MATH 441
Probability

REVIEW EXERCISES

Show all work here:

11.1

1. A restaurant offers 20 appetizers and 40 main courses. In how many ways can a person order a two-course meal?
2. A popular brand of pen comes in red, green, blue, or black ink. The writing tip can be chosen from extra bold, bold, regular, fine, or micro. How many different choices of pens do you have with this brand?
3. In how many ways can first and second prize be awarded in a contest with 100 people, assuming that each prize is awarded to a different person?
4. You are answering three multiple-choice questions. Each question has five answer choices, with one correct answer per question. If you select one of these five choices for each question and leave nothing blank, in how many ways can you answer the questions?
5. A stock can go up, go down, or stay unchanged. How many possibilities are there if you own five stocks?
6. A person can purchase a condominium with a choice of five kinds of carpeting, with or without a pool, with or without a porch, and with one, two, or three bedrooms. How many different options are there for the condominium?

11.2

7. Six acts are scheduled to perform in a variety show. How many different ways are there to schedule their appearances?
8. In how many ways can five airplanes line up for departure on a runway?
9. You need to arrange seven of your favorite books along a small shelf. Although you are not arranging the books by height, the tallest of the books is to be placed at the left end and the shortest of the books at the right end. How many different ways can you arrange the books?

In Exercises 10–13, evaluate each factorial expression.

10. $\frac{16!}{14!}$
11. $\frac{800!}{799!}$
12. $5! - 3!$
13. $\frac{11!}{(11-3)!}$

In Exercises 14–15, use the formula for ${}_nP_r$ to evaluate each expression.

14. ${}_{10}P_6$
15. ${}_{100}P_2$

Use the formula for ${}_nP_r$ to solve Exercises 16–17.

16. A club with 15 members is to choose four officers—president, vice-president, secretary, and treasurer. In how many ways can these offices be filled?
17. Suppose you are asked to list, in order of preference, the five favorite CDs you purchased in the past 12 months. If you bought 20 CDs over this time period, in how many ways can the five favorite be ranked?

Use the formula for the number of permutations with duplicate items to solve Exercises 18–19.

18. In how many distinct ways can the letters of the word TORONTO be arranged?

19. In how many ways can the digits in the number 335,557 be arranged?

11.3

In Exercises 20–22, does the problem involve permutations or combinations? Explain your answer. (It is not necessary to solve the problem.)

20. How many different 4-card hands can be dealt from a 52-card deck?

21. How many different ways can a director select from 20 male actors to cast the roles of Mark, Roger, Angel, and Collins in the musical *Rent*?

22. How many different ways can a director select 4 actors from a group of 20 actors to attend a workshop on performing in rock musicals?

In Exercises 23–24, use the formula for ${}_nC_r$ to evaluate each expression.

23. ${}_{11}C_7$ **24.** ${}_{14}C_5$

Use the formula for ${}_nC_r$ to solve Exercises 25–28.

25. An election ballot asks voters to select four city commissioners from a group of ten candidates. In how many ways can this be done?

26. How many different 5-card hands can be dealt from a deck that has only hearts (13 different cards)?

27. From the 20 CDs that you've bought during the past year, you plan to take 3 with you on vacation. How many different sets of three CDs can you take?

28. A political discussion group consists of 12 Republicans and 8 Democrats. In how many ways can 5 Republicans and 4 Democrats be selected to attend a conference on politics and social issues?

11.4

In Exercises 29–32, a die is rolled. Find the probability of rolling

29. a 6.

30. a number less than 5.

31. a number less than 7.

32. a number greater than 6.

In Exercises 33–37, you are dealt one card from a 52-card deck. Find the probability of being dealt

33. a 5.

34. a picture card.

35. a card greater than 4 and less than 8.

36. a 4 of diamonds.

37. a red ace.

In Exercises 38–40, suppose that you reach into a bag and randomly select one piece of candy from 15 chocolates, 10 caramels, and 5 peppermints. Find the probability of selecting

38. a chocolate.

39. a caramel.

40. a peppermint.

41. Tay-Sachs disease occurs in 1 of every 3600 births among Jews from central and eastern Europe, and in 1 in 600,000 births in other populations. The disease causes abnormal accumulation of certain fat compounds in the spinal cord and brain, resulting in paralysis, blindness, and mental impairment. Death generally occurs before the age of five. If we use t to represent a Tay-Sachs gene and T a healthy gene, the table below shows the four possibilities for the children of one healthy, TT, parent, and one parent who carries the disease, Tt, but is not sick.

a. Find the probability that a child of these parents will be a carrier without the disease.

b. Find the probability that a child of these parents will have the disease.

		Second Parent	
		T	t
First Parent	T	TT	Tt
	T	TT	Tt

The table shows the employment status of the U.S. civilian labor force in 2004, by gender. Use the data in the table, expressed in millions, to solve Exercises 42–44.

EMPLOYMENT STATUS OF THE U.S. LABOR FORCE, IN MILLIONS, IN 2004

	Employed	Unemployed	Total
Male	74.5	33.2	107.7
Female	64.7	51.0	115.7
Total	139.2	84.2	223.4

Source: U.S. Bureau of Labor Statistics

Find the probability, expressed as a decimal rounded to three places, that a randomly selected person from the civilian labor force represented in the table

42. is employed.

43. is female.

44. is an unemployed male.

11.5

45. If cities A, B, C, and D are visited in random order, each city visited once, find the probability that city D will be visited first, city B second, city A third, and city C last.

In Exercises 46–49, suppose that six singers are being lined up to perform at a charity. Call the singers A, B, C, D, E, and F. The order of performance is determined by writing each singer's name on one of six cards, placing the cards in a hat, and then drawing one card at a time. The order in which the cards are drawn determines the order in which the singers perform. Find the probability that

46. singer C will perform last.

47. singer B will perform first and singer A will perform last.

48. the singers will perform in the following order: F, E, A, D, C, B.

49. the performance will begin with singer A or C.

50. A lottery game is set up so that each player chooses five different numbers from 1 to 20. If the five numbers match the five numbers drawn in the lottery, the player wins (or shares) the top cash prize. What is the probability of winning the prize

   a. with one lottery ticket?

   b. with 100 different lottery tickets?

51. A committee of four people is to be selected from six Democrats and four Republicans. Find the probability that

   a. all are Democrats.

   b. two are Democrats and two are Republicans.

52. If you are dealt 3 cards from a shuffled deck of red cards (26 different cards), find the probability of getting exactly 2 picture cards.

11.6

In Exercises 53–57, a die is rolled. Find the probability of

53. not rolling a 5.

54. not rolling a number less than 4.

55. rolling a 3 or a 5.

56. rolling a number less than 3 or greater than 4.

57. rolling a number less than 5 or greater than 2.

In Exercises 58–63, you draw one card from a 52-card deck. Find the probability of

58. not drawing a picture card.

59. not drawing a diamond.

60. drawing an ace or a king.

61. drawing a black 6 or a red 7.

62. drawing a queen or a red card.

63. drawing a club or a picture card.

In Exercises 64–69, it is equally probable that the pointer on the spinner shown will land on any one of the six regions, numbered 1 through 6, and colored as shown. If the pointer lands on a borderline, spin again. Find the probability of

64. not stopping on 4.

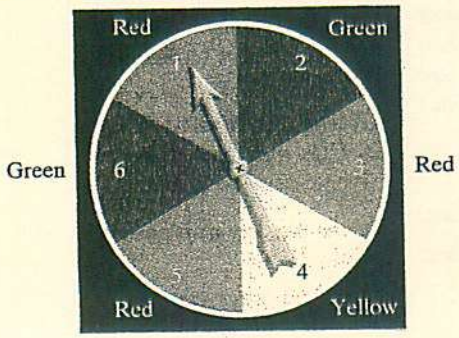
65. not stopping on yellow.

66. not stopping on red.

67. stopping on red or yellow.

68. stopping on red or an even number.

69. stopping on red or a number greater than 3.



Use this information to solve Exercises 70–71. At a workshop on police work and the African-American community, there are 50 African-American male police officers, 20 African-American female police officers, 90 white male police officers, and 40 white female police officers. If one police officer is selected at random from the people at the workshop, find the probability that the selected person is

70. African American or male.

71. female or white.

Suppose that a survey of 350 college students is taken. Each student is asked the type of college attended (public or private) and the family's income level (low, middle, high). Use the data in the table to solve Exercises 72–75. Express probabilities as simplified fractions.

	Public	Private	Total
Low	120	20	140
Middle	110	50	160
High	22	28	50
Total	252	98	350

Find the probability that a randomly selected student in the survey

72. attends a public college.

73. is not from a high-income family.

74. is from a middle-income or a high-income family.

75. attends a private college or is from a high-income family.

76. One card is randomly selected from a deck of 52 cards. Find the odds in favor and the odds against getting a queen.

77. The winner of a raffle will receive a two-year scholarship to any college of the winner's choice. If 2000 raffle tickets were sold and you purchased 20 tickets, what are the odds against your winning the scholarship?

78. The odds in favor of a candidate winning an election are given at 3 to 1. What is the probability that this candidate will win the election?

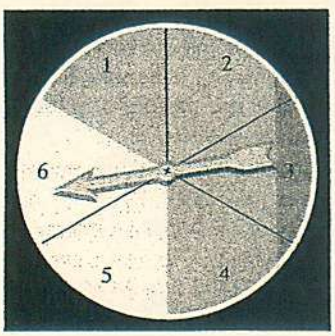
11.7

Use the spinner shown to solve Exercises 79–83. It is equally likely that the pointer will land on any one of the six regions, numbered 1 through 6, and colored as shown. If the pointer lands on a borderline, spin again. If the pointer is spun twice, find the probability it will land on

79. yellow and then red.

80. 1 and then 3.

81. yellow both times.



If the pointer is spun three times, find the probability it will land on

82. yellow and then 4 and then an odd number.

83. red every time.

84. What is the probability of a family having five boys born in a row?

85. The probability of a flood in any given year in a region prone to flooding is 0.2.

a. What is the probability of a flood two years in a row?

b. What is the probability of a flood for three consecutive years?

c. What is the probability of no flooding for four consecutive years?

d. What is the probability of a flood at least once in the next four years?

In Exercises 86–87, two students are selected from a group of four psychology majors, three business majors, and two music majors. The two students are to meet with the campus cafeteria manager to voice the group's concerns about food prices and quality. One student is randomly selected and leaves for the cafeteria manager's office. Then, a second student is selected. Find the probability of selecting

86. a music major and then a psychology major.

87. two business majors.

88. A final visit to the box of chocolates: It's now grown to a box of 50, of which 30 are solid chocolate, 15 are filled with jelly, and 5 are filled with cherries. The story is still the same: They all look alike. You select a piece, eat it, select a second piece, eat it, and help yourself to a final sugar rush. Find the probability of selecting a solid chocolate followed by two cherry-filled chocolates.

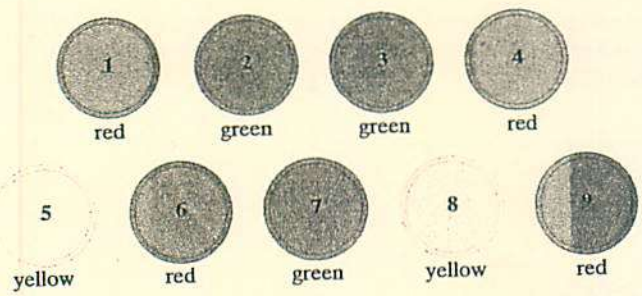
89. A single die is tossed. Find the probability that the tossed die shows 5, given that the outcome is an odd number.

90. A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes k.

91. The numbers shown below are each written on a colored chip. The chips are placed into a bag and one chip is selected at random. Find the probability of selecting

a. an odd number, given that a red chip is selected.

b. a yellow chip, given that the number selected is at least 3.



Chapter 11 Review Exercises

1. 800 **2.** 20 **3.** 9900 **4.** 125 **5.** 243 **6.** 60 **7.** 720 **8.** 120 **9.** 120 **10.** 240 **11.** 800 **12.** 114 **13.** 990
14. 151,200 **15.** 9900 **16.** 32,760 **17.** 1,860,480 **18.** 420 **19.** 60 **20.** combinations **21.** permutations **22.** combinations
23. 330 **24.** 2002 **25.** 210 **26.** 1287 **27.** 1140 **28.** 55,440 **29.** $\frac{1}{6}$ **30.** $\frac{2}{3}$ **31.** 1 **32.** 0 **33.** $\frac{1}{13}$ **34.** $\frac{3}{13}$ **35.** $\frac{3}{13}$
36. $\frac{1}{52}$ **37.** $\frac{1}{26}$ **38.** $\frac{1}{2}$ **39.** $\frac{1}{3}$ **40.** $\frac{1}{6}$ **41. a.** $\frac{1}{2}$ **b.** 0 **42.** 0.623 **43.** 0.518 **44.** 0.149 **45.** $\frac{1}{24}$ **46.** $\frac{1}{6}$ **47.** $\frac{1}{30}$
48. $\frac{1}{720}$ **49.** $\frac{1}{3}$ **50. a.** $\frac{1}{15{,}504} \approx 0.0000645$ **b.** $\frac{100}{15{,}504} \approx 0.00645$ **51. a.** $\frac{1}{14}$ **b.** $\frac{3}{7}$ **52.** $\frac{3}{26}$ **53.** $\frac{5}{6}$ **54.** $\frac{1}{2}$ **55.** $\frac{1}{3}$
56. $\frac{2}{3}$ **57.** 1 **58.** $\frac{10}{13}$ **59.** $\frac{3}{4}$ **60.** $\frac{2}{13}$ **61.** $\frac{1}{13}$ **62.** $\frac{7}{13}$ **63.** $\frac{11}{26}$ **64.** $\frac{5}{6}$ **65.** $\frac{5}{6}$ **66.** $\frac{1}{2}$ **67.** $\frac{2}{3}$ **68.** 1 **69.** $\frac{5}{6}$
70. $\frac{4}{5}$ **71.** $\frac{3}{4}$ **72.** $\frac{18}{25}$ **73.** $\frac{6}{7}$ **74.** $\frac{3}{5}$ **75.** $\frac{12}{35}$ **76.** in favor: 1:12; against: 12:1 **77.** 99:1 **78.** $\frac{3}{4}$ **79.** $\frac{2}{9}$ **80.** $\frac{1}{36}$ **81.** $\frac{1}{9}$
82. $\frac{1}{36}$ **83.** $\frac{8}{27}$ **84.** $\frac{1}{32}$ **85. a.** 0.04 **b.** 0.008 **c.** 0.4096 **d.** 0.5904 **86.** $\frac{1}{9}$ **87.** $\frac{1}{12}$ **88.** $\frac{1}{196}$ **89.** $\frac{1}{3}$ **90.** $\frac{3}{10}$
91. a. $\frac{1}{2}$ **b.** $\frac{2}{7}$ **92.** $\frac{27}{29}$ **93.** $\frac{4}{29}$ **94.** $\frac{144}{145}$ **95.** $\frac{11}{20}$ **96.** $\frac{11}{135}$ **97.** $\frac{1}{125}$ **98.** $\frac{1}{232}$ **99.** $\frac{19}{1044}$ **100.** $\frac{26{,}098}{30{,}242} \approx 0.863$
101. $\frac{11{,}586}{30{,}242} \approx 0.383$ **102.** $\frac{27{,}818}{30{,}242} \approx 0.920$ **103.** $\frac{15{,}892}{30{,}242} \approx 0.525$ **104.** $\frac{4186}{30{,}242} \approx 0.138$ **105.** $\frac{3887}{26{,}098} \approx 0.149$ **106.** $\frac{2225}{2424} \approx 0.918$
107. 3.125 **108. a.** \$0.50; In the long run, the average cost of a claim is \$0.50. **b.** \$10.00 **109.** \$4500; Answers will vary.
110. −\$0.25; In the long run, a person can expect to lose an average of \$0.25 for each game played.