FORMULA SHEET FOR MATH 442 FINAL EXAM

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$Area = \frac{1}{2}ab\sin C$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Compound interest formula:

$$= P\left(1 + \frac{r}{n}\right)^{nt}$$

Formulas for sequences:

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Standard deviation:

$$s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$$

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SCARSDALE HIGH SCHOOL
Mathematics Department

Math 442 Final Exam — **Name:**________________

June 20, 2014 — **Teacher:** ________________
12:45 – 2:45 PM

Directions: A scientific calculator may be used on this test. Graphing calculators are not permitted. Follow the directions for each part of the test.

Part I

Directions for Part I: Answer **all** of the multiple choice questions and write your answer on the line to the left. 2 points each.

______ 1. Expressed in simplest form, $\csc x \cdot \tan x \cdot \cos x$ is equivalent to

A. 1 B. $\sin x$ C. $\cos x$ D. $\tan x$

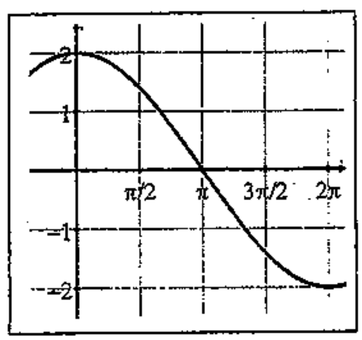
______ 2. Which equation is represented in the graph below?

A. $y = 2\sin\left(\frac{1}{2}x\right)$

B. $y = \frac{1}{2}\sin(2x)$

C. $y = 2\cos\left(\frac{1}{2}x\right)$

D. $y = \frac{1}{2}\cos(2x)$



______ 3. If $\tan\theta = -\frac{3}{5}$ and $\cos\theta < 0$, what is the value of $\sin\theta$?

A. $-\frac{6\sqrt{2}}{2}$ B. $\frac{6\sqrt{2}}{2}$ C. $-\frac{3\sqrt{34}}{34}$ D. $\frac{3\sqrt{34}}{34}$

SCARSDALE HIGH SCHOOL
Mathematics Department

Math 442 Final Exam **Name:**________________

June 20, 2014 **Teacher:** ______________
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Part I

Directions for Part I: Answer **all** of the multiple choice questions and write your answer on the line to the left. 2 points each.

A

1. Expressed in simplest form, $\csc x \cdot \tan x \cdot \cos x$ is equivalent to

A. 1 B. $\sin x$ C. $\cos x$ D. $\tan x$

C

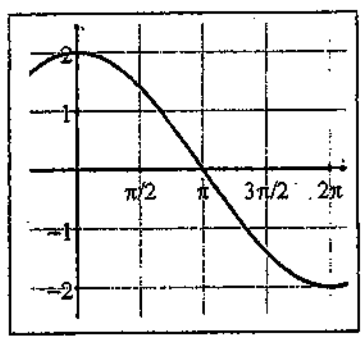
2. Which equation is represented in the graph below?

A. $y = 2\sin\left(\frac{1}{2}x\right)$

B. $y = \frac{1}{2}\sin(2x)$

C. $y = 2\cos\left(\frac{1}{2}x\right)$

D. $y = \frac{1}{2}\cos(2x)$



D

3. If $\tan\theta = -\frac{3}{5}$ and $\cos\theta < 0$, what is the value of $\sin\theta$?

A. $-\frac{6\sqrt{2}}{2}$ B. $\frac{6\sqrt{2}}{2}$ C. $-\frac{3\sqrt{34}}{34}$ D. $\frac{3\sqrt{34}}{34}$

_______ 4. If $f(x)=x+2$ and $g(x)=x^2-4$, for which value(s) of x does $(g\circ f)(x)=0$?

A. 0 only B. −4 only C. 2 and −2 D. 0 and −4

_______ 5. Janet scored an 87 on her biology test. The class scores had a mean of 77 and a standard deviation of 5.2. What is Janet's z-score rounded to the nearest hundredth?

A. −1.92 B. 1.92 C. −10 D. 10

_______ 6. Determine the domain of $f(x)=\dfrac{x-2}{\sqrt{3-x}}$

A. $x\neq 3$ B. $(3,\infty)$ C. $(-\infty,3]$ D. $(-\infty,3)$

_______ 7. Emily has $500 in her bank account. She decided that she will donate $8 to charity each week. Which equation below represents the number of dollars, n, that she will have in her bank account after w weeks?

A. $n=500+8w$ B. $n=500w+8$ C. $n=-8w+500$ D. $n=8w-500$

_______ 8. If θ is an angle in standard position and P(-3,4) is a point on the terminal side of θ, what is the value of $\cos\theta$?

A. $-\frac{3}{5}$ B. $\frac{3}{5}$ C. $-\frac{4}{5}$ D. $\frac{4}{5}$

D

4. If $f(x)=x+2$ and $g(x)=x^2-4$, for which value(s) of x does $(g\circ f)(x)=0$?

A. 0 only B. −4 only C. 2 and −2 D. 0 and −4

B

5. Janet scored an 87 on her biology test. The class scores had a mean of 77 and a standard deviation of 5.2. What is Janet's z-score rounded to the nearest hundredth?

A. −1.92 B. 1.92 C. −10 D. 10

D

6. Determine the domain of $f(x)=\frac{x-2}{\sqrt{3-x}}$

A. $x\neq 3$ B. $(3,\infty)$ C. $(-\infty,3]$ D. $(-\infty,3)$

C

7. Emily has $500 in her bank account. She decided that she will donate $8 to charity each week. Which equation below represents the number of dollars, n, that she will have in her bank account after w weeks?

A. $n=500+8w$ B. $n=500w+8$ C. $n=-8w+500$ D. $n=8w-500$

A

8. If θ is an angle in standard position and P(-3,4) is a point on the terminal side of θ, what is the value of $\cos\theta$?

A. $-\frac{3}{5}$ B. $\frac{3}{5}$ C. $-\frac{4}{5}$ D. $\frac{4}{5}$

Part II

Directions for Part II: Answer 13 of the following 14 questions. Show all of your work in order to receive full credit. 4 points, each. Write "**OMIT**" for the questions you plan on omitting.

9. A. List all the possible rational roots of the equation:

   $3x^3 - 25x^2 + 131x - 41 = 0.$

   B. Show that $\frac{1}{3}$ is a rational root.

10. Determine the domain of

    $f(x) = \sqrt{x-3} + \sqrt{x+4}$

    Express your answer in interval notation. Show your work.

11. Find this sum. You must use a formula and show your work.

    $\sum_{i=1}^{50}(10i - 16)$

12. Verify that the following is an identity:

    $\frac{\sin^2 x - \cos^2 x + 1}{2\sin x \cos x} = \tan x$

Part II

Directions for Part II: Answer 13 of the following 14 questions. Show all of your work in order to receive full credit. 4 points, each. Write "**OMIT**" for the questions you plan on omitting.

9. A. List all the possible rational roots of the equation:

$3x^3 - 25x^2 + 131x - 41 = 0.$

$p = \pm 1, \pm 41$

$q = \pm 1, \pm 3$

$\frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm 41, \pm \frac{41}{3}$

B. Show that $\frac{1}{3}$ is a rational root.

$\frac{1}{3}$ | 3 -25 +131 -41
↓ +1 -8 +41
3 -24 +123 | +0

10. Determine the domain of

$f(x) = \sqrt{x-3} + \sqrt{x+4}$

Express your answer in interval notation. Show your work.

$(x - 3 \geq 0) \wedge (x + 4 \geq 0)$

$(x \geq 3) \wedge (x \geq -4)$

$[3, \infty)$

11. Find this sum. You must use a formula and show your work.

$\sum_{i=1}^{50} (10i - 16) = -6 + 4 + 14 + \cdots + 484$

$S_n = \frac{n}{2}[a_1 + a_n]$

$S_{50} = \frac{50}{2}[-6 + 484]$

$= 25[478]$

$S_{50} = 11{,}950$

12. Verify that the following is an identity:

$\frac{\sin^2 x - \cos^2 x + 1}{2 \sin x \cos x} = \tan x$

$\frac{\sin^2 x - (1 - \sin^2 x) + 1}{2 \sin x \cos x} = \frac{\sin x}{\cos x}$

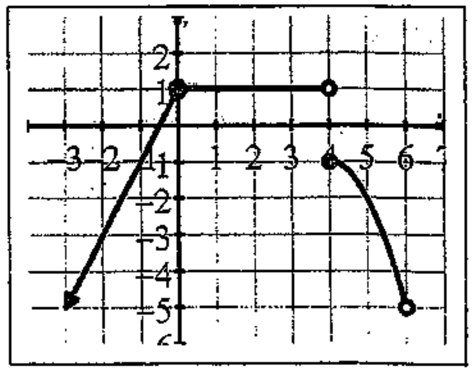
$\frac{\sin^2 - 1 + \sin^2 x + 1}{2 \sin x \cos x}$

$\frac{2 \sin^2 x}{2 \sin x \cos x}$

$\frac{\sin x}{\cos x} = \frac{\sin x}{\cos x}$

13. In triangle ABC,
$a = 10$, $b = 16$, and $m\angle A = 30°$. How many distinct triangles can be formed given these measurements?
**Justify your work!

14. Below is the graph of function f. Answer parts a – c.

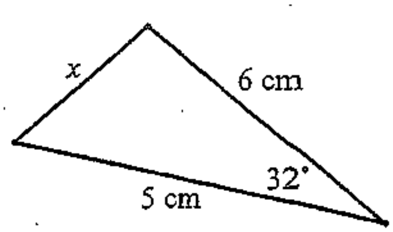


a. Evaluate $f(4)$

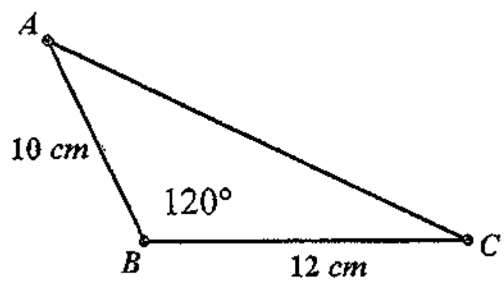
b. Evaluate $(f \circ f)(5)$

c. For which value(s) of x is $f(x) > -1$?

15. Find the value of x in the triangle below. Round your answer to the nearest hundredth.



16. Find the **EXACT** area of triangle ABC.



13. In triangle ABC,
$a = 10$, $b = 16$, and $m\angle A = 30°$. How many distinct triangles can be formed given these measurements?
**Justify your work!

	A	B	C
ΔI	30°	53°	97°
ΔII	30°	127°	23°

$\frac{a}{\sin A} = \frac{b}{\sin B}$, $\frac{10}{\sin 30} = \frac{16}{\sin B}$

$10 \sin B = 16 \sin 30°$

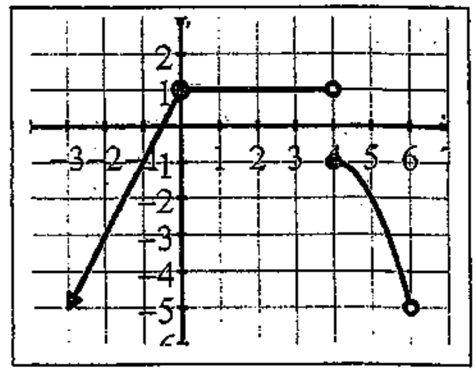
$\sin B = \frac{16 \sin 30°}{10}$

2 triangles

$\sin B = .8$

$m\angle B = 53°$ or $127°$

14. Below is the graph of function f. Answer parts a – c.



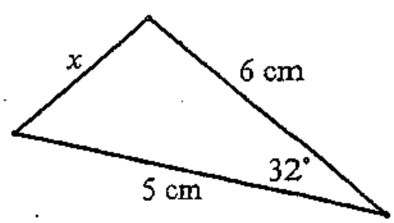
a. Evaluate $f(4)$ $= -1$

b. Evaluate $(f \circ f)(5)$ $= f(-2) = -3$

c. For which value(s) of x is $f(x) > -1$?

$-1 < x < 4$ or $(-1, 4)$

15. Find the value of x in the triangle below.
Round your answer to the nearest hundredth.

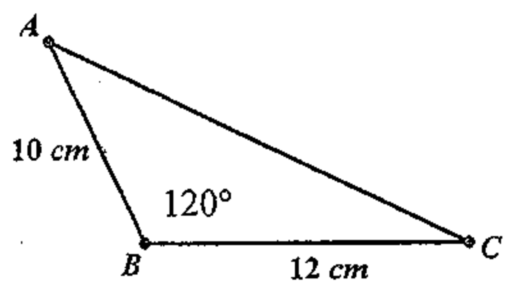


$x^2 = 6^2 + 5^2 - 2(6)(5)\cos 32°$

$x^2 = 10.1124$

$x = 3.18$ cm

16. Find the **EXACT** area of triangle ABC.



$K = \frac{1}{2} ac \sin B$

$K = \frac{1}{2}(12)(10) \sin 120°$

$K = 60 \cdot \left(\frac{\sqrt{3}}{2}\right)$

$K = 30\sqrt{3}$

17. Find the sum of the first 11 terms of this sequence: You must use a formula and show your work.

2, -8, 32, -128,…

18. Evaluate the following:

$$\sum_{k=1}^{4}(-1)^{k+1}\cdot(2k-1)$$

19. Solve the following equation for all values of θ in the interval $0^\circ \le \theta < 360^\circ$.

$$2\sin^2\theta + \sin\theta - 1 = 0$$

20. Perform the division below. State the quotient and the remainder.

$$(6 + a^3 - 3a) \div (a+3)$$

Quotient: ________________

Remainder: ________________

17. Find the sum of the first 11 terms of this sequence: You must use a formula and show your work.
2, -8, 32, -128,...

$a_1 = 2, r = -4$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{11} = \frac{2[1-(-4)^{11}]}{1-(-4)}$

$S_{11} = \frac{2[1-(-4194304)]}{1+4}$

$S_{11} = \frac{2[4194305]}{5}$

$\boxed{S_{11} = 1677722}$

18. Evaluate the following:

$\sum_{k=1}^{4}(-1)^{k+1}\cdot(2k-1)$

$(-1)^{1+1}[2(1)-1] + (-1)^{2+1}[2(2)-1] + (-1)^{3+1}[2(3)-1] + (-1)^{4+1}[2(4)-1]$

$1 - 3 + 5 - 7$

$\boxed{-4}$

19. Solve the following equation for all values of θ in the interval $0^\circ \le \theta < 360^\circ$.
$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0 \vee \sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2} \vee \sin\theta = -1$

$\theta = 30^\circ, 150^\circ \vee \theta = 270^\circ$

$\{30^\circ, 150^\circ, 270^\circ\}$

20. Perform the division below. State the quotient and the remainder.

$(6 + a^3 - 3a) \div (a + 3)$

-3	1	+0	-3	+6
	↓	-3	+9	-18
	1	-3	+6	-12

Quotient: $a^2 - 3a + 6$

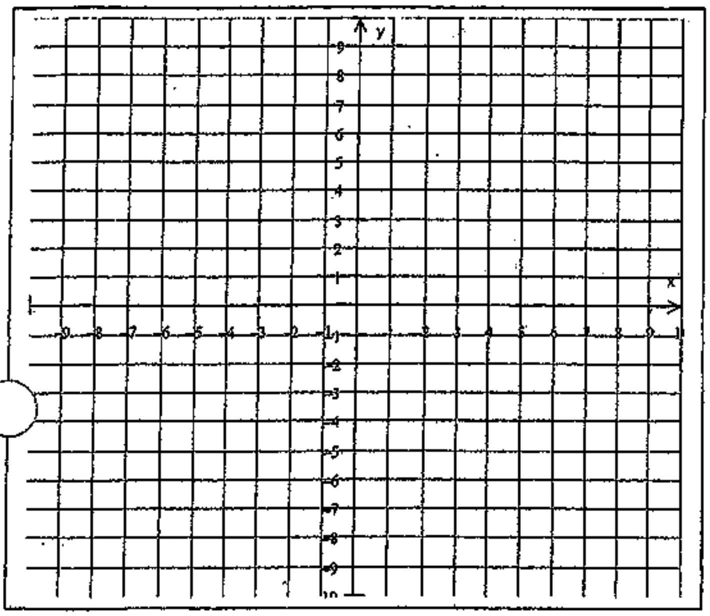
Remainder: -12

21. $f(x)=\begin{cases} 1 & \text{if } x<-4 \\ -x & \text{if } -4\le x<0 \\ x^2-1 & \text{if } x\ge 0 \end{cases}$

Graph the above piecewise function. Determine the domain and range and write your answer in interval notation.

Domain: ______________

Range: ________________



22. The angle of elevation to the top of a building is 38°. At a point 50 feet closer, the angle of elevation to the top of the same building is 45°. Find the height of the building. Round your answer to the nearest foot.

Extra space:

21. $f(x)=\begin{cases}1 & \text{if } x<-4\\ -x & \text{if } -4\le x<0\\ x^2-1 & \text{if } x\ge 0\end{cases}$

Graph the above piecewise function. Determine the domain and range and write your answer in interval notation.

Domain: ____________

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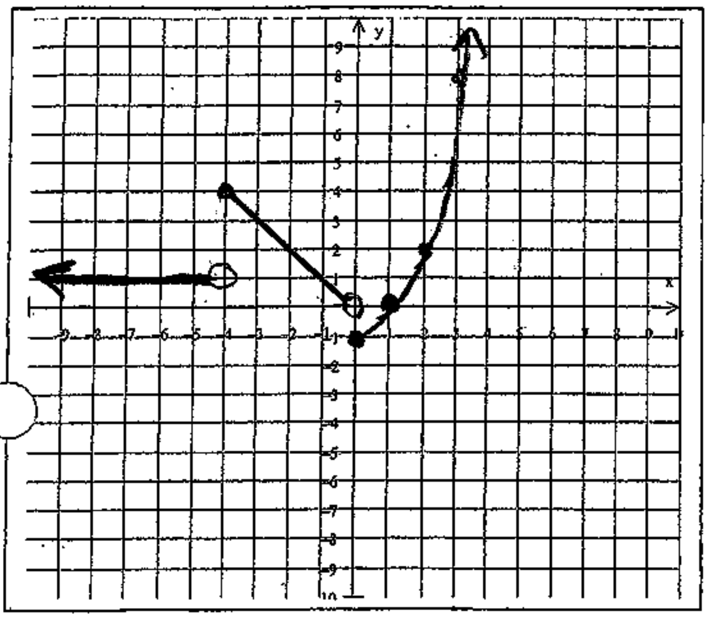

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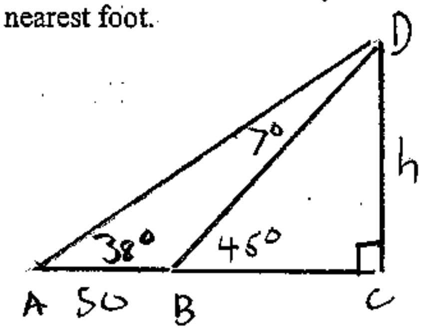


$\frac{50}{\sin 7°} = \frac{BD}{\sin 38°}$

$BD = \frac{50 \cdot \sin 38°}{\sin 7°}$ $BD = 252.59$

$\frac{252.59}{\sin 90°} = \frac{h}{\sin 45°}$

$h = \frac{252.59 \sin 45°}{\sin 90°} = 178.607$

$h = 179 ft$

Extra space:

Part III

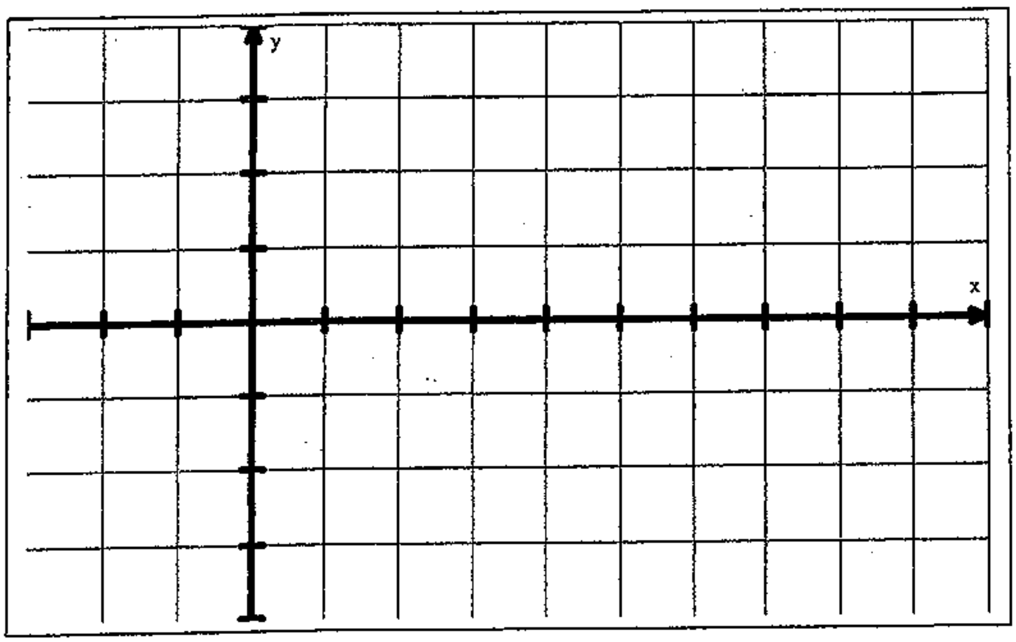
Directions for Part III: Answer 4 of the following 5 questions. Show all of your work in order to receive full credit. 8 points, each.

23.

a. On the same set of axes, sketch the graphs of f and g in the interval $0 \le x \le 2\pi$. REMEMBER TO LABEL THE AXES!!!

$f(x) = 2\cos x$

$g(x) = 3\sin\left(\frac{1}{2}x\right)$



b. From the graphs drawn in part a, determine the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $f(x) = g(x)$. ____________

Part III

Directions for Part III: Answer 4 of the following 5 questions. Show all of your work in order to receive full credit. 8 points, each.

23.

a. On the same set of axes, sketch the graphs of f and g in the interval $0 \le x \le 2\pi$. REMEMBER TO LABEL THE AXES!!!

$f(x) = 2\cos x$

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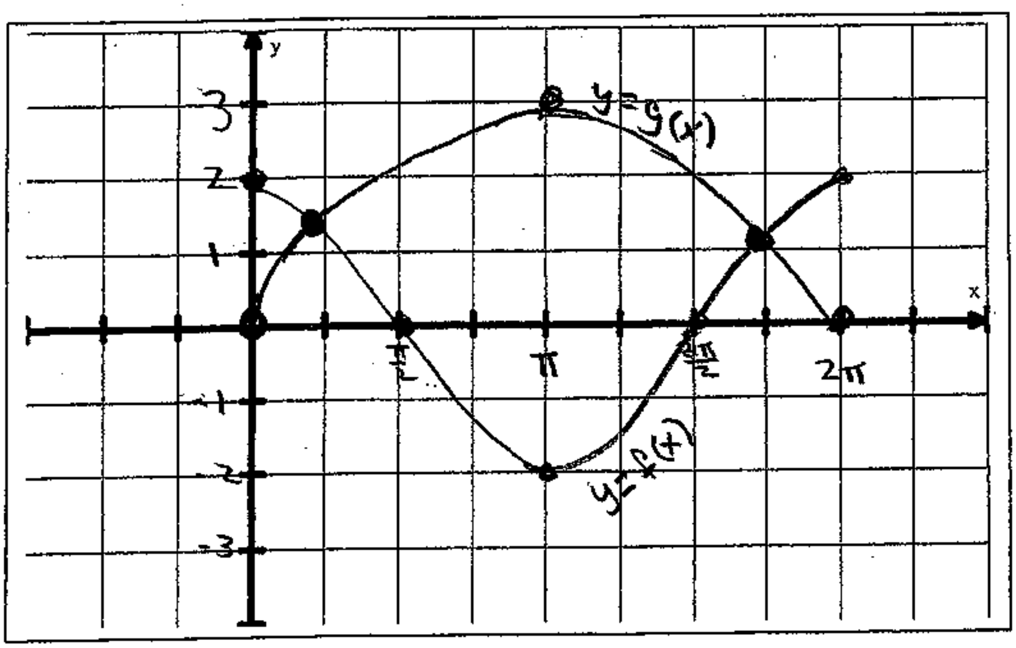
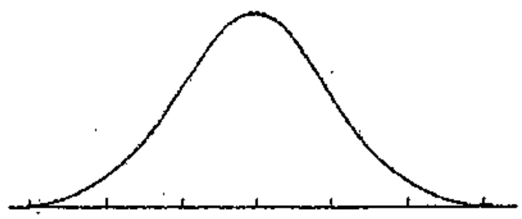


b. From the graphs drawn in part a, determine the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $f(x) = g(x)$. 2

24. Answer **ALL** parts of this problem.

A survey of the soda drinking habits of the population in a high school revealed the mean number of cans of soda consumed per person per week to be 20 with a standard deviation of 3.5. Assuming a normal distribution,

a) Label the normal curve for this data.



Use the 68-95-99.7 Rule or the attached table to answer the following questions.

b) What percent of the population drinks between 13 and 27 cans of soda per week?

c) What percent of the population drinks more than 23.5 cans of soda per week?

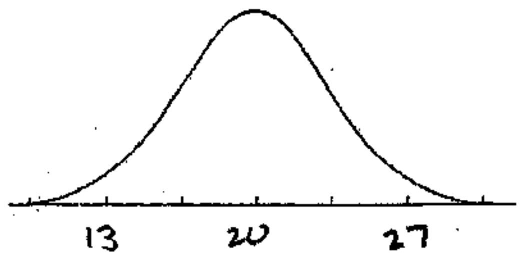
d) What percent of the population drinks less than 9.5 cans of soda per week?

e) What are you more likely to find? A student who drinks 14 or fewer cans of a soda per week OR a student who drinks at least 26 cans of soda per week? Justify.

24. Answer **ALL** parts of this problem.

A survey of the soda drinking habits of the population in a high school revealed the mean number of cans of soda consumed per person per week to be 20 with a standard deviation of 3.5. Assuming a normal distribution,

a) Label the normal curve for this data.



Use the 68-95-99.7 Rule or the attached table to answer the following questions.

b) What percent of the population drinks between 13 and 27 cans of soda per week?

95%

c) What percent of the population drinks more than 23.5 cans of soda per week?

16%

d) What percent of the population drinks less than 9.5 cans of soda per week?

virtually nobody

e) What are you more likely to find? A student who drinks 14 or fewer cans of a soda per week OR a student who drinks at least 26 cans of soda per week? Justify.

Both are equally like since the are the same magnitude of z-score

$z_{13} = \frac{14-20}{3.5}$ $z_{26} = \frac{26-20}{3.5}$

$z_{13} = -1.714$ $z_{26} = 1.714$

25. Part A. Prove the following identity.

$$\tan x \sin x + \cos x = \sec x$$

Part B. Solve the following equation on the interval $0^\circ \le x < 360^\circ$.

$$2\sin^2 x + 3\cos x - 3 = 0$$

25. Part A. Prove the following identity.

$$\tan x \sin x + \cos x = \sec x$$

$$\frac{\sin x}{\cos x} \cdot \sin x + \cos x \cdot \frac{\cos x}{\cos x} = \frac{1}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$\frac{1}{\cos x} = \frac{1}{\cos x}$$

Part B. Solve the following equation on the interval $0^\circ \leq x < 360^\circ$.

$$2\sin^2 x + 3\cos x - 3 = 0$$

$$2(1-\cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$-2\cos^2 x + 3\cos x - 1 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$2\cos x - 1 = 0 \quad \vee \quad \cos x - 1 = 0$$

$$\cos x = \tfrac{1}{2} \quad \vee \quad \cos x = 1$$

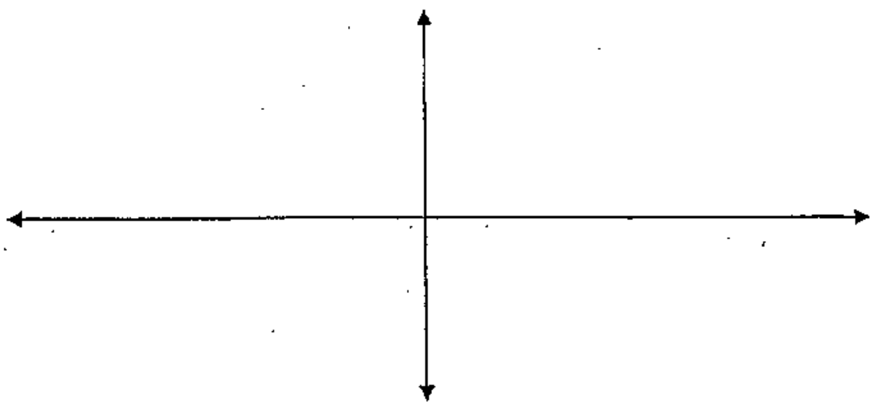
$$x = 60^\circ, 300^\circ \quad \vee \quad x = 0^\circ$$

$$\{0^\circ, 60^\circ, 300^\circ\}$$

26. Answer BOTH parts of this question.

Part A. Sketch a graph of f. State the degree, leading coefficient and y - intercept.

$$f(x) = -2(2x-3)(x+5)^2(x-4)^3$$



Degree: ________

Leading Coefficient: ________

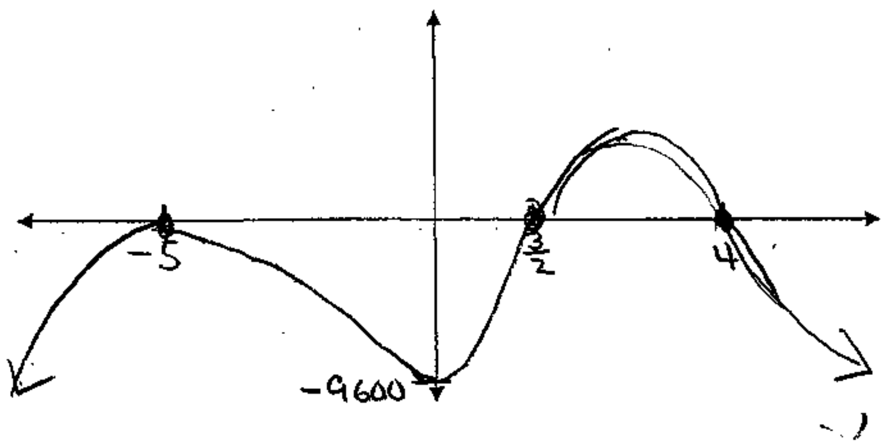
y – intercept: ______

Part B. Find all the roots of the equation: $x^3 - 4x^2 + 13x + 50$, given that $3 + 4i$ is one of the roots.

26. Answer BOTH parts of this question.

Part A. Sketch a graph of f. State the degree, leading coefficient and y - intercept.

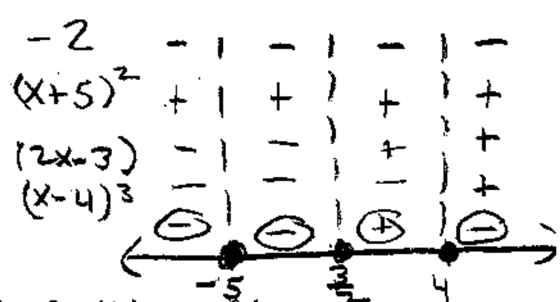
$$f(x) = -2(2x-3)(x+5)^2(x-4)^3$$



Degree: 6

Leading Coefficient: -2

y – intercept: 9600



Part B. Find all the roots of the equation: $x^3 - 4x^2 + 13x + 50$, given that $3+4i$ is one of the roots.

$3+4i$ | $1 \quad -4 \quad +13 \quad +50$

$\downarrow \quad +3+4i \quad -19+8i \quad -50$

$3-4i$ | $1 \quad -1+4i \quad -6+8i \quad | +0$

$\downarrow \quad 3-4i \quad +6-8i$

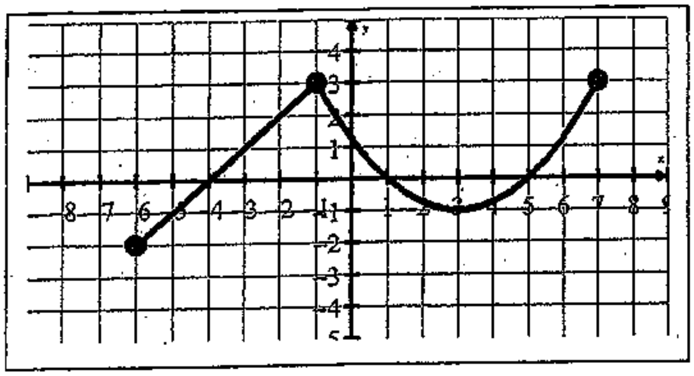
$1 \quad 2 \quad | 0$

$x+2=0$

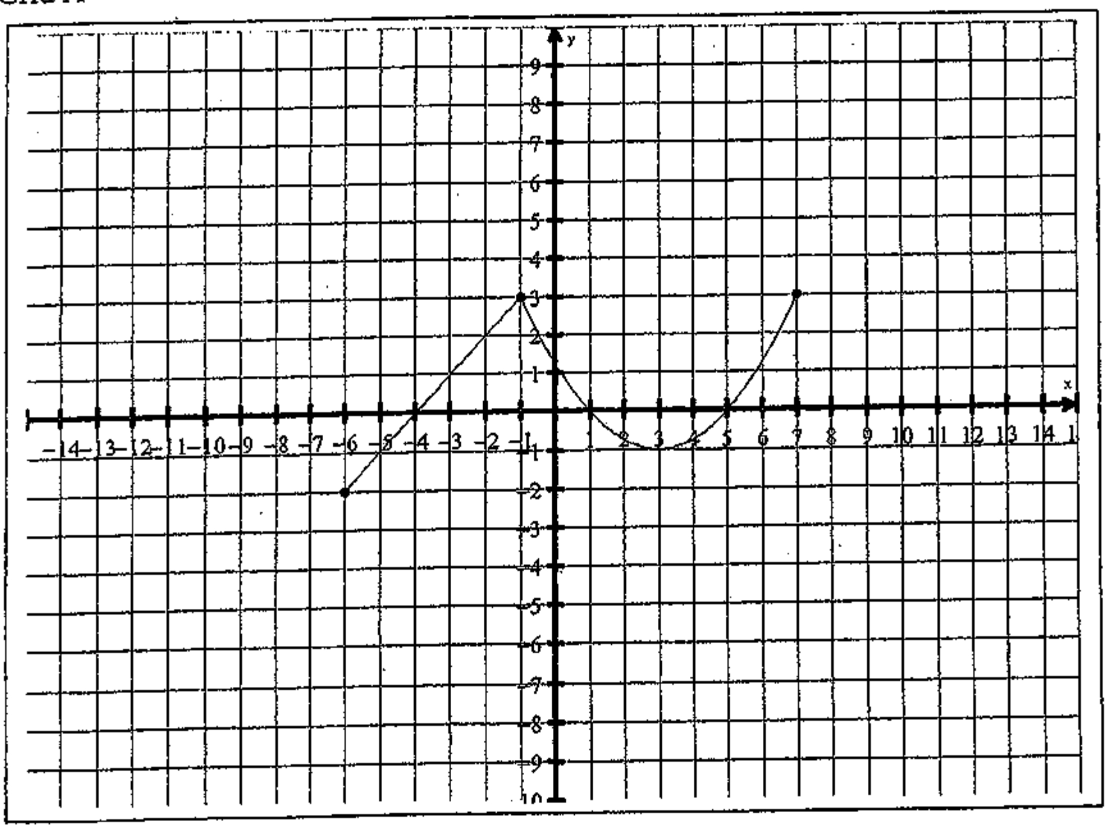
$x=-2$

$\{3 \pm 4i, -2\}$

27. Part A. Below is the graph of $y = f(x)$. On Grid A sketch a graph of $y = -2f(x) + 1$.



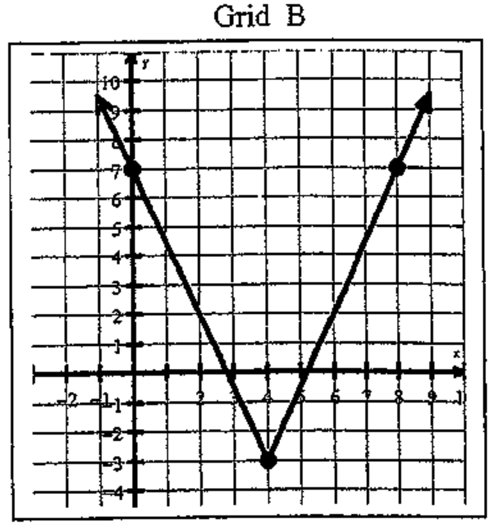
Grid A



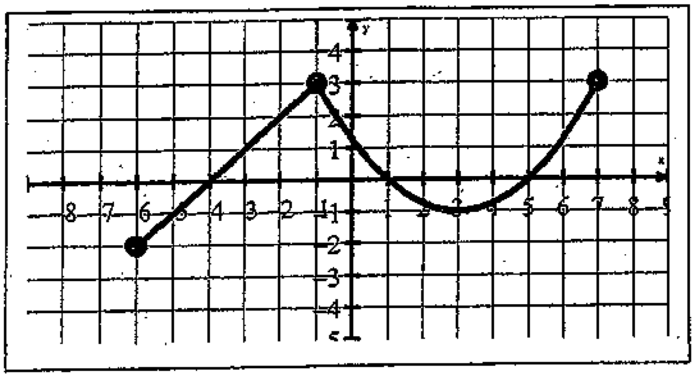
Part B. Determine the equation of the function graphed on Grid B.

$g(x) =$ ____________

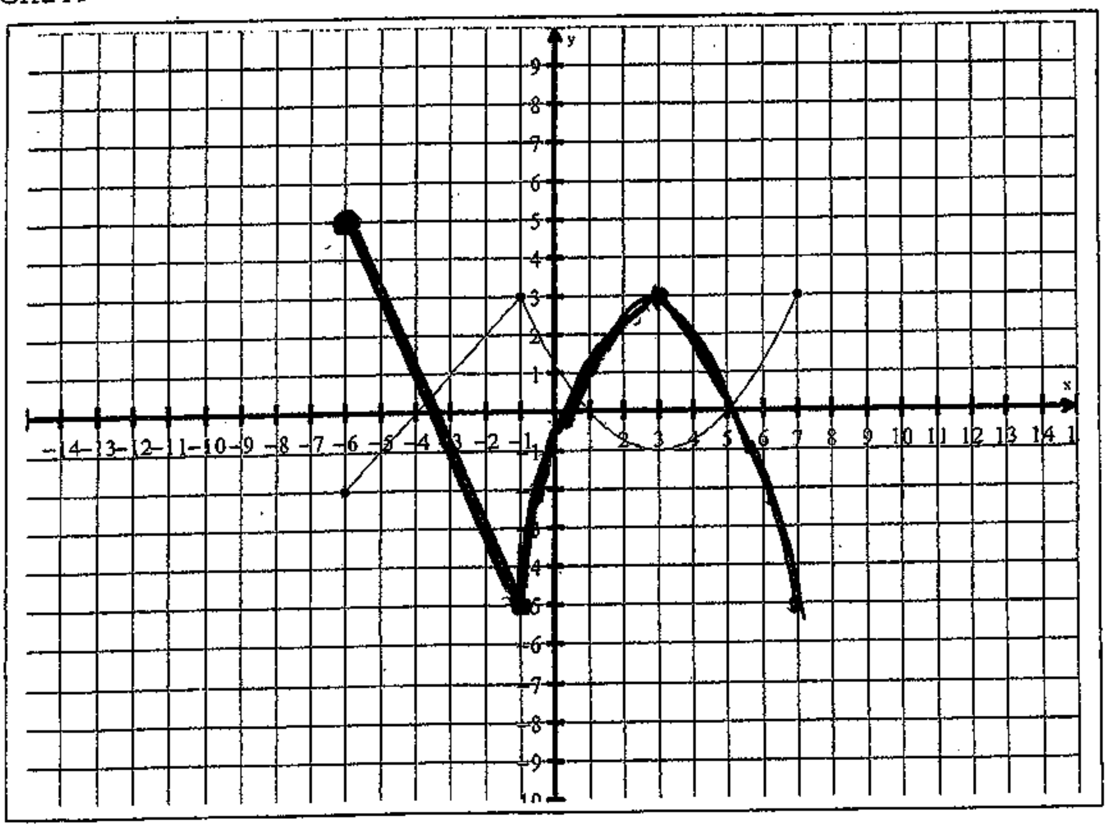
Grid B



27. Part A. Below is the graph of $y = f(x)$. On Grid A sketch a graph of $y = -2f(x) + 1$.



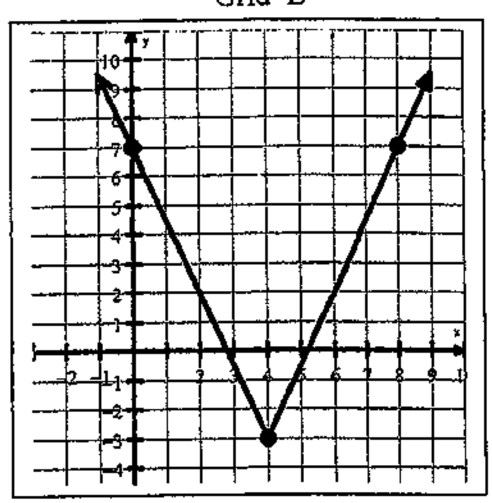
Grid A



① vert stretch
② rx
③ 1 up

Part B. Determine the equation of the function graphed on Grid B.

Grid B



$g(x) = \begin{cases} -\frac{5}{2}x + 7, & x < 4 \\ \frac{5}{2}x - 13, & x \geq 4 \end{cases}$

or

$g(x) = \left|\frac{5}{2}x - 10\right| - 3$