}$$

$b = 33.027$

$\boxed{b = 33}$

10. Find all solutions in the interval $0° \le \theta < 360°$. Round your answers to the nearest degree.

$\cos^2\theta + 2\cos\theta - 3 = 0$

$(\cos\theta + 3)(\cos\theta - 1) = 0$

$\cos\theta + 3 = 0 \;\; \vee \;\; \cos\theta - 1 = 0$

$\cos\theta = -3 \;\; \vee \;\; \cos\theta = 1$

reject

$\boxed{\theta = 0°}$

11. Emily has \$500 in her bank account. She decided that she will donate \$8 to charity each week. Write an equation representing the number of dollars, *n*, that she will have in her bank account after *w* weeks

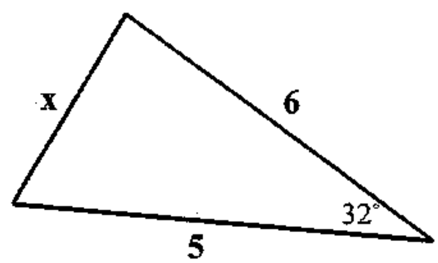
12. Find the first term and the common difference of an arithmetic sequence if $a_{21} = -14$ and $a_{51} = 226$.

First term: ____________________

Common difference: ___________

13.A ladder 6 feet long leans against a wall and makes and angle of 71 degrees with the ground. Find to the nearest tenth of a foot, how high up the wall the ladder will reach.

14.Find the value of x in the triangle below. Round your answer to the nearest hundredth.



11. Emily has \$500 in her bank account. She decided that she will donate \$8 to charity each week. Write an equation representing the number of dollars, *n*, that she will have in her bank account after *w* weeks

$n(0) = 500$

$n(1) = 500 - 8 \cdot 1$

$n(2) = 500 - 8 \cdot 2$

$n(3) = 500 - 8 \cdot 3$

$\vdots$

$\boxed{n(w) = 500 - 8w}$

12. Find the first term and the common difference of an arithmetic sequence if $a_{21} = -14$ and $a_{51} = 226$.

First term: -174

Common difference: 8

$\underset{a_1}{?}\ \underset{a_2}{\_\_}\ \cdots \left[\underset{\substack{a_{21}\\ *a_1}}{-14}\ \underset{a_{22}}{\_\_}\ \cdots\ \underset{\substack{a_{51}\\ *a_{31}}}{226}\right]$

$a_{31} = a_1 + (n-1)d$

$226 = -14 + (31-1)d$

$240 = 30d$

$\boxed{d = 8}$

$a_{21} = a_1 + (n-1)d$

$-14 = a_1 + (21-1)d$

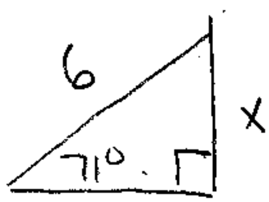
$-14 = a_1 + 20 \cdot 8$

$-14 = a_1 + 160$

$\boxed{a_1 = -174}$

13. A ladder 6 feet long leans against a wall and makes and angle of 71 degrees with the ground. Find to the nearest tenth of a foot, how high up the wall the ladder will reach.

Method I:



$\frac{\sin 71^\circ}{1} = \frac{x}{6}$

$x = 6 \cdot \sin 71^\circ$

$x = 5.673$ $\boxed{x = 5.7}$

Method II:

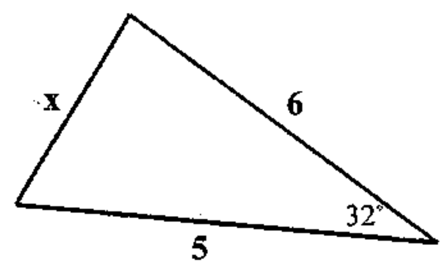
$\frac{6}{\sin 90^\circ} = \frac{x}{\sin 71^\circ}$

$x \cdot \sin 90^\circ = 6 \cdot \sin 71^\circ$ $(\sin 90^\circ = 1)$

$x = 6 \cdot \sin 71^\circ$

$\boxed{x = 5.7}$

14. Find the value of *x* in the triangle below. Round your answer to the nearest hundredth.



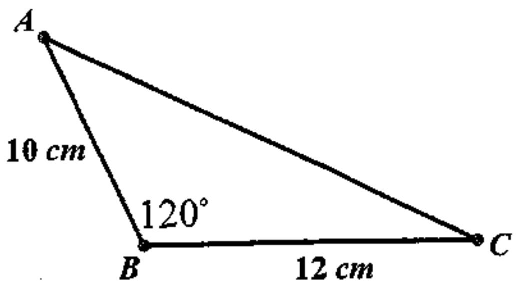
$x^2 = 5^2 + 6^2 - 2(5)(6)\cos 32^\circ$

$x^2 = 10.117$

$x = 3.1807$

$\boxed{x = 3.18}$

15. Find the exact area of triangle ABC.



Area = ______________________

16. Scores on a recent quiz were:

7, 8,10, 10,12, 15, 22

The standard deviation of these scores is 5.1316.

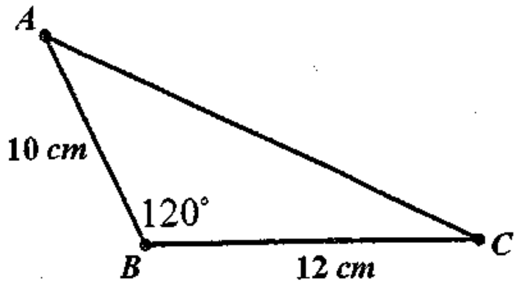
a. Calculate the z – score of 15.

b. Determine the range of the scores.

17. Two sides of a triangular plot measure 30 feet and 18 feet respectively. If the angle opposite the 30-foot side measures $58°$, find to the nearest degree, the measure of the angle opposite the 18-foot side.

18. A hang glider at the edge of a 175 foot cliff determines the angle of depression out to a distant farmhouse to be $28°$. To the nearest foot, what is the distance of the hang glider to the farmhouse? Draw a diagram to receive full credit.

15. Find the exact area of triangle ABC.



$K = \frac{1}{2} ac \sin B$

$K = \frac{1}{2}(12)(10)\sin 120^\circ$

$K = 60\left(\frac{\sqrt{3}}{2}\right)$

$K = 30\sqrt{3}$

Area = $30\sqrt{3}$

16. Scores on a recent quiz were:

7, 8, 10, 10, 12, 15, 22

The standard deviation of these scores is 5.1316.

$\bar{x} = \frac{7+8+10+10+12+15+22}{7}$

$\bar{x} = \frac{84}{7}$ $\boxed{\bar{x} = 12}$

a. Calculate the z – score of 15.

$z = \frac{x - \bar{x}}{\sigma} = \frac{15-12}{5.1316}$

$\boxed{z = 0.58}$

b. Determine the range of the scores.

range = high − low

range = 22 − 7

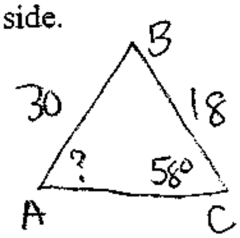
$\boxed{\text{range} = 15}$

17. Two sides of a triangular plot measure 30 feet and 18 feet respectively. If the angle opposite the 30-foot side measures 58°, find to the nearest degree, the measure of the angle opposite the 18-foot side.

since $a < c$

$m\angle A < m\angle C$

So, $\angle A$ is acute



$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{18}{\sin A} = \frac{30}{\sin 58^\circ}$

$30 \sin A = 18 \cdot \sin 58^\circ$

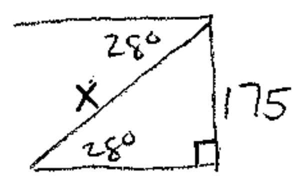
$\sin A = \frac{18 \cdot \sin 58^\circ}{30}$

$\sin A = .5088$

$m\angle A = 30.583$ $\boxed{m\angle A = 31}$

18. A hang glider at the edge of a 175 foot cliff determines the angle of depression out to a distant farmhouse to be 28°. To the nearest foot, what is the distance of the hang glider to the farmhouse? Draw a diagram to receive full credit.

Method I:



$\frac{\sin 28^\circ}{1} = \frac{175}{x}$

$x \cdot \sin 28^\circ = 175$

$x = \frac{175}{\sin 28^\circ}$ $x = 372.76$ $\boxed{x = 373}$

Method II:

$\frac{x}{\sin 90^\circ} = \frac{175}{\sin 28^\circ}$

$x \cdot \sin 28^\circ = 175 \sin 90^\circ$

$x = \frac{175 \cdot \sin 90^\circ}{\sin 28}$, $x = 372.76$

$\boxed{x = 373}$

19. If $\cos\theta = -\frac{2}{3}$ and $\tan\theta < 0$, find the EXACT value of $\sin\theta$.

20. Maria scored an 88 on her math test. The mean grade on the math test was an 85 and the standard deviation was 2. In her Biology class, Maria earned an 89. In Biology, the mean score was an 84 and the standard deviation was 3. In which class did she get the better grade, statistically? (Support your work with statistic evidence.)

21.

a. Convert 210° to radian measure.

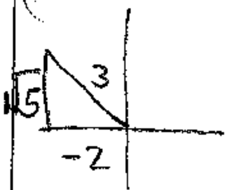
b. Convert $\frac{11\pi}{12}$ to degree measure.

22. Find this sum:

$$\sum_{i=1}^{7} 4(-3)^{i}$$

19. If $\cos\theta = -\frac{2}{3}$ and $\tan\theta < 0$, find the EXACT value of $\sin\theta$.

$\cos\theta < 0, \tan\theta < 0$ in QII



$$\boxed{\sin\theta = \frac{\sqrt{5}}{3}}$$

20. Maria scored an 88 on her math test. The mean grade on the math test was an 85 and the standard deviation was 2. In her Biology class, Maria earned an 89. In Biology, the mean score was an 84 and the standard deviation was 3. In which class did she get the better grade, statistically? (Support your work with statistic evidence.)

Math	Bio
$x = 88$	$x = 89$
$z = \frac{88-85}{2}$	$z = \frac{89-84}{3}$
$z = 1.5$	$z = 1.67$

She did better in Biology because she had a higher z-score

21.

a. Convert 210° to radian measure.

$$210 \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{6}}$$

b. Convert $\frac{11\pi}{12}$ to degree measure.

$$\frac{11\pi}{12} \cdot \frac{180}{\pi} = \boxed{165^\circ}$$

22. Find this sum:

$$\sum_{i=1}^{7} 4(-3)^i = -12 + 36 - 108 + \ldots$$

you want to find the finite sum of a geometric series

$a_1 = -12 \quad r = -3 \quad n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{-12[1-(-3)^7]}{1-(-3)}$$

$$S_7 = \frac{-12[1+2187]}{4}$$

$$S_7 = -3[2187]$$

$$\boxed{S_7 = -6,561}$$

23. Evaluate $$\sum_{m=1}^{\infty} 3\left(\frac{-1}{2}\right)^{m-1}$$	Extra space: **Make sure you clearly marked your omitted question.

23. Evaluate $\sum_{m=1}^{\infty} 3\left(\frac{-1}{2}\right)^{m-1} = 3 - \frac{3}{2} + \frac{3}{4} - \ldots$ infinite geometric series $a_1 = 3, \quad r = -\frac{1}{2}$ $S_\infty = \frac{a_1}{1-r} = \frac{3}{1-(-\frac{1}{2})} = \frac{(3)}{(\frac{3}{2})}\frac{2}{2}$ $\boxed{S_\infty = 2}$	Extra space: **Make sure you clearly marked your omitted question.

Part 3. Answer four of the following six questions. You must OMIT two questions. Clearly indicate which question you are omitting by writing "OMIT" in the space provided for that question. Make sure your answers are clearly marked. Partial credit may be awarded. [6 points each]

24. For all values of x for which the expressions are defined, prove the following is an identity.

$$\sec x \csc x = \cot x + \tan x$$

25. Five students took an exam and got the following scores:

75, 80, 85, 90, 95

Find the mean and standard deviation. Show all calculations and round to ***one*** decimal place.

Part 3. Answer four of the following six questions. You must OMIT two questions. Clearly indicate which question you are omitting by writing "OMIT" in the space provided for that question. Make sure your answers are clearly marked. Partial credit may be awarded. [6 points each]

24. For all values of x for which the expressions are defined, prove the following is an identity.

$$\sec x \csc x = \cot x + \tan x$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$= \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

25. Five students took an exam and got the following scores:

75, 80, 85, 90, 95

Find the mean and standard deviation. Show all calculations and round to ***one*** decimal place.

X	f	X·i	$\bar{X}$	$X-\bar{X}$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
75	1	75	85	-10	100	100
80	1	80	85	-5	25	25
85	1	85	85	0	0	0
90	1	90	85	5	25	25
95	1	95	85	10	100	100
	5	425				250

$$\bar{X} = \frac{425}{5} = 85 \qquad \sigma = \sqrt{\frac{250}{4}} \qquad \sigma = 7.906 \qquad \boxed{\sigma = 7.9}$$

26. For question answer all of the parts.

a) Which of the following is NOT a Fibonacci Number?

A. 8   B. 17   C. 34   D. 55

b) Demonstrate how the Golden Ratio is approximated

c) Give two different examples where the Golden Ratio is found in nature.

d) Give an example where the Golden Ratio used in design.

26. For question answer all of the parts.

a) Which of the following is NOT a Fibonacci Number?

A. 8 B. 17 C. 34 D. 55

b) Demonstrate how the Golden Ratio is approximated

c) Give two different examples where the Golden Ratio is found in nature.

Not relevant

d) Give an example where the Golden Ratio used in design.

27. Projections for the U.S. population, ages 85 and older, are shown in the following table.

Year	2000	2010	2020	2030	2040	2050
Projected Population in millions	4.2	5.9	8.3	11.6	16.2	22.7

Actual 2000 population

a. Divide the population for each decade by the population in the preceding decade. Show that the projected population is approximately a geometric sequence.

b. Write the general term of the geometric sequence describing the U.S. population ages 85 and older, in millions, n decades after 1990.

c. Use the model in part b) above to project the U.S. population, ages 85 and older, in 2080.

27. Projections for the U.S. population, ages 85 and older, are shown in the following table.

Year	2000	2010	2020	2030	2040	2050
Projected Population in millions	4.2	5.9	8.3	11.6	16.2	22.7

Actual 2000 population

a. Divide the population for each decade by the population in the preceding decade. Show that the projected population is approximately a geometric sequence.

$\frac{5.9}{4.2} \sim 1.405 \quad \frac{8.3}{5.9} \sim 1.407 \quad \frac{11.6}{8.3} \sim 1.398$

b. Write the general term of the geometric sequence describing the U.S. population ages 85 and older, in millions, n decades after 1990.

$a_n = 4.2(1.4)^{n-1}$

c. Use the model in part b) above to project the U.S. population, ages 85 and older, in 2080.

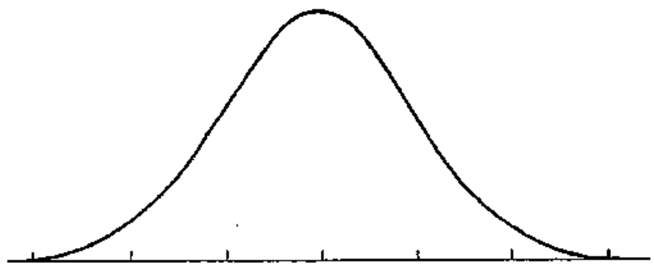
$a_{80} = 4.2(1.4)^{9-1}$

$= 4.2(1.4)^8$

$= 61.983$ million

28. A survey of the soda drinking habits of the population in a high school revealed the mean number of cans of soda consumed per person per week to be 20 with a standard deviation of 3.5. Assuming a Normal distribution,

a. Label the Normal curve for this data, using the 68%-95%-99.7% Rule.



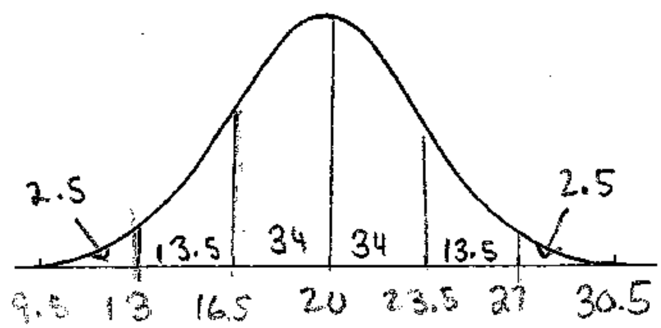
b. Based on the Normal curve, what percent of the population drinks between 13 and 27 cans of soda per week?

c. Based on the Normal curve, what percent of the population drinks more than 23.5 cans of soda per week?

d. What percent of the population drinks less than 9.5 cans of soda per week?

28. A survey of the soda drinking habits of the population in a high school revealed the mean number of cans of soda consumed per person per week to be 20 with a standard deviation of 3.5. Assuming a Normal distribution,

a. Label the Normal curve for this data, using the 68%-95%-99.7% Rule.



b. Based on the Normal curve, what percent of the population drinks between 13 and 27 cans of soda per week?

95%

c. Based on the Normal curve, what percent of the population drinks more than 23.5 cans of soda per week?

16%

d. What percent of the population drinks less than 9.5 cans of soda per week?

Virtually 0%

29. Answer all parts of this question.

a) You are deciding between two different banks to invest your money. Show the steps you would take to determine which is a better investment:

Bank A who pays 4.25% nominal annual interest compounded quarterly.

Bank B who pays $4\frac{1}{8}\%$ nominal annual interest compounded monthly.

b) You invested \$2500 which grew to \$3345.56 in five years. What was the Effective Annual Yield?

c) Once you get your first credit card, what should you do each time your statement comes in order to avoid interest and late payments?

29. Answer all parts of this question.

a) You are deciding between two different banks to invest your money. Show the steps you would take to determine which is a better investment:

Bank A who pays 4.25% nominal annual interest compounded quarterly.

Assume a $1000 investment

$$P = 1000\left(1 + \frac{.0425}{4}\right)^4 = \$1043.18$$

Bank B who pays $4\frac{1}{8}\%$ nominal annual interest compounded monthly.

$$P = 1000\left(1 + \frac{.04125}{12}\right)^{12} = \$1042.04$$

b) You invested $2500 which grew to $3345.56 in five years. What was the Effective Annual Yield?

$$2500(1+r)^5 = 3345.56$$

$$(1+r)^5 = \frac{3345.56}{2500}$$

$$1+r = \sqrt[5]{\frac{3345.56}{2500}}$$

$$r = \sqrt[5]{\frac{3345.56}{2500}} - 1$$

r is about 6%

$$r = .05999$$

c) Once you get your first credit card, what should you do each time your statement comes in order to avoid interest and late payments?

Pay it in full