Fourth Grade - Mathematics
Kentucky Core Academic Standards with Targets
Student Friendly Targets
Pacing Guide

Anderson County Schools

Safe, Prepared, Responsible
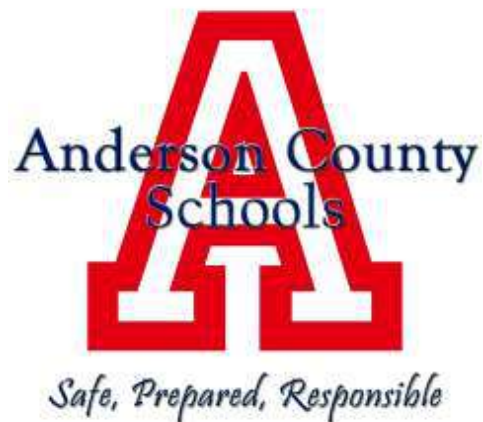
College and Career Readiness Anchor Standards for Math

The K-5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to eight mathematical practices: 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, and 8) Look for express regularity in repeated reasoning.
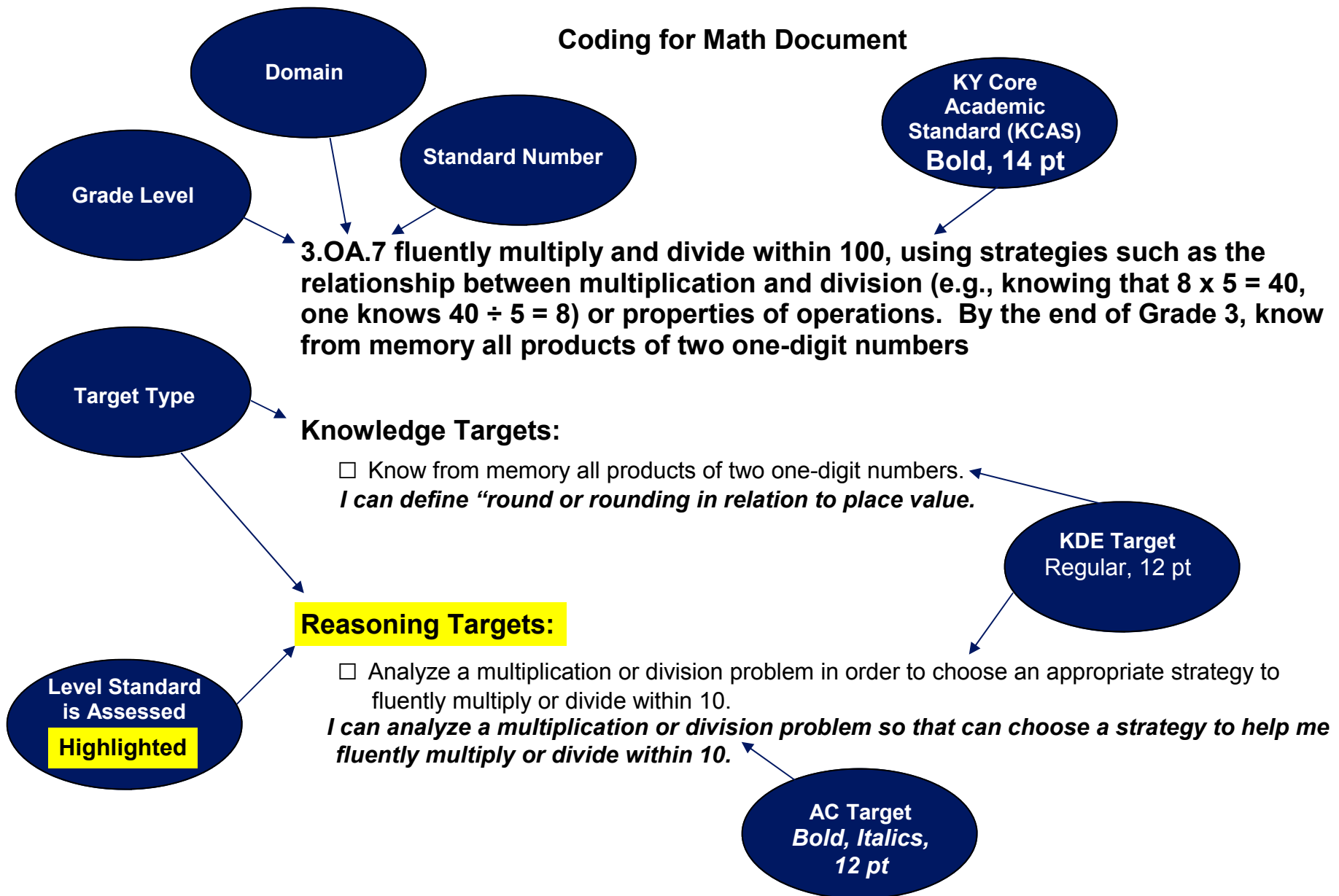
Mathematics is divided into five domains: 1) Counting and Cardinality (CC), 2) Operations and Algebraic Thinking (OA), 3) Number and Operations in Base Ten (NBT), 4) Measurement and Data (MD), and 5) Geometry (G).

Development of Pacing Document

During the summer 2011, Anderson County teachers and administrators developed learning targets for each of the Kentucky Core Content Standards. In winter 2012, curriculum resource teachers verified the congruency of the standards and targets and recommended revisions. Teachers refined the work and began planning the development of common assessments to ensure students learn the intended curriculum. Anderson County Schools would like to thank each of our outstanding teachers and administrators who contributed to this important math curriculum project. Special thanks to Robin Arnzen, Stephanie Barnes, Traci Beasley, Julie Bowen, Tony Calvert, Linda Dadisman, Amanda Ellis, Leslie Fields, Amy Gritton, Lauren Hamel, Linda Hill, Sharon Jackman, Lesley Johnson, Steve Karsner, Chris Kidwell, Joel Maude, Melissa Montgomery, Matt Ogden, Kim Penn, Wayne Reese, Monica Rice, Chrystal Rowland, Kim Ruble, Jennifer Sallee, Amy Satterly, Krista Sawyer, Francine Sloan, Jeanna Slusher, Shayla Smith, T.J. Spivey, Rebecca Stevens, Emily Thacker, Lori Wells, Shannon Wells, Tim Wells, and Jamie White. Thanks also to Tony Calvert (EBW), Brian Edwards (ACHS), and Alex Hunter (ACMS) for providing comments to the work.

North Carolina State Board of Education created a most helpful document entitled "Common Core Instructional Support Tools - Unpacking Standards". The document answers the question "What do the standards mean that a student must know and be able to do?" The "unpacking" is included in our "What Does This Standard Mean?" section. The complete North Carolina document can be found at
http://www.dpi.state.nc.us/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf
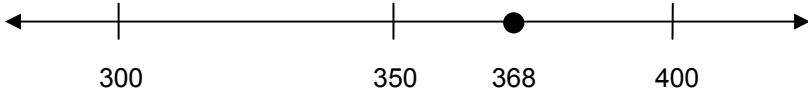
Coding for Math Document

Domain

Standard Number

KY Core Academic Standard (KCAS)
Bold, 14 pt

Grade Level

3.OA.7 fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers

Target Type

Knowledge Targets:

☐ Know from memory all products of two one-digit numbers.
I can define "round or rounding in relation to place value.

KDE Target
Regular, 12 pt

Reasoning Targets:

☐ Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 10.
I can analyze a multiplication or division problem so that can choose a strategy to help me fluently multiply or divide within 10.

Level Standard is Assessed
Highlighted

AC Target
Bold, Italics, 12 pt

Anderson County Elementary
Pacing Guide

```
Mathematics
Grade 4
```

Unit: Place Value, Rounding and Estimation, Addition and Subtraction		
Standard	**Dates Taught**	**Dates Taught**
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* <mark>Knowledge Targets</mark> ☐ Recognize a multi-digit number (up to one million). (underpinning target) *I can recognize how a number increases or decreases in a base 10 system. (underpinning target)* ☐ A digit in one place represents 10 times as much as it represents in the place to its right. *I can explain that a digit in one place is 10 times as much as the place to its right. That means, 7 in the hundreds place is 10 times more than a 7 in the tens place.*	This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with. Example: How is the 2 in the number 582 similar to and different from the 2 in the number 528?	1st Nine Weeks and 17 Days for Unit Aug.17-Sept. 9

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Knowledge Targets** ☐ Read and write multi-digit whole numbers (up to 1,000,000) using base-ten numerals, number names, and expanded form. *I can read numbers to 1,000,000 using numerals, number names, and expanded form.* *I can write numbers to 1,000,000 using numerals, number names, and expanded form.* <mark>**Reasoning Targets**</mark> ☐ Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. *I can write numbers to 1,000,000 using (<, >, =) to record the results of comparisons.*	This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. Students should also be able to compare two multi-digit whole numbers using appropriate symbols.	
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place using place value. <mark>**Knowledge Targets**</mark> ☐ Round multi-digit whole numbers (up to 1,000,000) to any place using place value. *I can round numbers (up to 1,000,000) to any place using place value.*	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Example: Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?	

	Student 1 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	**Student 2** First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300-60 = 240, so we need about 240 more bottles.

Example:
Round 368 to the nearest hundred. This will either be 300 or 400, since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400

| 300 | 350 | 368 | 400 |

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm. **<mark>KnowledgeTargets</mark>** □ Fluently add and subtract multi-digit numbers less than or equal to 1,000,000 using the standard algorithm. □ Round multi-digit whole numbers (up to 1,000,000) to any place using place value. ***I can fluently add and subtract whole numbers (up to 1,000,000.)***	Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use. When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

3892
+ 1567

Student explanation for this problem
  1. Two ones plus seven ones is nine ones.
  2. Nine tens plus six tens is 15 tens.
  3. I am going to write down five tens and think of the10 tens as
     one more hundred.(notates with a 1 above
     the hundreds column)
  4. Eight hundreds plus five hundreds plus the extra hundred from
     adding the tens is 14 hundreds.
  5. I am going to write the four hundreds and think of the 10
     hundreds as one more 1000. (notates with a 1
     above the thousands column)
  6. Three thousands plus one thousand plus the extra thousand
     from the hundreds is five thousand.

3546
- 928

Student explanation for this problem:
  1. There are not enough ones to take 8 ones from 6 ones so I
     have to use one ten as 10 ones. Now I have 3
     tens and 16 ones. (Marks through the 4 and notates with a 3
     above the 4 and writes a 1 above the ones
     column to be represented as 16 ones.)
  2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones
     column of answer.)
  3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens
     column of answer.)
  4. There are not enough hundreds to take 9 hundreds from 5
     hundreds so I have to use one thousand as 10
     hundreds. (Marks through the 3 and notates with a 2 above it.
     (Writes down a 1 above the hundreds
     column.) Now I have 2 thousand and 15 hundreds.
  5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in
     the hundreds column of the answer).
  6. I have 2 thousands left since I did not have to take away any
     thousands. (Writes 2 in the thousands place
     of answer.)
Students should know that it is mathematically possible to subtract a
larger number from a smaller number but
that their work with whole numbers does not allow this as the
difference would result in a negative number.

Unit: Multiplication and Division

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Knowledge Target
- ☐ Multiply a whole number of up to four digits by a one-digit whole number.
- *I can multiply four-digit numbers by a one-digit number.*
- ☐ Multiply two two-digit numbers.
- *I can multiply two-digit numbers by a two-digit number.*

<mark>Reasoning Targets</mark>
- ☐ Use strategies based on place value and the properties of operations to multiply whole numbers.
- *I can use strategies based on place value and the properties of operations to multiply numbers.*
- ☐ Illustrate and explain calculations by using written equations, rectangular arrays and/or area models.
- *I can explain my answer using written equations.*
- *I can explain my answer using rectangular arrays.*
- *I can explain my answer using area models.*
- *I can explain my answer using illustrations.*

Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?
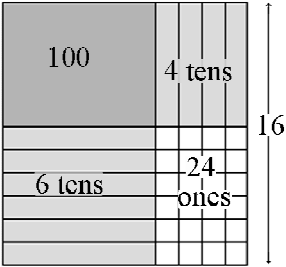
Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

Example:
What would an array area model of 74 x 38 look like?

	70	4
30	70 x 30 = 2100	4 x 30 = 120
8	70 x 8 = 560	4 x 8 = 32

| | Example:
To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.
The area model below shows the partial products. 14 x 16 = 224

$100 + 40 + 60 + 24 = 224$

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.
25
<u>x24</u>

400 (20 x 20)
100 (20 x 5)
80 (4 x 20)
20 (4 x 5)
600 | |
| **4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**

Knowledge Target
☐ Find whole-number quotients and remainders with | In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?
• Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
• Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
• Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65 | |

up to four-digit dividends and one-digit divisors.
I can divide up to four-digit dividends by one-digit divisors.

Reasoning Target

☐ Use the strategies based on place value, the properties of operations, and/or the relationship between multiplication and division (to find whole-number quotients).
I can explain my answer using place value strategies.
I can explain my answer using the properties of operations.
I can explain my answer using the relationship between multiplication and division.

☐ Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
I can explain my answer using written equations.
I can explain my answer using rectangular arrays.
I can explain my answer using area models.
I can explain my answer using illustrations.

This standard calls for students to explore division through various strategies.

Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

Student 1	Student 2			Student 3
592 divided by 8. There are 70 8's in 560. 592 - 560 = 32 There are 4 8's in 32. 70 + 4 = 74	592 divided by 8. I know that 10 8's is 80. If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160. 192 - 160 = 32. 8 goes into 32 4 times. I have none left. I took out 50, then 20 more, then 4 more. That's 74.	592 -400 / 192 -160 / 32 -32 / 0	50 / 20 / 4	I want to get to 592. 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

Example:
Using an Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

Example: 150 ÷ 6
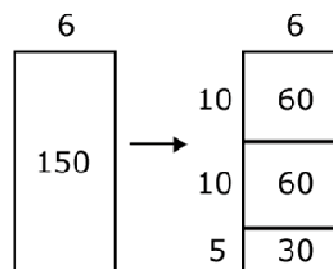
Students make a rectangle and write 6 on one of its sides. They

express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.

3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:

   a.
   ```
      150            150 ÷ 6 = 10 + 10 + 5 = 25
    - 60  (6 x 10)
      90
    - 60  (6 x 10)
      30
    - 30  (6 x 5)
       0
   ```

   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

Example:
1917 ÷ 9
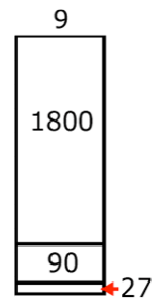
A student's description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **Knowledge Target** ☐ Know multiplication strategies. (Underpinning target) ***I can explain multiplication strategies. (Underpinning target)*** ==**Reasoning**== ☐ Interpret a multiplication equation as a comparison (e.g., interpret 18 = 3 times as many as 6 and 6 times as many as 3). ***I can interpret multiplication equations by comparing 18 = 3 x 6 and 6 x 3.*** ☐ Represent verbal statements of multiplicative comparisons as multiplication equations. ***I can verbalize comparisons using multiplication as an equation.***	A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given opportunities to write and identify equations and statements for multiplicative comparisons. Example: *5 x 8 = 40.* Sally is five years old. Her mom is eight times older. How old is Sally's Mom? *5 x 5 = 25* Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?	
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **Knowledge Target** ☐ Multiply or divide to solve word problems. (Underpinning target)	This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Refer to Glossary, Table 2(page 89) For more examples (table included at the end of this document for your convenience) Examples: Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p). Group Size Unknown: A book costs $18. That is 3 times more than a	

I can multiply or divide to solve word problems. (Underpinning target) ☐ Describe multiplicative comparison. (Underpinning target) *I can describe multiplicative comparison. That means, A rabbit can go two feet in one jump. A kangaroo can go five times as far as a rabbit. How far can a kangaroo go in one jump? (Underpinning target)* ☐ Describe additive comparison. That means repeated addition. (Underpinning target) *I can describe additive comparison. (Underpinning target)* **Reasoning Target** ☐ Determine appropriate operation and solve word problems involving multiplicative comparison. *I can analyze and solve word problems involving multiplicative comparison.* ☐ Determine and use a variety of representations to model a problem involving multiplicative comparison. *I can choose and use representations to model problems involving multiplicative comparison.* ☐ Distinguish between multiplicative comparison and additive comparison (repeated addition). *I can explain the difference between multiplicative comparison and additive comparison.*	DVD. How much does a DVD cost? (18 ÷ p = 3 or 3 x p = 18). Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18). When distinguishing multiplicative comparison from additive comparison, students should note that • additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, "How many more?" • multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is "How many times as much?" or "How many times as many?"	
4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or	This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the	

composite.

Knowledge Target
☐ Define prime and composite numbers. (underpinning target)
I can define prime and composite numbers. (underpinning target)
☐ Know strategies to determine whether a whole number is prime or composite.
I can evaluate whole numbers to determine if they are prime or composite.
☐ Identify all factor pairs for any given number 1-100.
I can list factors for any number up to 100.
☐ Recognize that a whole number is a multiple of each of its factors.
I can tell that a whole number is a multiple of each of its factors. That means 12 is a multiple of 2, 3, 4, and 6.

<mark>Reasoning Target</mark>
☐ **Determine if a given whole number (1-100) is a multiple of a given one-digit number.**
I can determine if a number up to 100 is a multiple of a one-digit number.

number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:
A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by
- building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
- finding factors of the number students should understand the process of finding factor pairs so they can do this for any number 1 -100

Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12. Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6, 8,12, 24
Multiples: 1, 2, 3, 4, 5…24
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
3, 6, 9, 12, 15, 18, 21, 24
4, 8, 12, 16, 20, 24
8, 16, 24
12, 24
24

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints
include the following:
- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

Unit: Fractions and Decimals

4.NF.1 Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Knowledge Target

- ☐ Identify equivalent fractions with unlike denominators. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can identify equivalent fractions with unlike denominators (2, 3, 4, 5, 6, 8, 10, 12, 100)

<mark>Reasoning Targets</mark>

- ☐ Explain why a/b is equal to (n x a)/(n x b) by using fraction models with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (ex: use fraction strips to show why ½ = 2/4 = 3/6 = 4/8). (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can explain why a/b is equal to (n x a)/ (n x b) using fraction models.
I can explain that the number and size of the parts differ in the two fractions but they are the same size (equivalent).

- ☐ Use visual fraction models to show why fractions are equivalent (ex.: ¾ = 6/8). (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can use visual fraction models to show why fractions are equivalent.

- ☐ Generate equivalent fractions using visual fraction models and explain why they can be called equivalent. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12

This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100)

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example：

$$½ = 2/4 = 6/12$$

1/2	

1/4	2/4		

1/12	4/12		
2/12	5/12		
3/12	6/12		

Technology Connection:
http://illuminations.nctm.org/activitydetail.aspx?id=80
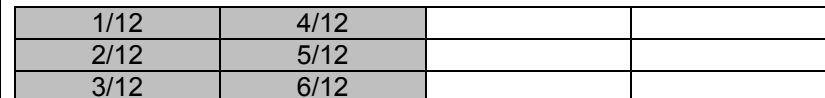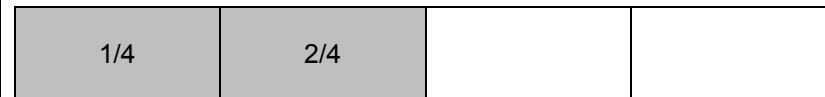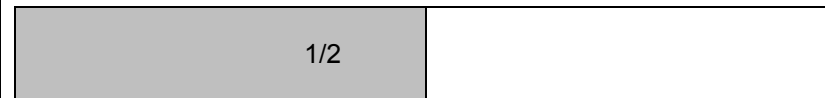
2nd Nine Weeks and 25 days for Unit Oct. 17-Nov. 22

and 100).
I can create equivalent fractions using fraction models and explain why they are called equivalent.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
 	
Knowledge Targets
☐ Recognize fractions as being greater than, less than, or equal to other fractions. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
I can compare fractions using greater than, less than, or equal to other fractions. Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
☐ Record comparison results with symbols: less than, greater than, equal. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
I can compare fractions using symbols. (<, >, =). (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
 	
☐ Use benchmark fractions such as ½ for comparison purpose. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
I can use benchmark fractions such as ½ to compare fractions.
☐ Make comparisons based on parts of the same whole. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and
	This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, . and 1/8 of two medium pizzas is very different from . of one medium and 1/8 of one large).
	
	Example:
	Use patterns blocks.
	    1. If a red trapezoid is one whole, which block shows 1/3?
	
	    2. If the blue rhombus is 1/3, which block shows one whole?
	
	    3. If the red trapezoid is one whole, which block shows 2/3?
	
	Mary used a 12 x 12 grid to represent 1 and Janet used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?
	
	Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so ¼ of each total number is different.

100).

I can compare fractions using parts of the same whole.

<mark>Reasoning Targets</mark>

☐ Compare two fractions with different numerators, e.g. by comparing to a benchmark fraction such as ½. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can compare two fractions with different numerators using a benchmark fraction such as ½.

☐ Compare two fractions with different denominators, e.g. by creating common denominators or by comparing to a benchmark fraction such as ½. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can compare two fractions with different denominators by creating common denominators.
I can compare two fractions with different denominators by comparing to a benchmark fraction such as ½.

☐ Justify the results of a comparison of two fractions, e.g. by using a visual fraction model. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).

I can justify the results of a comparison of two fractions using a visual fraction model.

Mary's Grid
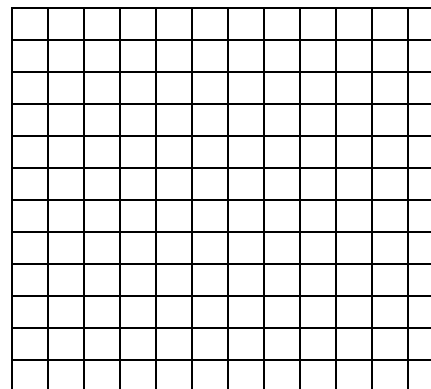
Janet's Grid
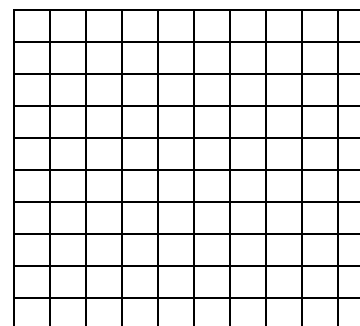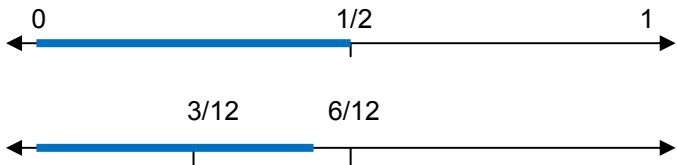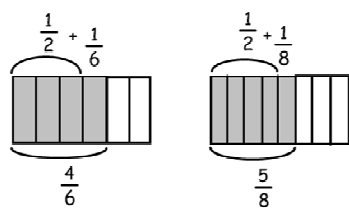
Example:
There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 of it left. Which cake has more left?
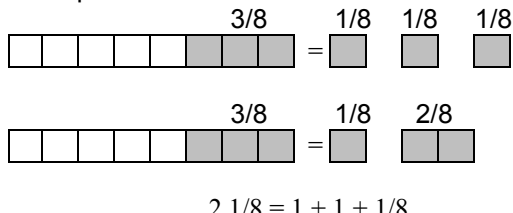
Student 1
Area model:
The first cake has more left over. The second cake has 5/12 left which is smaller than ½.

Student 2
Number Line Model

Student 3
Verbal Explanation:
I know that 6/12 equal ½. Therefore, the second cake which has 5/12 left is less than ½.

Example:
When using the benchmark of ½ to compare 4/6 and 5/8, you could use diagrams such as these:

| 4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
    a. **Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.**

Knowledge Targets
☐ Accumulating unit fractions (1/b) results in a fraction (a/b) where a is greater than 1. (Denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100).
I can explain how fractions accumulate. That means, ¼ + ¼ + ¼ accumulates to ¾. | A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

Example: 2/3 = 1/3 + 1/3
Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example:
1 ¼ - ¾ = ☐   4/4 + 1/4 = 5/4   5/4 – ¾ = 2/4 or ½

Example of word problem: | |

☐ From the introduction: students extend previous understandings about how fractions are built from unit fractions, composing (joining) fractions from unit fractions, and decomposing (separating) fractions into unit fractions. ***I can explain how fractions are put together (composed or joined) which results in addition. I can explain how fractions are broken down (decomposed or separated) which results in subtraction.*** <mark>**Reasoning Targets**</mark> ☐ Using fraction models, reason that addition of fractions is joining parts that are referring to the same whole. ***I can use fraction models to reason that addition of fractions is joining parts that are referring to the same whole.*** ☐ Using fraction models, reason that subtraction of fractions is separating parts that are referring to the same whole. ***I can use fraction models and explain that subtraction of fractions is separating parts that are referring to the same whole.***	Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together? Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza.	
4.NF.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. **b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*** **Knowledge Targets**	Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models. Example: $2\ 1/8 = 1 + 1 + 1/8$	

☐ Add and subtract fractions with like denominators.
I can add fractions with like denominators.
I can subtract fractions with like denominators.

☐ Recognize multiple representations of one whole using fractions with the same denominator.
I can recognize multiple representations of one whole using fractions with the same denominator.

$$2\ 1/8 = 8/8 + 8/8 + 1/8$$

☐ Using visual fraction models, decompose a fraction into the sum of fractions with the same denominator in more than one way.
I can use visual fraction models to break down a fraction into the sum of fractions with the same denominator. That means, 3/8 = 1/8 + 1/8 + 1/8.

☐ Record decompositions of fractions as an equation and explain the equation using visual fraction models.
I can write the equation that shows the break down of fractions, build a visual fraction model to match and explain the equation.
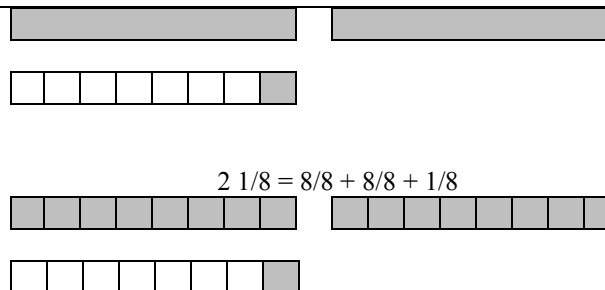
4.NF.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

      c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Knowledge Targets
☐ Add and subtract mixed numbers with like denominators by using properties of operations and the relationship between addition and subtraction.

I can add and subtract mixed numbers with like denominators using the following strategies: 1) properties of operations or 2) relationship between addition and subtraction.

☐ Replace mixed numbers with equivalent fractions, using visual fraction models.

I can replace mixed numbers with equivalent fractions, using visual fractioins.

☐ Replace improper fractions with a mixed number, using visual fraction models.

I can replace mixed numbers with equivalent fractions using visual fraction models.

<mark>Reasoning Target</mark>
☐ Add and subtract mixed numbers by replacing each mixed number with an equivalent fraction.

I can add and subtract mixed numbers by replacing each mixed number with an equivalent fraction.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:
Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot.

Example:
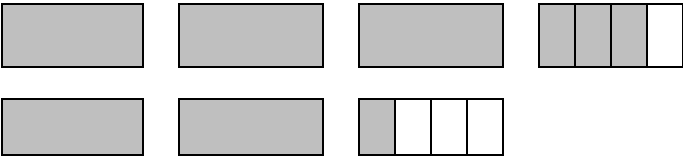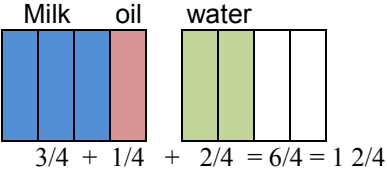Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend?

Possible solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas.
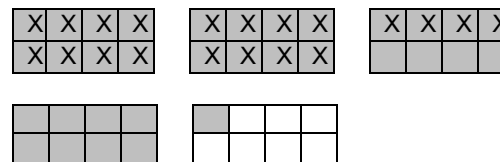
Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions.

Example:

While solving the problem, 3 . + 2 . students could do the following:

Student 1
3 + 2 = 5 and ¾ + ¼ = 1 so 5 + 1 = 6

Student 2
3 ¾ + 2 = 5 ¾ so 5 ¾ + ¼ = 6

Student 3
3 ¾ = 15/4 and 2 ¼ = 9/4 so 15/4 + 9/4 = 24/4 = 6

4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. 　　**d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.** **Knowledge Targets** ☐ Add and subtract fractions with like denominators. **I *can add and subtract fractions with like denominators.*.** <mark>**Reasoning Target**</mark> ☐ Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g.., by using visual fraction models and equations to represent the problem. **I *can solve word problems involving addition and subtraction of fractions by using visual fraction models and equations.***	A cake recipe calls for you to use 3/4 cup of milk, 1/4 cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake? Milk　oil　water 3/4 + 1/4 + 2/4 = 6/4 = 1 2/4

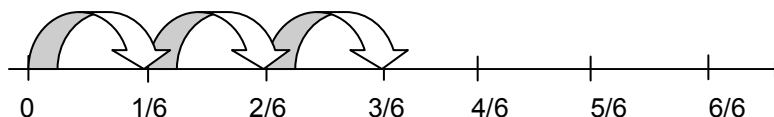| **4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**
     a. **Understand a fraction *a*/*b* as a multiple of 1/*b*. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).**

 # Knowledge Targets
 ☐ Represent a fraction a/b as a multiple of 1/b (unit fractions). For example, represent 5/4 as an accumulation of five 1/4s.
 I can write a fraction as a multiple. That means (5/4 is an accumulation of five 1/4s;
 5/4 = 5 x (1/4)).

 ☐ From the introduction: Students extend previous understandings about how fractions are built from unit fractions, using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
 I can explain that fractions are built from unit fractions using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

 <mark>Reasoning Target</mark>
 ☐ Apply multiplication of whole numbers to multiplication of a fraction by a whole number using visual fraction models. (For example, just as students know that four 3's can be represented by 4 x 3, students know that five ¼'s is 5 x ¼ which is 5/4.)
 I can compare multiplication of whole numbers to multiplication of fractions by a whole number. | This standard builds on students' work of adding fractions and extending that work into multiplication.

 Example:
 3/6 = 1/6 + 1/6 + 1/6 = 3 x (1/6)

 Number line:

 Area model:
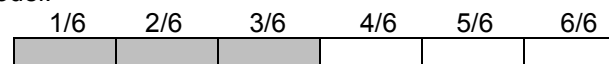
 | |
| b. **Understand a multiple of *a*/*b* as a multiple of** | This standard extended the idea of multiplication as repeated | |

1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

Knowledge Targets
☐ From the Introduction: Extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposinig fractions into unit fractions, and using the meaning for fractions and the meaning of multiplication to multiply by a whole number.
I can .

<mark>Reasoning Targets</mark>
☐ Explain that a multiple of a/b is a multiple of 1/b (unit fraction) using a visual fraction model.
I can explain that a multiple of a fraction is a multiple of the unit fraction using visual fraction model. That means, 4/5 is a multiple of 1/5.

☐ Multiply a fraction by a whole number by using the idea that a/b is a multiple of 1/b. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5) recognizing this product as (6/5).
I can multiply a fraction by a whole number using the idea that a/b is a multiple of 1/b.

addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/5 = 6/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

---
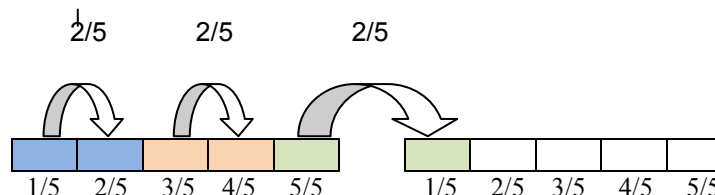
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers*

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.
Example:
    In a relay race, each runner runs . of a lap. If there are 4 team members how long is the race?

Student 1
Draws a number line shows 4 jumps of 1/2

does your answer lie?

Knowledge Targets
☐ Multiply a fraction by a whole number.
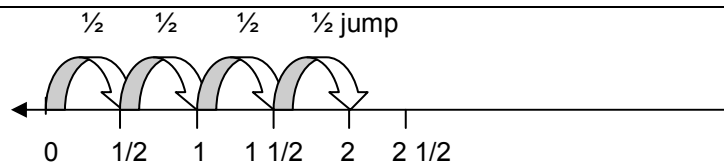*I can multiply a fraction by a whole number.*

☐ Use fraction models and equations to represent the problem.
I can use fraction models and equations to show the word problem.

☐ Solve word problems involving multiplication of a fraction by a whole number.
I can solve word problems by multiplying fractions by a whole number.
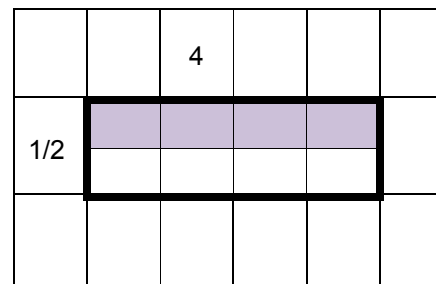
½   ½   ½   ½ jump

0   1/2   1   1 1/2   2   2 1/2

Student 2
Draws and area model showing 4 pieces of 1/2 joined together to equal 2.

1/2	1/2

1/2	1/2

Student 3

Draws and area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier.
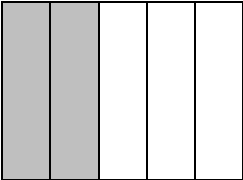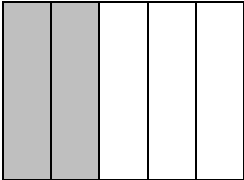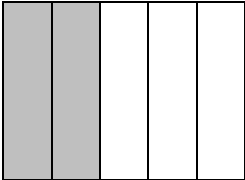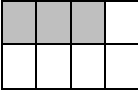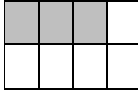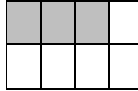

Example:
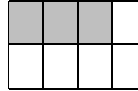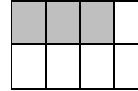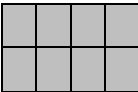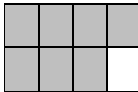Heather bought 12 plums and ate 1/3 of them. Paul bought 12 plums and ate ¼ of them. Which statement is true?

Draw a model to explain your reasoning.
   a. Heather and Paul ate the same number of plums.
   b. Heather at 4 plums and Paul ate 3 plums.
   c. Heather ate 3 plums and Paul ate 4 plums.
   d. Heather had 9 plums remaining.

Example:
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fractions lend themselves to modeling and examining patterns.

Examples:

2/5

1/5 1/5

2/5

1/5 1/5

2/5

1/5 1/5

If each person at a party eats 3/8 of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:

3/8   3/8   3/8   3/8   3/8

3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8

| **4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in** | This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms.

Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100. This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade. | |

general is not a requirement at this grade.)

Knowledge Targets
☐ Rename and recognize a fraction with a denominator of 10 as a fraction with a denominator of 100.
I can rename fractions with a denominator of 10 as a fraction with a denominator of 100.
I can recognize fractions with a denominator of 10 as a fraction with a denominator of 100.
☐ Recognize that two fractions with unlike denominators can be equivalent.
I can describe how two fractions with unlike denominators can be equivalent.

<mark>Reasoning Target</mark>
☐ Use knowledge of renaming tenths to hundredths to add two fractions with denominators 10 and 100.
I can add two fractions with denominators 10 and 100 by renaming tenths to hundredths.

Example:

Ones	.	Tenths	Hundredths

Tenths Grid   Hundredths Grid
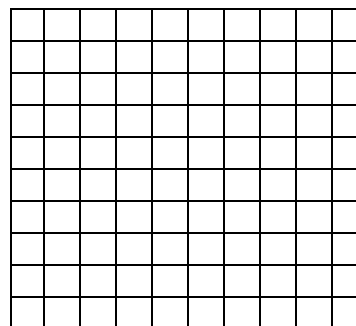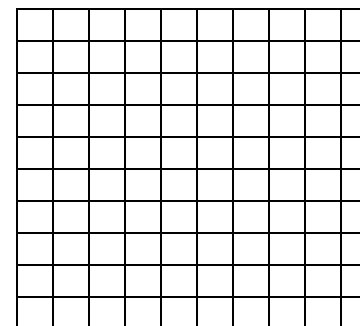
.3 = 3 tenths = 3/10   .30 = 30 hundredths = 30/100

Example:

Represent 3 tenths and 30 hundredths on the models below.
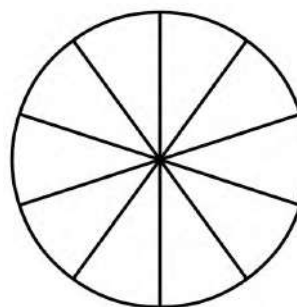
10ths circle   100ths circle
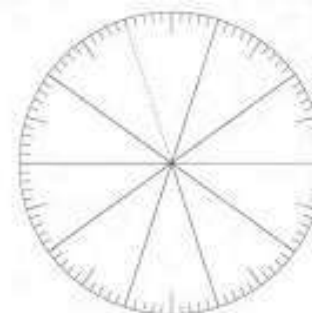
4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Knowledge Targets
☐ Explain the values of digits in the decimal places.

Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32

I can explain the values of digits in the decimal places.

☐ Read and write decimals through hundredths.
I can read and write decimals through hundredths.

☐ Rename fractions with 10 and 100 in the denominator as decimals.
I can rename fractions with 10 and 100 in the denominator as decimals.

☐ Recognize multiple representations of fractions with denominators of 10 or 100.
I can recognize multiple representations of fractions with denominators of 10 or 100.

Reasoning Targets

☐ Represent fractions with denominators 10 or 100 with multiple representations and decimal notation.
I can show fractions with denominators 10 or 100 with visual models and decimal notation.

☐ **Explain how decimals and fractions relate.**
I can explain how decimals and fractions are related.

or represent it on a place value model as shown below.

Hundreds	Tens	Ones	.	Tenths	Hundredths
			.	3	2

Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.

.32

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
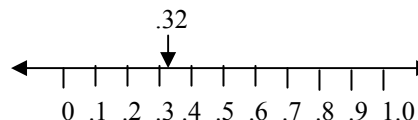
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Knowledge Targets
☐ Recognize that comparisons are valid only when the two decimals refer to the same whole.

I can tell that comparisons are correct when two decimals refer to the same whole.

<mark>Reasoning Targets</mark>
☐ Compare two decimals to hundredths by reasoning about their size.

I can compare two decimals to hundredth by thinking about their size.

☐ Record the results of comparisons with the *symbols (<, >, =)*.

I can record comparison results using symbols (<, >, =)

☐ Justify the conclusions using visual models and other methods.

I can support the conclusions of comparisons using visual models and other methods.

Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.
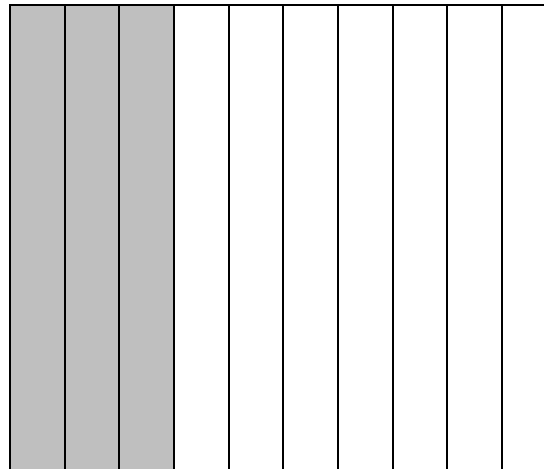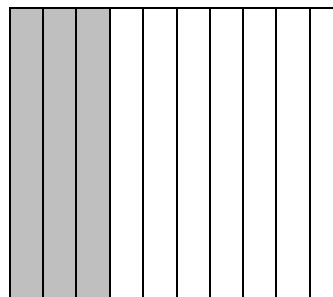
When the holes are the same, the decimals or fractions can be compared.

Example:

Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show
the area that represents three-tenths is smaller than the area that represents five-tenths.
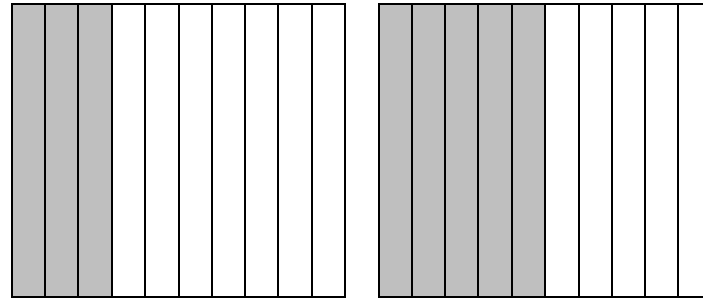
Unit: Graphing and Data

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Knowledge Targets
☐ Add and subtract fractions.
I can add and subtract fractions.

Reasoning Targets
☐ Analyze and interpret a line plot to solve problems involving addition and subtraction of fractions.
I can analyze and interpret a line plot to solve addition and subtraction problems of fractions.

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:
Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 inch. They displayed their data collected on a line plot. How many objects measured 1/4 inch? 1/2 inch? If you put all the objects together end to end what would be the total length of **all** the objects.
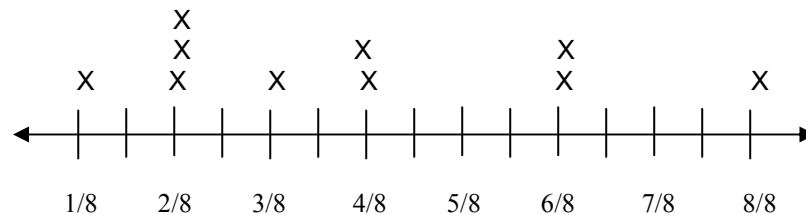
2nd Nine Weeks and 15 days for Unit Nov. 28-Dec. 16

☐ Create a line plot to display a data set of measurements given in fractions of a unit.

I can create a line plot to display a data involving measurements of fractions of a unit.

Unit: Geometry

| **4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.**

Knowledge Targets
☐ Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.
I can draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines.

Reasoning Targets
☐ Analyze two-dimensional figures to identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.
I can analyze two-dimensional figures to identify points; lines; line segments; rays; right, acute, and obtuse angles; and perpendicular and parallel lines. | This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines.
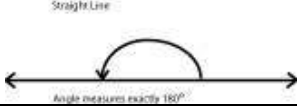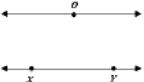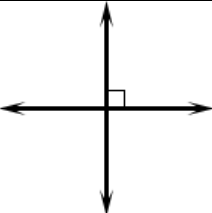
Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract.

<table><tr><td>Right angle</td><td>⌐ 90°</td></tr><tr><td>Acute angle</td><td>O, T, X</td></tr><tr><td>Obtuse angle</td><td></td></tr></table> | 3rd Nine Weeks and 19 days for Unit   Jan. 2-Jan.27 |
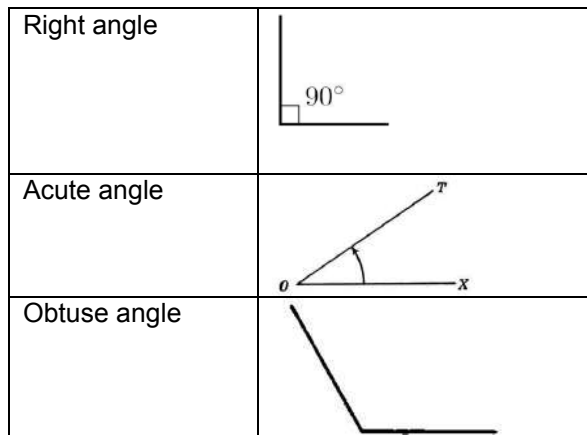
Straight angle	Straight Line Angle measures exactly 180°
Segment	A B
line	
ray	● RAY →
parallel lines	o x y
perpendicular lines	

Example:
Draw two different types of quadrilaterals that have two pairs of parallel sides?
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.

Example:
How many acute, obtuse and right angles are in this shape?

Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

| **4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.**

Knowledge Targets
☐ Identify parallel or perpendicular lines in two-dimensional figures.
I can identify parallel or perpendicular lines in two-dimensional figures.
☐ Recognize acute, obtuse and right angles.
I can recognize acute, obtuse and right angles.
☐ Identify right angles.
I can identify right angles.

==**Reasoning Targets**==
☐ Classify two-dimensional figures based on parallel or perpendicular lines and size of angles.
I can classify two-dimensional figures based on parallel or perpendicular lines and size of angles.
☐ Classify triangles as right triangles or not right.
I can classify triangles as right triangles or not right. | Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90°).

Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:
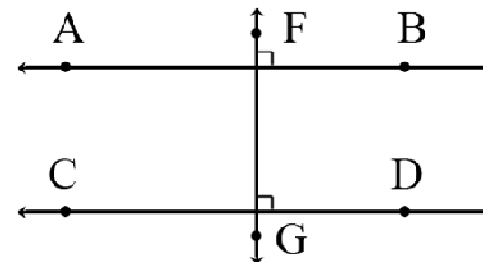
This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.
Example: | |
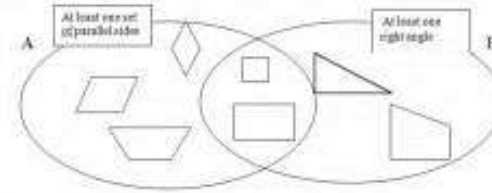
Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example.
    • A parallelogram with exactly one right angle.
    • An isosceles right triangle.
    • A rectangle that is *not* a parallelogram. *(impossible)*
    • Every square is a quadrilateral.
    • Every trapezoid is a parallelogram.

Example:
    Identify which of these shapes have perpendicular or parallel sides and justify your selection.
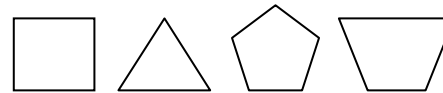
A possible justification that students might give is:
The square has perpendicular lines because the sides meet at a corner, forming right angles.

Angle Measurement:
This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1.

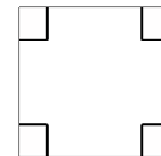|	Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.	
	Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.	
4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. ==**Knowledge Targets**== ☐ Recognize lines of symmetry for a two-dimensional figure. *I can identify lines symmetry for a two-dimensional figure.* ☐ Recognize a line of symmetry as a line across a figure that when folded along creates matching parts. *I can identify a line of symmetry as a line across a figure.* ☐ Draw lines of symmetry for two-dimensional figures. *I can draw lines of symmetry for two-dimensional figures.* ☐ **Identify line-symmetric figures.** *I can identify line-symmetric figures.*	Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry. This standard only includes line symmetry not rotational symmetry. Example: For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions. Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex. 	
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:	This standard brings up a connection between angles and circular measurement (360 degrees). The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2	

a. **An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.**

b. **An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.**

Knowledge Targets
☐ Define angle.
I can define angle.
☐ Recognize a circle as a geometric figure that has 360 degrees.
I can tell a circle as a geometric figure that has 360 degrees.
☐ Recognize and identify an angle as a geometric shape formed from 2 rays with a common endpoint.
I can identify an angle as a geometric shape formed from 2 rays with a common endpoint.
☐ Recognize that an angle is a fraction of a 360 degree circle.
I can identify an angle as a fraction of a 360 degree circle.
☐ Explain the angle measurement in terms of degrees.
I can explain the angle measurement in terms of degrees.

<mark>Reasoning Targets</mark>
☐ Compare angles to circles with the angle's point at the center of the circle to determine measure of the

rays is different for both circles yet the angle measure is the same.
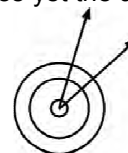
This standard calls for students to explore an angle as a series of "one-degree turns."

A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

angle.

I can compare angles to circles with the angle's point at the center of the circle to determine the measure of the angle.

☐ Calculate angle measurement using the 360 degrees of a circle.

I can calculate the angle measurement using 360 degrees of a circle.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Knowledge Targets

☐ Recognize that angles are measured in degrees (º).

I can tell that angles are measured in degrees (º).

☐ Read a protractor.

I can read a protractor.

Reasoning Targets

☐ Determine which scale on the protractor to use, based on the direction the angle is open.

I can choose which scale on the protractor to use, based on the direction the angle is open.

☐ Determine the kind of angle based on the specified measure to decide reasonableness of the sketch.

I can choose the kind of angle based on the specified measure to determine the reasonableness of the sketch.

<mark>Performance Skill Targets</mark>

☐ Measure angles in whole number degrees using a protractor.

I can measure angles in whole number degrees using a protractor.

☐ Sketch angles of specified measure.

I can sketch angles of given measure.

Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should measure angles and sketch angles.

120 degrees   135 degrees

4.MD.7 Recognize angle measure as additive.

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.

When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

KnowledgeTargets
☐ Recognize that an angle can be divided into smaller angles.
I can tell that an angle can be divided into smaller angles.

Reasoning Targets
☐ Solve addition and subtraction equations to find unknown angle measurements on a diagram.
I can solve addition and subtraction equations to find unknown angle measurements.
☐ Find an angle measure by adding the measurements of the smaller angles that make up the larger angle.
I can find an angle measure by adding the measurements of the smaller angles that make up the larger angle.
☐ Find an angle measure by subtracting the measurements of the smaller angle from the larger angle.
I can find an angle measure by subtracting the measurements of the smaller angle from larger scale.
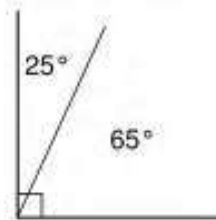
Example:
A lawn water sprinkler rotates 65 degress and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:
If the two rays are perpendicular, what is the value of m?
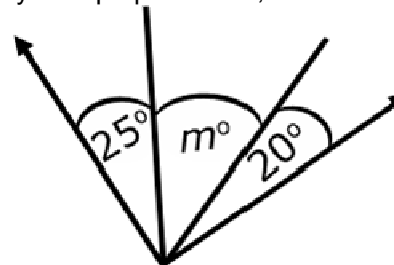
Unit: Measurement

4.MD.1 Know rela easurement	The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters,	3rd Nine Weeks and
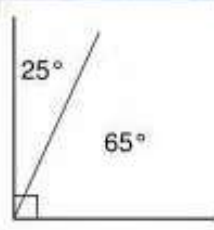
units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

Knowledge Targets

☐ Know relative size of measurement units (km, m, kg, g, lb, oz, L, mL, hrs, min, sec).

I can tell relative size of measurement units (km, m, kg, g, lb, oz, L, mL, hrs, min, sec).

<mark>Reasoning Targets</mark>

☐ Compare the different units within the same system of measurement (e.g. 1 ft = 12 in; 1 lb = 16 oz.)

I can compare different units within the same system of measurement. That means I can compare ft. and inches, pounds and ounces, etc.

☐ Convert larger units of measurement within the same system to smaller units and record conversions in a two column table.

I can convert larger units of measurement to smaller units and record conversions in a two column table.

and seconds.

Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time.

Students did not convert measurements.
Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.
Example:
Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	n + 3

Foundational understandings to help with measure concepts:
- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and The number of units needed (compensatory principle.)

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Knowledge Targets
- ☐ Add, subtract, multiply and divide fractions and decimals.
- *I can add fractions and decimals.*
- *I can subtract fractions and decimals*
- *I can multiply fractions and decimals.*
- *I can divide fractions and decimals.*
- ☐ Express measurements given in a larger unit in terms of a smaller unit.
- *I can write measurements in larger units in terms of smaller units.*

<mark>Reasoning Targets</mark>
- ☐ Solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money.
- *I can solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money.*
- ☐ Solve word problems involving measurement that includes simple fractions or decimals.
- *I can solve word problems involving measurement, including simple fractions and decimals.*
- ☐ Solve word problems that require expressing

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?

Possible Solution: Charlie plus 10 friends = 11 total people
      11 people x 8 ounces (glass of milk) = 88 total ounces
            1 quart = 2 pints = 4 cups = 32 ounces

            Therefore 1 quart = 2 pints = 4 cups = 32 ounces
            2 quarts = 4 pints = 8 cups = 64 ounces
            3 quarts = 6 pints = 12 cups = 96 ounces

If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.
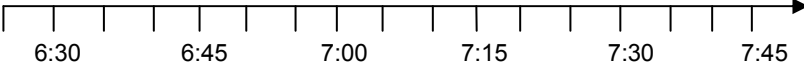
Additional Examples with various operations:
Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)
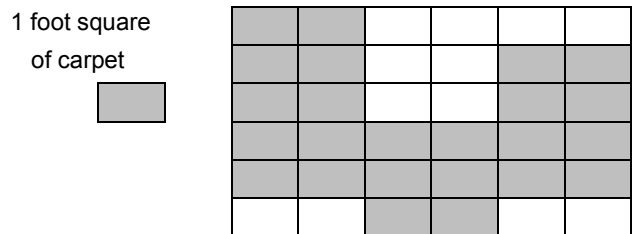
Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto

	brought 450 milliliters. How many total milliliters of lemonade did the boys have?	
measurements given in a larger unit in terms of a smaller unit. ***I can solve word problems involving measurements in larger units and write them in smaller units.*** ☐ Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. ***I can show measurement quantities using diagrams. That means I can use number line diagrams.***	Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container. Example: At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem. 6:30 6:45 7:00 7:15 7:30 7:45	
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **Knowledge Targets** ☐ Know that the formula for the perimeter of a rectangle is 2L + 2W or L+L+W+W. ***I can tell that the formula for perimeter of a rectangle. That means Perimeter = 2L + 2W or L+L+W+W.***	Students developed understanding of area and perimeter in 3rd grade by using visual models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is *l* x *w* and the answer will always be in square units. The formula for perimeter can be 2 *l* + 2 *w* or 2 (*l* + *w*) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization. Example: Mr. Rutherford is covering the miniature golf course with an artificial	

	grass. How many 1-foot squares of carpet will he need to cover the entire course?	
☐ Know that the formula for the area of a rectangle is L x W. *I can tell that the formula for area of a rectangle. That means Area = L x W* <mark>**Reasoning Targets**</mark> ☐ Apply the formula for perimeter of a rectangle to solve real world and mathematical problems. *I can use the formula for perimeter of a rectangle to solve real world and math problems.* ☐ Apply the formula for area of a rectangle to solve real world and mathematical problems. *I can use the formula for area of a rectangle to solve real world and math problems.* ☐ Solve area and perimeter problems in which there is an unknown factor (n). *I can solve area and perimeter problems where there is an unknown. That means a missing value.*	1 foot square of carpet	

Unit: Patterns and Algebra		
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental	The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations. Example: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? Some typical estimation strategies for this problem:	3rd Nine Weeks and 24 days for Unit Feb. 27-Mar. 30

computation and estimation strategies including rounding.

Knowledge Target
- ☐ Divide whole numbers including division with remainders.

I can divide whole numbers with and without remainders.

<mark>**Reasoning Target**</mark>
- ☐ **Represent multi-step word problems using equations with a letter standing for an unknown quantity.**

I can write equations to represent multi-step word problems with variables. That means a letter stands for the unknown quantity or number.

- ☐ Interpret multi-step word problems (including problems in which remainders must be interpreted) and determine the appropriate operation(s) to solve.

I can interpret multi-step word problems (with and without remainders) and choose the correct operation to solve.

- ☐ Assess the reasonableness of an answer in solving a multi-step word problem using mental math and estimation strategies (including rounding).

I can determine whether my answer is reasonable using mental math, estimation strategies and rounding.

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Examples continued on the next page.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50	First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40.

plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	40+20=60. 300-60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation.
ways to address remainders:
- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving 44 ÷ 6 = ? where the answers are best represented as:
Problem A: 7
Problem B: 7 r 2
Problem C: 8
Problem D: 7 or 8
Problem E: 7
6
2

Possible solutions:
Problem A: 7. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? *44 ÷ 6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

Problem B: 7 r 2. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? *44 ÷ 6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

Problem C: 8. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? *44 ÷ 6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

Problem D: 7 or 8. Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? *44 ÷ 6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends*

	received 8 pencils.
	Problem E: 7 2/6 Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ 6 = *p*; *p* = 7 2/6
	Example: There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*
	Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over.
	Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end, • making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and • multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on • how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., • rounding to factors and grouping numbers together that have round sums like 100 or 1000), • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or • decimals to determine an estimate).
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the*	Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce

starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Knowledge Target
- ☐ Identify a number or shape pattern.
- *I can identify number or shape patterns.*

<mark>**Reasoning Targets**</mark>
- ☐ Generate a number or shape pattern that follows a given rule.
- *I can create a number or shape pattern that follows a given rule.*
- ☐ Analyze a pattern to determine features not apparent in the rule (always odd or even, alternates between odd and even, etc.)
- *I can analyze a pattern to determine features not apparent in the rule (That means always odd, or even, alternates between odd and even, etc.)*

facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28, …	Start with 3, add 5	The numbers alternately end with a 3 or 8
5, 10, 15, 20 …	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers. Students write 1, 3, 9, 27, 8, 243.

Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, etc.)

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	Operation	Beans
0	3 x 0 + 4	4
1	3 x 1 + 4	7
2	3 x 2 + 4	10
3	3 x 3 + 4	13
4	3 x 4 + 4	16
5	3 x 4 + 5	17

Unit: Maintenance, Testing, Snow Day Makeup

| All standards | | 4th Nine Weeks Apr. 9-May 24 |

Some examples used in this document are from the Arizona Mathematics Education Department

Standards	Explanations and Examples
Students are expected to:	
4.MP.1. Make sense of problems and persevere in solving them.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
4.MP.2. Reason abstractly and quantitatively.	Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
4.MP.3. Construct viable arguments and critique the reasoning of others.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

Standards	Explanations and Examples
Students are expected to:	
4.MP.5. Use appropriate tools strategically.	Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
4.MP.6. Attend to precision.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
4.MP.7. Look for and make use of structure.	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
4.MP.8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Math Accountable Talk

Teach students to use one of the following when discussing each other's math work.

I agree with _____ because _____.

I like the way _____ used _____ because as his/her reader, it helps me _____.

I disagree with _____ because _____.

I got a different answer than _____ because _____.

I can add to _____'s thoughts: _____

I got the same answer as _____ but my strategy was different.

I have a question for _____.

I don't understand why _____ got the answer of _____ because _____.

Glossary

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[2]
Put together/Take apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$

		5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2 Common multiplication and division situations[1]

	Unknown Product **3 x 6 = ?**	**Group Size Unknown** **("How many in each group?"** **Division)** **3 x ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** **("How many groups?"** **Division)** **? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether.	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[2] **Area**[3]	There are 3 rows of apples with 6 apples in each row. How man apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has an area 18 square centimeters. If one	If 18 apples are arranged into 3 equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has an area 18 square centimeters. If one

Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	side is 3 cm long, how long is a side next to it? A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost? *Measurement example.* A rubber band is 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	side is 6 cm long, how long is a side next to it? A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band is 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p and p ÷ b = ?

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3 The properties of operation

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$

Multiplicative identity property of 1	*a x 1 = 1 x a = a*
Distributive property of multiplication over addition	*a x (b + c) = a x b + a x c*

REFERENCES

Burns, M. (2000). *About teaching mathematics*. White Plains, NY: Math Solutions.

Fosnot, C., Dolk, M. (2001). *Young mathematicians at work: constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.

Richardson, K. (2002). *Assessing math concepts: hiding assessment.* Bellingham, WA: Math Perspectives.

Van de Walle, J., Lovin, L. (2006). *Teaching student-centered mathematics*. Boston: Pearson.