SCARSDALE HIGH SCHOOL
Mathematics Department

Math 424 Final Exam
June 12, 2013
12:45 – 2:45 P.M.

Name: Answer Key
Teacher: _____

SCIENTIFIC CALCULATORS ARE PERMITTED

Instructions: Read the directions at the beginning of each part. Show all work for possible partial credit in parts II, III and IV.

PART I

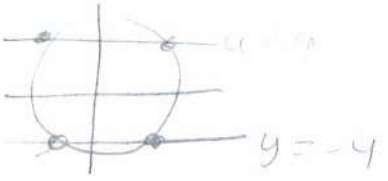
Directions: Answer 12 out of 15 questions. Omit 3 questions. Write the letter of the best choice in the space provided. Partial credit is not allowed. (3 points each)

a 1. Which of the following elements does not have a multiplicative inverse in Z_{24}?

(a)9 (c)13
(b)11 (d)23

b 2. Which of the following is a point that is 5 units away from the origin and 4 units away from the $x-$axis?

$(a)(4,3)$ $(c)(-4,-3)$
$(b)(-3,4)$ $(d)(0,4)$



C 3. A chord is 8 cm from the center of a circle whose radius is 17. Which of the following represents the length of the chord?

(a)15 (c)30
(b)25 (d)34

C 4. Which of the following equals $-\frac{5}{8}$ in Z_9?

$-5 \cdot \frac{1}{8}$

$4 \cdot 8 \quad 32$

(a)3 (c)5
(b)4 (d)no solution

a

5. If $M(3,4)$ is the midpoint of $\overline{AB}$, where $A(2,4)$ and $B(|k+1|,4)$. Which of the following could be the value of k?

$(a)\ -5$ $(c)\ -1$

$(b)\ -4$ $(d)\ -3$

b

6. If the operation $\blacklozenge$ is defined as $x \blacklozenge y = 2x + y$ for $\forall x, y \in Q$, what is the value of a in $2 \blacklozenge a = a \blacklozenge 3$?

$(a) -1$ $(c) 0$

$(b) 1$ $(d) 1.5$

7. Which of the following could lie on the vertex of a triangle?

(a) orthocenter (c) centroid

(b) circumcenter (d) incenter

a

8. The perimeter of an isosceles right triangle is 8. Which of the following is the length of a leg of the triangle?

$(a)\ 8-4\sqrt{2}$ $(c)\ 2\sqrt{2}$

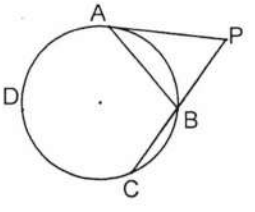
$(b)\ 16-8\sqrt{2}$ $(d) 4\sqrt{2}$

d

9. Using the accompanying diagram, $\overline{PA}$ is tangent to the circle at A, $m\angle PAB = x$, and $AP = AB$. which of the following represents $m\widehat{ADC}$ in terms of x?

$(a) 90+x$ $(c) 180-\frac{1}{2}x$

$(b) 90-x$ $(d) 180+x$



b 10. In isosceles $\triangle ABC$, the measure of one side of the triangle is 14. Which of the following can be the measures of the other sides?

(a) 7 and 7 (c) 14 and 28

(b) 10 and 20 (d) 29 and 29

d 11. Which of the following is a factor of $y^2 - x^2 + 6x - 9$?

(a) $y - x$ (c) $y + x$

(b) $y - x - 3$ (d) $y + x - 3$

b 12. Which of the following is the inverse of $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$?

$(a)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$ $(c)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$

$(b)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ $(d)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

C 13. If the diagonals of two similiar rectangles are 8 and 12 cm respectively. Which of the following is the ratio of the areas of the smaller rectangle to the larger?

(a) 8:12 (b) 12:8 (c) 4:9 (d) 9:4

b 14. Which of the following is logically equivalent to the statement "I completed the school year and I'm still not ready for summer"?

(a) It's not the case that If I complete the school year then I'm ready for summer."

(b) I didn't complete the school year or I'm ready for summer."

(c) ~~(b)~~ If I complete the school year, then I'm not ready for summer."

(d) It is not the case that I completed the school year or I'm ready for summer."

b 15. The diagonals of a quadrilateral $ABCD$ are 28 cm and 36 cm. Which of the following could be the perimeter of the quadrilateral formed by joining, in succession, the midpoints of the sides of $ABCD$?

(a) 58 cm (c) 100 cm

(b) 64 cm (d) 120 cm

PART II

Directions: Answer 8 out of 12 questions. Omit 4 questions. Show all work in the space provided for possible partial credit. Unless otherwise specified, answers should be given in simplest radical form. (4 points each)

Write the question number to be omitted in the space below.

_____, _____, _____, _____

1. Solve for $x \in \Re$: $x^2 \le 2(x+12)$

$x^2 \le 2x+24$

$x^2-2x-24 \le 0$

$(x-6)(x+4) \le 0$

$[(x-6) \le 0 \wedge x+4 \ge 0] \vee [(x-6 \ge 0 \wedge x+4 \le 0]$

$[x \le 6 \wedge x \ge -4] \vee [x \ge 6 \wedge x \le -4]$

$\emptyset$

2. Find the center and area of a circle whose equation is $x^2+y^2+14x-8y+29=0$.

$x^2+14x+49+y^2-8y+16 = -29+49+16$

$(x+7)^2+(y-4)^2 = 36$

$C: (-7, 4)$ $r=6$

$A_{\odot} = 36\pi$

3. Solve for $x \in \Re$: $|4x-7| = 3x-21$

$4x-7 = 3x-21$ or $4x-7 = 21-3x$

$x = -14$ $7x = 28$

$x = 4$

$\{\}$

$|4(-14)-7| = 3(-14)-21$

$|-56-7| = -42-21$

$|-63| = -63$

$63 \neq -63$

$|4(4)-7| = 3(4)-21$

$|16-7| = 12-21$

$|9| = -9$

$9 \neq -9$

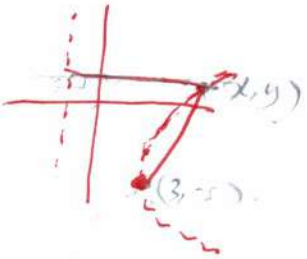
4. Simplify the expression: $\dfrac{\dfrac{7a}{a-b}}{\dfrac{a^2}{a^3-b^3}}$

$\left[\dfrac{7a}{a-b}\right] \cdot (a-b)(a^2+ab+b^2)$

$\left[\dfrac{a^2}{(a-b)(a^2+ab+b^2)}\right](a-b)(a^2+ab+b^2)$

$\dfrac{(7a)(a^2+ab+b^2)}{a^2}$ $\qquad$ $\dfrac{7(a^2+ab+b^2)}{a}$

5. Find the equation of the locus of points equidistant from $x=-1$ and $(3,-5)$.



$\sqrt{(x-3)^2+(y+5)^2} = x+1$

$(x-3)^2+(y+5)^2 = (x+1)^2$

$x^2-6x+9+y^2+10y+25 = x^2+2x+1$

$y^2+10y+33 = 8x$

$\frac{1}{8}(y^2+10y+33) = x$

6. $\triangle ABC$ has vertices $A(6,7), B(-3,2)$, and $C(5,-8)$. Write in standard form the equation of the line that passes through the median drawn from vertex A.

Midpt BC $= \left(\dfrac{-3+5}{2}, \dfrac{2-8}{2}\right) = \left(\dfrac{2}{2}, \dfrac{-6}{2}\right)$

$= (1,-3)$

slope $= \dfrac{7+3}{6-1} = \dfrac{10}{5} = 2$

$y+3 = 2(x-1)$

$y+3 = 2x-2$

$y-2x = -5$

or

$2x-y = 5$

7. Solve for x in $(Z_{11}, +, \cdot)$: $x^2 + 4x + 1 = 0$. Use only elements found in Z_{11} and the operations of addition and multiplication?

$x^2+4x+1=0$

$x^2+4x-21=0$

~~$(x+3)(x-7)$~~

~~$(x+3)(x+8)$~~

$(x+7)(x+8)=0$

$x+7=0$ or $x+8=0$

$+4\ +4$ $+3\ +3$

$x=4$ $x=3$

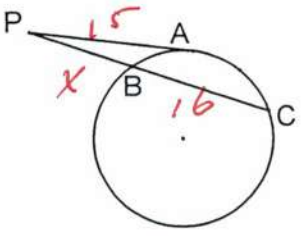
8. Simplify the expression: $\frac{13a+5}{a^2-25}+\frac{7}{5-a}+\frac{2}{a+5}$

$\frac{13a+5}{(a+5)(a-5)} - \frac{7}{a-5} + \frac{2}{a+5}$

$\frac{13a+5}{(a+5)(a-5)} - \frac{7(a+5)}{(a-5)(a+5)} + \frac{2(a-5)}{(a+5)(a-5)}$

$\frac{13a+5-7a-35+2a-10}{(a+5)(a-5)}$ $\frac{8a-40}{(a+5)(a-5)}$ $\frac{8(a-5)}{(a+5)(a-5)} = \frac{8}{a+5}$

9. In the accompanying diagram, $\overline{PA}$ is a tangent to the circle and $\overline{PBC}$ is a secant. If $PA = 15$, $PB = x$, and $BC = 16$, find the value of x.



$(PA)^2 = PB \cdot PC$

$15^2 = x(x+16)$

$225 = x^2+16x$

$x^2+16x-225=0$

$(x+25)(x-9)=0$

$x=-25$ or $x=9$

$\{9\}$

10. Let $a * b$ be defined as $a * b = ab\sqrt{2}$ for $\forall a, b \in \Re$. Find, if possible, the inverse of 6.

$a * e = a$

$a * e = ae\sqrt{2}$

$ae\sqrt{2} = a$

$ae\sqrt{2} - a = 0$

$a(e\sqrt{2} - 1) = 0$

$a = 0$ or $e\sqrt{2} - 1 = 0$

$a = 0$ reject $\quad e = \frac{1}{\sqrt{2}}$

$e = \frac{\sqrt{2}}{2}$

$6 * 6_{INV} = \frac{\sqrt{2}}{2}$

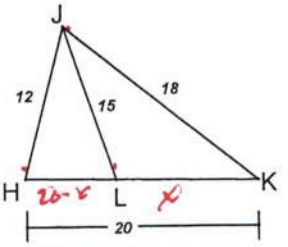
let $x = 6_{INV}$

$6 * x = \frac{\sqrt{2}}{2}$

$6x\sqrt{2} = \frac{\sqrt{2}}{2}$

$x = \frac{1}{12}$

11a. In the accompanying diagram, $\overline{JL}$ bisects $\angle HJK$. Find LK.



not drawn to scale

$\frac{12}{18} = \frac{20-x}{x}$ $\quad \frac{2}{3} = \frac{20-x}{x}$

$2x = 60 - 3x$

$5x = 60$

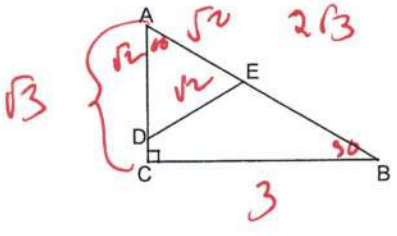
$x = 12$

12

11b. Which is the smallest angle in $\triangle HLJ$?

$\angle HJL$

12. In the accompanying diagram, equilateral triangle ADE has sides of $\sqrt{2}$ and $AC = \sqrt{3}$. Find the perimeter of $DEBC$. (*Leave your answer in simplest radical form.*)



$\sqrt{3}$ $\quad \sqrt{2}$ $\quad 2\sqrt{3}$

$\sqrt{2}$ $\quad \sqrt{2}$ $\quad 3$

$DC = (\sqrt{3} - \sqrt{2}) + 3 + \sqrt{2} + (2\sqrt{3} - \sqrt{2})$

$3 + 3\sqrt{3} - \sqrt{2}$

PART III

Directions: **Answer two (2) questions from this part. Omit 1 problem. Show all work.** **(8 pts. each)**

Write the question number to be omitted in the space below.

_____

1. Given: $\sim(p \rightarrow \sim r)$

$(n \vee \sim t) \rightarrow \sim k$

$\sim r \vee k$

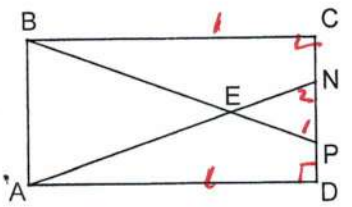
Prove: $n \rightarrow \sim t$

Statement	Reason
(1) $\sim(p \rightarrow \sim r)$	(1) Given
(2) $p \wedge \sim(\sim r)$	(2) Negation of a conditional (1)
(3) $\sim(\sim r)$	(3) Defn of a true conjunction (2)
(4) $\sim r \vee k$	(4) Given
(5) k	(5) law of disjunctive inference (3,4)
(6) $(n \vee \sim t) \rightarrow \sim k$	(6) Given
(7) $\sim(\sim k)$	(7) Double Negation (5)
(8) $\sim(n \vee \sim t)$	(8) modus tollens (6,7)
(9) $\sim n \wedge \sim(\sim t)$	(9) DeMorgan's law (8)
(10) $\sim n$	(10) Defn of a true conjunction (9)
(11) $\sim n \vee \sim t$	(11) Disjunctive addition
(12) $n \rightarrow \sim t$	(13) Disjunctive equivalent of a conditional (11)

QED

2. Given: $ABCD$ is a rectangle; $\overline{AN} \cong \overline{BP}$

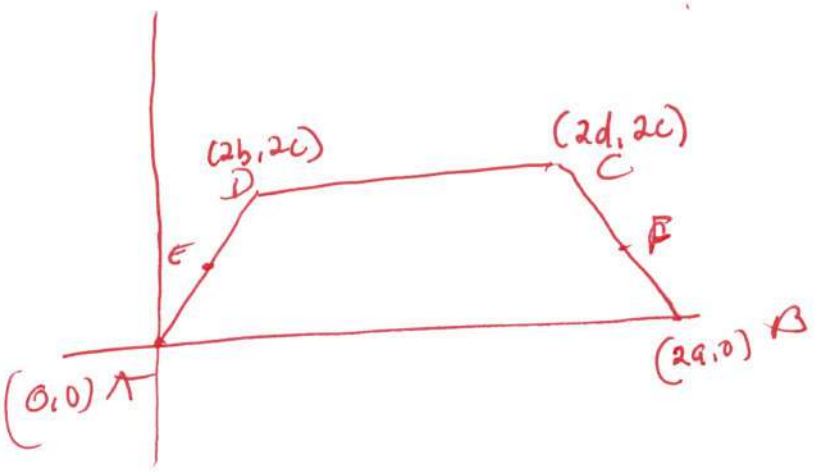
Prove: $\overline{AE} \cong \overline{BE}$



Statement	Reason
(1) ABCD is a rectangle	(1) Given
(2) $\overline{AD} \cong \overline{BC}$	(2) opp. sides of a rectangle are $\cong$.
(3) $\overline{AN} \cong \overline{BP}$	(3) Given
(4) $\angle C$ + $\angle D$ are rt $\angle$s	(4) Defn of a rectangle
(5) $\triangle BCP$ + $\triangle ADN$ are right $\triangle$'s	(5) Defn of a right $\triangle$.
(6) $\triangle ADN \cong \triangle BCP$	(6) HL theorem
(7) $\angle 1 \cong \angle 2$	(7) CPCTC
(8) $\overline{EN} \cong \overline{EP}$	(8) If 2 $\angle$s of a $\triangle$ are $\cong$, then the sides opp. are $\cong$.
(9) $AE + EN = AN$ $BE + EP = BP$	(9) Segment addition Post.
(10) $AN = BP$, $EN = EP$	(10) Defn of congruency.
(11) $AE + EN = BE + EP$	(11) substitution
(12) $AE = BE$	(12) subtraction
(13) $\overline{AE} \cong \overline{BE}$	(13) Defn of congruency

QED

3. Prove using coordinate geometry: The median of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.



Slope $AB = 0$

Slope $DC = \frac{2c-2c}{2d-2b} = \frac{0}{2d-2b} = 0$

midpoint E (b, c)

midpoint F $(a+d, c)$

Slope $EF = \frac{c-c}{a+d, -b} = 0$

$\therefore \overline{AB} \parallel \overline{EF} \parallel \overline{DC}$

$\frac{1}{2}(AB + DC) \stackrel{?}{=} EF$

$EF = a+d-b$

$AB = 2a$

$DC = |2b-2d| = 2d-2b$

$\frac{1}{2}(2a + 2d - 2b) \stackrel{?}{=} a+d-b$

$a+d-b = a+d-b$ ✓

$\frac{1}{2}(AB+DC) = EF$ QED!

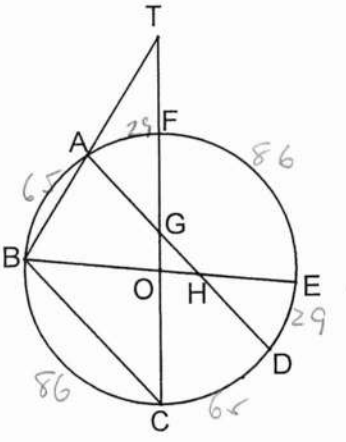
PART IV

Directions: Answer any two (2) questions. Omit 2 problems. Show all work. Final answers must be in simplest radical form. (8 points each)

Write the question number to be omitted in the space below.

_____, _____

1. In the accompanying diagram, $\overline{TAB}$ and $\overline{TFGOC}$ are secants drawn to $\odot O$ from T. Chords $\overline{AD}$ and $\overline{BC}$ are parallel. Chords $\overline{AGHD}$ and $\overline{BOHE}$ intersect at H and chords $\overline{AGHD}$ and $\overline{FGOC}$ intersect at G. If $m\overset{\frown}{CD} = 65, m\overset{\frown}{BC} = 86$, answer the following:



(a) If $AT = 8, AB = 12, AG = 10$, find BC

$\frac{8}{10} = \frac{20}{BC}$

$BC = 25$

(b) Find $m\angle T$ $\frac{1}{2}(\overset{\frown}{BC} - \overset{\frown}{AF})$

$= 28.5$

(c) Find $m\angle AHB$ $\frac{1}{2}(\overset{\frown}{AB} + \overset{\frown}{ED})$

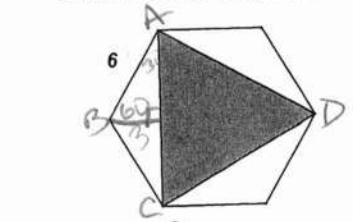
$= 47$

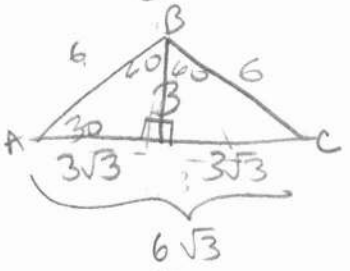
(d) Find $m\angle TAG$ supplement of $\angle DAB$

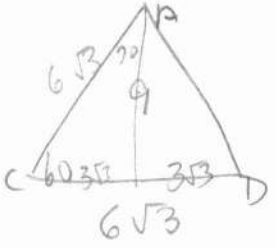
$180 - \frac{1}{2}(86+65)$

$180 - \frac{151}{2} = 104.5$

2*a*. Find the area of the triangle inscribed in a regular hexagon below.







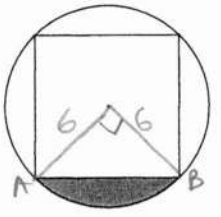
Δ is equilateral

Side of $\Delta = 6\sqrt{3}$

ht of $\Delta = 9$

$\frac{1}{2} \cdot 6\sqrt{3} \cdot 9 = 27\sqrt{3}\ u^2$

2*b*. In the accompanying diagram, a square is inscribed in a circle of radius 6. Find the perimeter of the bounded shaded region.



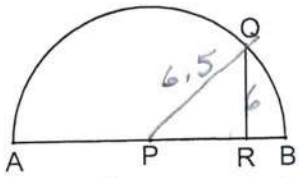
$AB = 6\sqrt{2}$

length $\overset{\frown}{AB} = \frac{1}{4} \cdot \pi(12) = 3\pi$

$6\sqrt{2} + 3\pi$

3. It takes a boy riding his bicycle 9 minutes longer to deliver the newspaper to the homes on his route than it does his father driving in his car. Working together, they can deliver the paper in 20 minutes. How long would it take each person working alone to deliver papers?

4. $\overline{APRB}$ is a diameter of $\odot P$, $QR = 6, AB = 13. \overline{QR} \perp \overline{AB}$. Find RB.

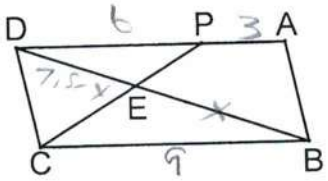


PR = 2.5 from Pythag. th.

RB = 6.5 − 2.5 =

(4)

4*b*. If $ABCD$ is a parallelogram with $PD = 6, PA = 3,$ and $BD = 7.5,$ find EB.



~ Δ'S

$\frac{x}{9} = \frac{7.5 - x}{6}$

(x = 4.5)