fourth
GRADE

Fractions and Decimals
MATH IN FOCUS

Unit 2 Curriculum Guide
November 12th – February 1st

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

Unit 2: Chapters 6-7

Eureka Module 5:

Fractions Equivalence, Ordering, and Operations
(TOPICS B, C, D ONLY)

In this Unit Students will:

- Add and subtract unlike fractions, rename improper fractions and mixed numbers, rename whole numbers when adding and subtracting fractions, find a fraction of a set, display data involving fractions of a unit in a line plot, solve problems using a line plot
- Read, write, and express decimals in expanded form, place value of decimals, patterns of decimals, compare and order decimals, round decimals, convert fractions to decimals and decimals to fractions

Essential Questions

- How can a fraction look different but still be the same?
- How can you use multiplication to find equivalent fractions?
- How can you write a pair of fractions as fractions with a common denominator?
- How can you use benchmarks to compare fractions?
- How can you compare fractions?
- How can you order fractions?
- How can I use models to help compare fractions?
- What patterns do you notice among numerators and denominators of equivalent fractions?
- How do fractions represent parts of a whole?
- How can you use fraction strips to add fractions?
- How can you add fractions with like denominators?
- How can you use fraction strips to subtract fractions?
- How do you subtract fractions with like denominators?
- How can you use a number line to add and subtract fractions?
- How are mixed numbers and improper fractions related?
- How do you use models to add mixed numbers?
- How do you add mixed numbers?
- How can we use addition to represent a fraction in a variety of ways?
- How can you describe a fraction using a unit fraction?
- How can you find the product of a fraction multiplied by a whole number?
- How can you locate points for decimals on a number line?
- How can you use equivalent fractions to change a fraction to a decimal?
- What are some ways to represent decimals?
- How do you compare decimals?
- How are decimals related to money?
- How can you draw a picture to solve a problem?

Enduring Understandings

- Use comparing, ordering, and equivalent fractions to extend understanding of fractions.
- Fractions can be represented visually and in written form.
- Comparisons are only valid when the two fractions refer to the same whole.
- Fractions and mixed numbers are composed of unit fractions and can be decomposed as a sum of unit fractions.
- Improper fractions and mixed numbers express the same value.
- Using students’ previous knowledge of the properties of whole numbers in addition and subtraction will aid in teaching of addition and subtractions of fractions.
- Addition and subtraction of fractions involves joining and separating parts referring to the same whole.
- Multiplying a fraction by a whole number is a logical step after multiplication of whole numbers.
- A product of a fractions times a whole number can be written as a multiple of a unit fraction.
- Decimal notation is another way to represent a fraction.
- Fractions with denominators of 10 can be expressed as an equivalent fraction with a denominator of 100.
- Fractions with denominators of 10 and 100 may be expressed when using decimal notation.
- When comparing two decimals to hundredths, the comparisons are only valid if they refer to the same whole.

MIF Pacing Guide

MIF Chapter 6-7 & Eureka Math Module 5 (TOPICS B, C, and D)			
Activity	**Common Core Standards**	**Estimated Time (# of block)**	**Lesson Notes**
6.1 Adding Fractions	4.NF.1, 4.NF.3a	1 Block	You may want to use Fraction Strips (TRD28) or Fraction Circles(TRD29) with small groups to model reviewing equivalent fractions, before teaching this lesson.
6.2 Subtracting Fractions	4.NF.1, 4.NF.3a	1 Block	Throughout this lesson have students identify similarities and differences between the addition and subtraction of fractions. Guide students to recognize that the procedures are similar because both require common denominators.
6.3 Mixed Numbers Day 1	4.MD.1, 4.NF.3a	1 Block	For Hands-On-Activities, you will need to make copies of (TRD29). Advanced learners can use Fraction Circles (TR29).
6.3 Mixed Numbers Day 2	4.MD.1, 4.NF.3a	1 Block	You may wish to review finding common factors, taught in Lesson 2.2, before teaching students how to simplify fractions.
6.4 Improper Fractions Day 1	4.NF.3a-b, 4.NF.4a	1 Block	Point out that a fraction with a numerator less than its denominator is now as a proper fraction. Throughout the lesson have students identify examples of proper fractions, mixed numbers, and improper fractions, and explain how they arrived at their answers.
6.4 Improper Fractions Day 2	4.NF.3a-b, 4.NF.4a	1 Block	
6.5 Renaming Improper Fractions and Mixed Numbers Day 1	4.NF.3b, 4.NF.4a	1 Block	As students rename improper fractions and mixed numbers, encourage them to check that the denominator in the mixed number is always the same as the denominator in the improper fraction before they simplify the

			fraction.
6.5 Renaming Improper Fractions and Mixed Numbers Day 2	4.NF.3b, 4.NF.4a	1 Block	
6.6 Renaming Whole Numbers When Adding and Subtracting Fractions Day 1	4.NF.1, 4.NF.3a, 4.NF.3c	1 Block	Throughout this lesson, students rename whole numbers with different denominators. Before beginning the lesson, provide opportunities for students to name wholes in a variety of ways using different denominators. For example, rename 1 whole with denominators of 4, 6, and 8.
6.6 Renaming Whole Numbers When Adding and Subtracting Fractions Day 2	4.NF.1, 4.NF.3a, 4.NF.3c	1 Block	
6.7 Fraction of a Set Day 1	4.nf.4.b-c	1 Block	Direct students to look for similarities and differences as they find fractional parts of sets and numbers. Guide students to understand that fractional parts of sets or numbers are always less than the original set or number.
6.7 Fraction of a Set Day 2	4.nf.4.b-c	1 Block	
6.8 Real -World Problems: Fractions Day 1	4.NF.3d, 4.NF.4c, 4.OA.2	1 Block	
6.8 Real -World Problems: Fractions Day 2	4.NF.3d, 4.NF.4c, 4.OA.2	1 Block	For additional practice in this lesson, after completing each Guided Learning with the class, have students replace the numbers in each problem with new numbers following the same procedure.
6.9 Line Plots with Fractions of a Unit	4.MD.4	1 Block	Students may forget to label the number lines. Remind them that the purpose of a graph is to clearly communicate data. If the label is missing, people will not know what data is shown.
Authentic Assessment #5 Chocolate Bar Fractions	4.NF.2, 4.NF.4	1/2 Block	
Chapter Test/Performance Task	4.NF.1, 4.NF.3a-d, 4.NF.4a-c, 4.MD.1, 4.MD.4. 4.OA.2	1/2 Block	

Eureka Math Module 5 (TOPICS B, C, and D)

Topic B: Fraction Equivalence using Multiplication and Division	Lesson 7	Use the area model and multiplication to show the equivalence of 2 fractions. https://www.youtube.com/watch?v
	Lesson 8	Use the area model and multiplication to show the equivalence of 2 fractions. https://www.youtube.com/watch?v
	Lesson 9	Use the area model and division to show the equivalence of 2 fractions. https://www.youtube.com/watch?v
	Lesson 10	Use the area model and division to show the equivalence of 2 fractions. https://www.youtube.com/watch?v
	Lesson 11	Explain fraction equivalence using a tape diagram and the number line and relate that to the use of multiplication and division. https://www.youtube.com/watch?v
Topic C: Fraction Comparison	Lesson 12	Reason using benchmarks to compare two fractions on the number line. https://www.youtube.com/watch?v
	Lesson 13	Reason using benchmarks to compare two fractions on the number line. https://www.youtube.com/watch?v
	Lesson 14	Find common units or number of units to compare two fractions. https://www.youtube.com/watch?v
	Lesson 15	Find common units or number of units to compare two fractions. https://www.youtube.com/watch?v

Topic D: Fraction Addition and Subtraction	Lesson 16	Use visual models to add and subtract two fractions with the same units. https://www.youtube.com/watch?v
	Lesson 17	Use visual models to add and subtract two fractions with the same units, including subtracting from one whole. https://www.youtube.com/watch?v
	Lesson 18	Add and subtract more than two fractions. https://www.youtube.com/watch?v
	Lesson 19	Solve word problems involving addition and subtraction of fractions. https://www.youtube.com/watch?v
	Lesson 20	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. https://www.youtube.com/watch?v
	Lesson 21	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. https://www.youtube.com/watch?v
Authentic Assessment #6 Raising Money	4.NF.2, 4.NF.4	1/2 Block

7.1 Understanding Tenths Day 1a (Broken down into two days/blocks)	4.NF.5, 4.NF.6	1 Block	7.1 Day 1a Teach reading and writing tenths in decimal and fractional forms.
7.1 Understanding Tenths Day 1b	4.NF.5, 4.NF.6	1 Block	7.1b Teach representing and interpreting tenths models.
7.1 Understanding Tenths Day 2	4.NF.5, 4.NF.6	1 Block	Some students may have difficulty changing mixed numbers to decimals. Remind students that a mixed number consists of two parts-a whole number and a fraction. Similarly, a decimal consists of two parts-a whole number and a decimal fraction.
7.2 Understanding Hundredths Day 1a (Broken down into two days)	4.NF.5, 4.NF.6	1 Block	Teach reading and writing hundredths in decimal and fractional forms.

7.2 Understanding Hundredths Day 1b	4.NF.5, 4.NF.6	1 Block	Teach representing and interpreting hundredths models.
7.2 Understanding Hundredths Day 2	4.NF.5, 4.NF.6	1 Block	
7.2 Understanding Hundredths Day 3	4.NF.5, 4.NF.6	1 Block	Students may have difficulty assigning value to unlabeled marks on the number lines. Remind students to use the numbers already provided to determine what each mark represents.
7.3 Comparing Decimals Day 1a (Broken down into two days)	4.NF.7, 4.OA.5	1 Block	Teach comparing decimals, tenths and hundredths, example .34 to .5.
7.3 Comparing Decimals Day 1b	4.NF.7, 4.OA.5	1 Block	
7.5 Fractions and Decimals	4.NF.1, 4.NF.3a, 4.NF.6, 4.NF.7	1 Block	Tenths and hundredths.

Common Core State Standards

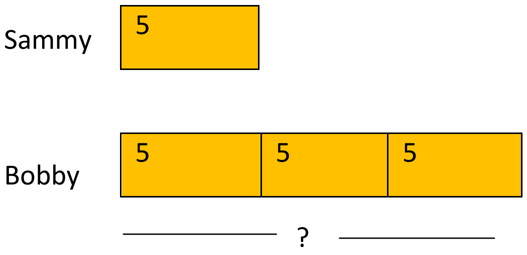
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.
Additive comparisons focus on the difference between two quantities. Multiplicative comparisons focus on comparing two quantities when one is a specified number of times greater or less than the given quantity. \

Product Unknown: ($3 \times 5 = t$)

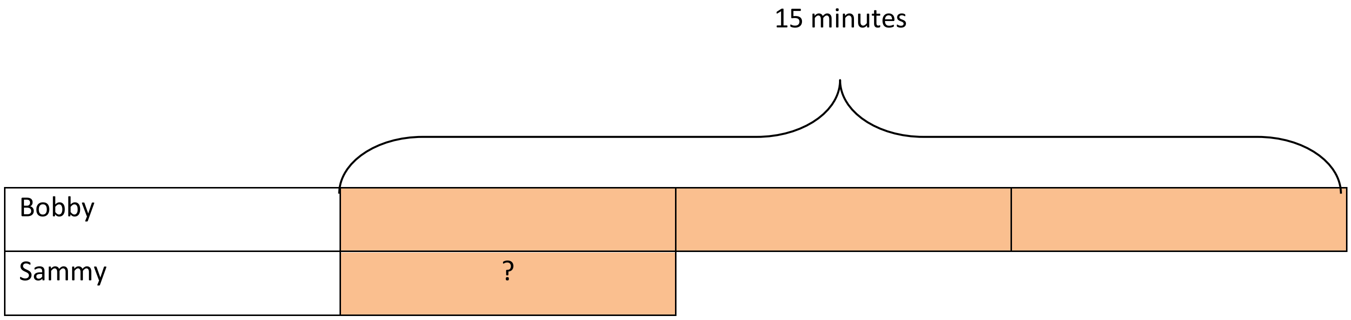
It takes Sammy 5 minutes to wash the dishes. It takes his little brother Bobby 3 times as long. How long does it take Bobby to wash the dishes?

Think: 5 minutes 3 times would be?



Factor Unknown (size of each group unknown) 3 x m = 15

It takes Bobby 15 minutes to wash the dishes. That is three times as long as it takes his brother Sammy. How long does it take Sammy to wash the dishes?



4.OA.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:
Pattern Rule Feature(s)
3, 8, 13, 18, 23, 28, … Start with 3, add 5 The numbers alternately end with a 3 or 8

5, 10, 15, 20 …
Start with 5, add 5 The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number.
The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

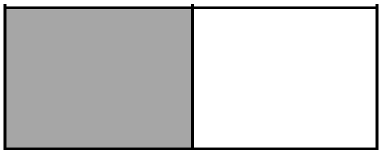
4.NF.1

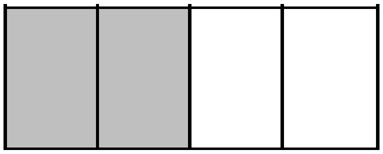
Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

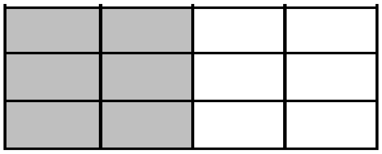
This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators. (5, 10, 12 and 100)

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example:







½ = 2/4 = 6/12

Students should begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.

4.NF.3a-d

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$

a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b) Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples:

$$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$$

$$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$$

c) Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d) Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

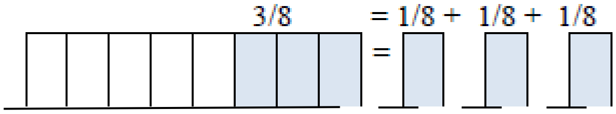
A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.
Example: 2/3 = 1/3 + 1/3
Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

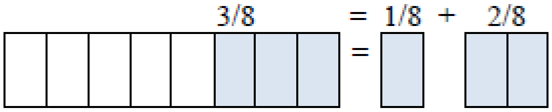
Example of word problem:
Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?

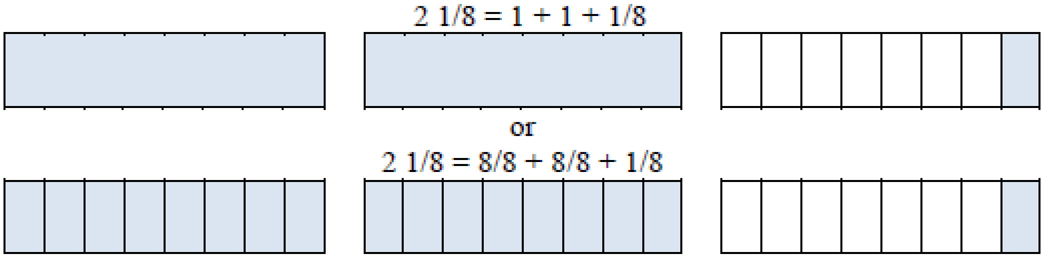
Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole.

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

Example: pizza







Similarly, converting an improper fraction to a mixed number is a matter of decomposing the fraction into a sum of a whole number and a number less than 1. Students can draw on their knowledge from third grade of whole numbers as fractions.
Example, knowing that 1 = 3/3, they see: $\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$

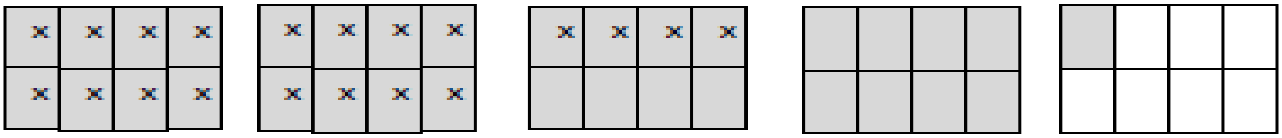
A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:
Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot.

Example:
Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend?
Possible solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend

which is 13/8 or 1 5/8 pizzas.



Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator.

Student 1

$$2+2=5\ and\ \frac{3}{4}+\frac{1}{4}=1\ so\ 5+1=6$$

Student 2

$$3\frac{3}{4}+2=5\frac{3}{4}\ so\ 5\frac{3}{4}+\frac{1}{4}=6$$

Student 3

$$3\frac{3}{4}=\frac{15}{4}\ and\ 2\frac{1}{4}=\frac{9}{4}\ so\ \frac{15}{4}+\frac{9}{4}=\frac{24}{4}=6$$

Fourth Grade students should be able to decompose and compose fractions with the same denominator.

Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract 5/6 from 17/6, they decompose.

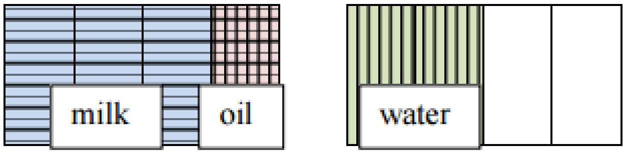
$$\frac{12}{6}+\frac{5}{6}, \quad \text{so} \quad \frac{17}{6}-\frac{5}{6}=\frac{17-5}{6}=\frac{12}{6}=2$$

Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalent fraction with the same denominator as the fraction. Example:

$$7\frac{1}{5} = 7 + \frac{1}{5} = \frac{35}{5} + \frac{1}{5} = \frac{36}{5}.$$

Students use this method to add mixed numbers with like denominators. Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition.

A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?



3/4 + 1/4 + 2/4 = 6/4 = 1 2/4

4.NF.4a-c

a) Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

b) Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)

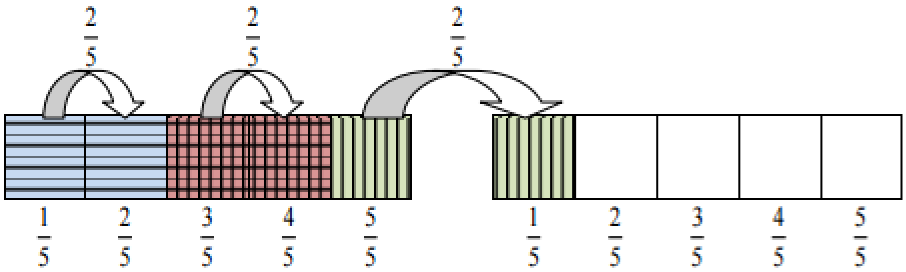
c) Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Students should see a fraction as the numerator times the unit fraction with the same denominator.

Example:

$$\frac{7}{5} = 7 \times \frac{1}{5}, \qquad \frac{11}{3} = 11 \times \frac{1}{3}.$$

This standard extended the idea of multiplication as repeated addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/5 = 6/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

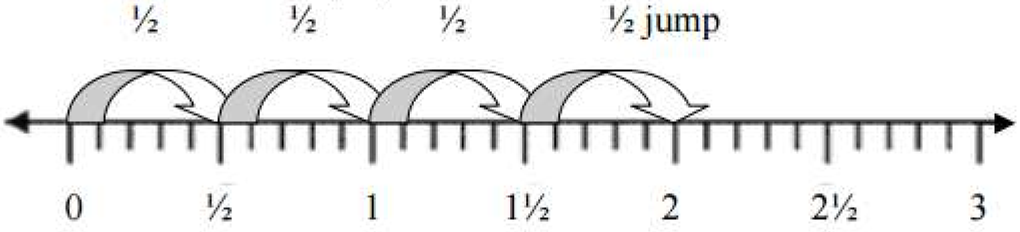


The same thinking, based on the analogy between fractions and whole numbers, allows students to give meaning to the product of whole number and a fraction. Example:

$$3 \times \frac{2}{5} \quad \text{as} \quad \frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{3 \times 2}{5}=\frac{6}{5}$$

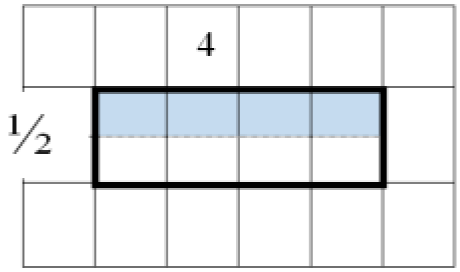
When introducing this standard make sure student use visual fraction models to solve word problems related to multiplying a whole number by a fraction. Example: In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

Student 1
Draws a number line shows 4 jumps of ½



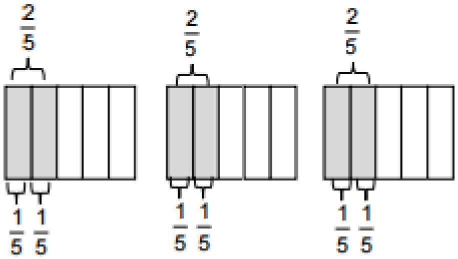
Student 2 Draws an area model showing 4 pieces of ½ joined together to equal 2.

Student 3 Draws an area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier

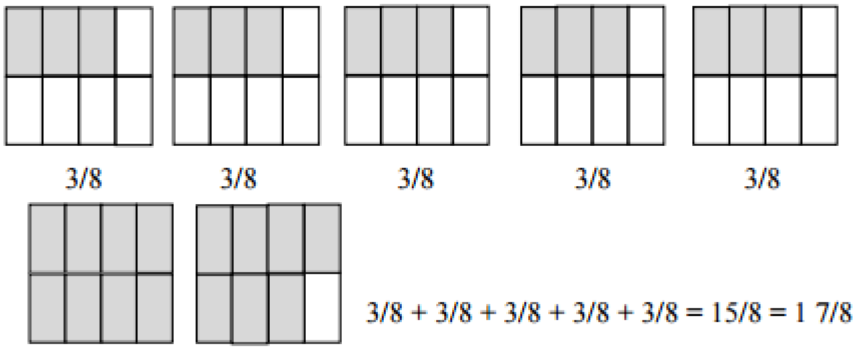


Example: Heather bought 12 plums and ate 3 1 of them. Paul bought 12 plums and ate 4 1 of them. Which statement is true? Draw a model to explain your reasoning. a. Heather and Paul ate the same number of plums. b. Heather ate 4 plums and Paul ate 3 plums. c. Heather ate 3 plums and Paul ate 4 plums. d. Heather had 9 plums remaining. Example: Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

Examples: 3 x (2/5) = 6 x (1/5) = 6/5



If each person at a party eats 3/8 of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie? A student may build a fraction model to represent this problem:



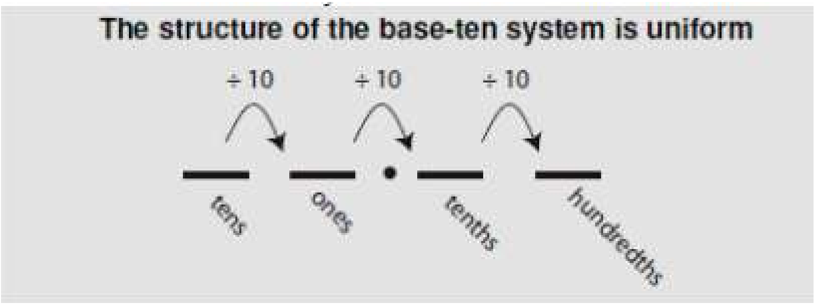
3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8

4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*.

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms.

Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

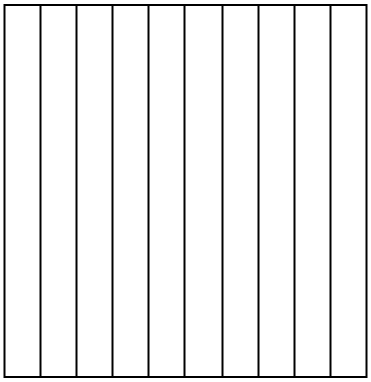
Students in fourth grade work with fractions having denominators 10 and 100. Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers.



This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.
Example:

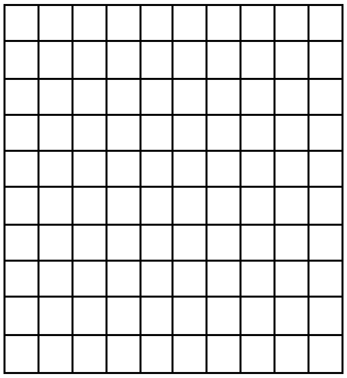
Ones	.	Tenths	Hundredths

Tenths Grid



.3 = 3 tenths = 3/10

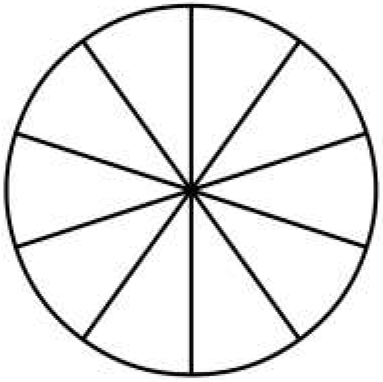
Hundredths Grid



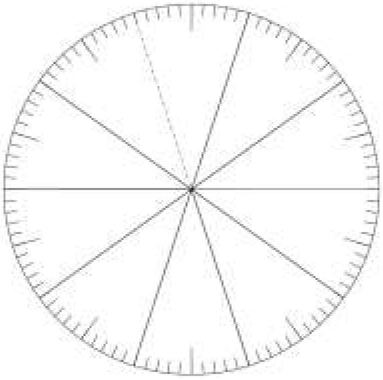
.30 = 30 hundredths = 30/100

Example: Represent 3 tenths and 30 hundredths on the models below.

10^{th}s circle



100^{th}s circle

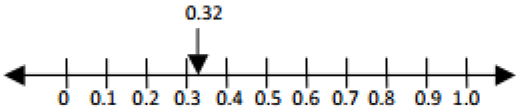


4.NF.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4 ,5, 6, 8, 10, 12, and 100.

Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.
Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.

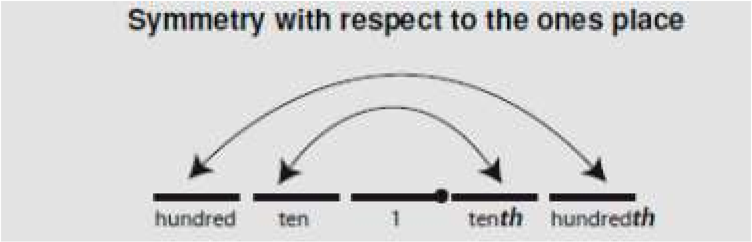


4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

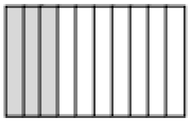
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

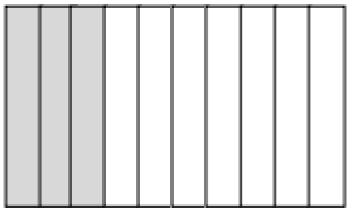
The decimal point is used to signify the location of the ones place, but its location may suggest there should be a “oneths" place to its right in order to create symmetry with respect to the decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place. Ways of reading decimals aloud vary. Mathematicians and scientists often read 0.15 aloud as “zero point one five" or “point one five." (Decimals smaller than one may be written with or without a zero before the decimal point.)Decimals with many non-zero digits are more easily read aloud in this manner. (For example, the number π, which has infinitely many non-zero digits, begins 3.1415 . ..)

Other ways to read 0.15 aloud are “1 tenth and 5 hundredths” and “15 hundredths,” just as 1,500 is sometimes read “15 hundred” or “1 thousand, 5 hundred.” Similarly, 150 is read “one hundred and fifty” or “a hundred fifty” and understood as 15 tens, as 10 tens and 5 tens, and as 100 + 50. Just as 15 is understood as 15 ones and as 1 ten and 5 ones in computations with whole numbers, 0.15 is viewed as 15 hundredths and as 1 tenth and 5 hundredths in computations with decimals. It takes time to develop understanding and fluency with the different forms. Layered cards for decimals can help students become fluent with decimal equivalencies such as three tenths is thirty hundredths.

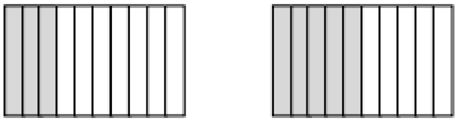


Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.





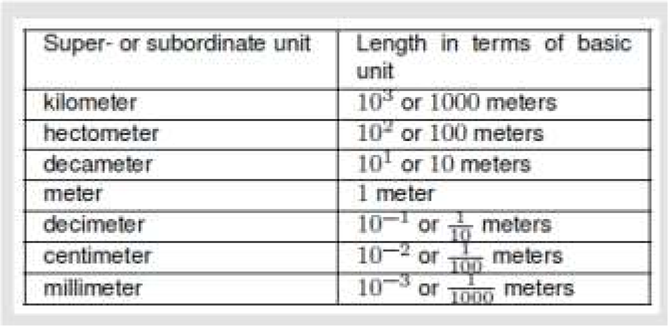
When the wholes are the same, the decimals or fractions can be compared. Example: Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.



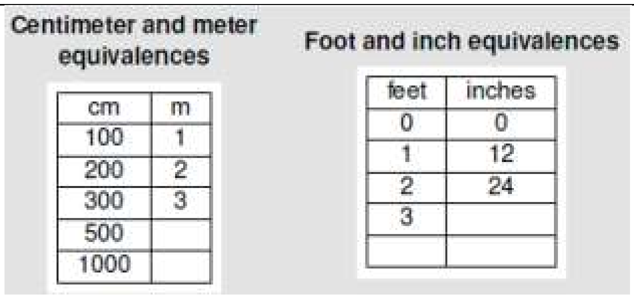
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, millimeter, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students develop benchmarks and mental images about a meter (e.g., about the height of a tall chair) and a kilometer (e.g., the length of 10 football fields including the end zones, or the distance a person might walk in about 12 minutes), and they also understand that "kilo" means a thousand, so 3000 m is equivalent to 3 km.

Expressing larger measurements in smaller units within the metric system is an opportunity to reinforce notions of place value. There are prefixes for multiples of the basic unit (meter or gram), although only a few (kilo-, centi-, and milli-) are in common use. Tables such as the one below are an opportunity to develop or reinforce place value concepts and skills in measurement activities. Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters. Relating units within the traditional system provides an opportunity to engage in mathematical practices, especially "look for and make use of structure" and "look for and express regularity in repeated reasoning" For example, students might make a table that shows measurements of the same lengths in feet and inches.



Super- or subordinate unit	Length in terms of basic unit
kilometer	10^3 or 1000 meters
hectometer	10^2 or 100 meters
decameter	10^1 or 10 meters
meter	1 meter
decimeter	10^{-1} or $\frac{1}{10}$ meters
centimeter	10^{-2} or $\frac{1}{100}$ meters
millimeter	10^{-3} or $\frac{1}{1000}$ meters



Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	
1000	

Foot and inch equivalences

feet	inches
0	0
1	12
2	24
3	

Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12. Example: Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	***n*** x 3

Foundational understandings to help with measure concepts:
Understand that larger units can be subdivided into equivalent units (partition).
Understand that the same unit can be repeated to determine the measure (iteration).
Understand the relationship between the size of a unit and the number of units needed (compensatory principal).
These Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity).

4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, and dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?

possible solution: Charlie plus 10 friends = 11 total people
11 people x 8 ounces (glass of milk) = 88 total ounces
1 quart = 2 pints = 4 cups = 32 ounces
Therefore 1 quart = 2 pints = 4 cups = 32 ounces
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces
If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional Examples with various operations:
Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?
Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)

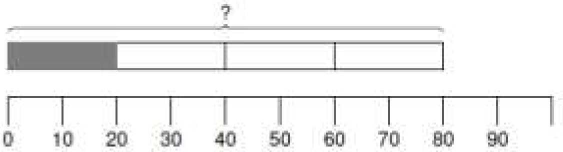
Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of the container.

Juan spent ¼ of his money on a game. The game cost $20. How much money did he have at first?

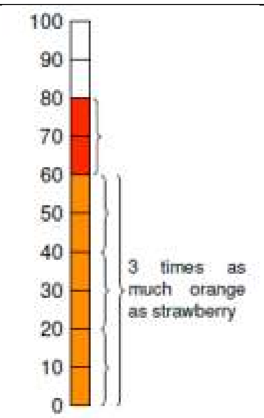


What time does Maria have to leave to be at her friend's house by a quarter after 3 if the trip takes 90 minutes?

Using a number line diagram to represent time is easier if students think of digital clocks rahter than round clocks. In the latter case, placing the numbers on the number line involves considering movements of the an minute hands.

Students also combine competencies from different domains as they solve measurement problems using all four arithmetic operations, addition, subtraction, multiplication, and division. Example: "How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?" Students may use tape or number line diagrams for solving such problems. Example:

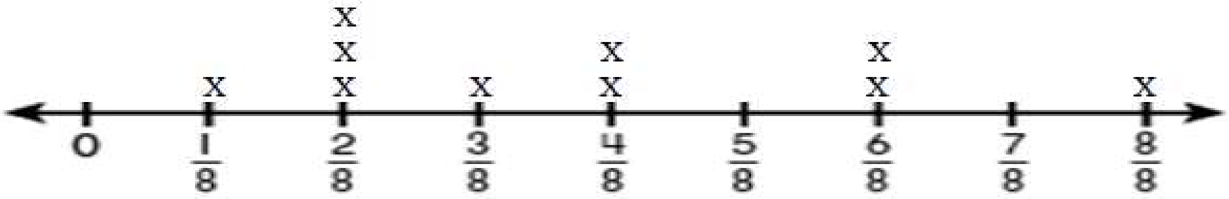
Lisa put two flavors of soda in a glass. There were 80 ml of soda in all. She put three times as much orange drink as strawberry. How many ml of orange did she put in?



Example: At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*.

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.
Example:
Students measured objects in their desk to the nearest 1/2, 1/4 or1/8 inch. They displayed their data collected on a line plot. How many objects measured an inch? If you put all the objects together end to end what would be the total length of all the objects?



M : Major Content S: Supporting Content A : Additional Content

MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	**Chapter Opener** Assessing Prior Knowledge *The Pre Test serves as a diagnostic test of readiness of the upcoming chapter*	**Teacher Materials** Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge **Student Materials** Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge	Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper Quick Check and the Pre Test can be done in the same block *(See Anecdotal Checklist; Transition Guide)* Recall Prior Knowledge – Quick Check – Pre Test
DIRECT ENGAGEMENT	**Direct Involvement/Engagement** Teach/Learn *Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions*	**Teacher Edition** 5-minute warm up Teach; Anchor Task **Technology** Digi **Other** Fluency Practice	• The Warm Up activates prior knowledge for each new lesson • Student Books are CLOSED; Big Book is used in Gr. K • Teacher led; Whole group • Students use concrete manipulatives to explore concepts • A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning • Teacher facilitates; Students find the solution
GUIDED LEARNING	**Guided Learning and Practice** Guided Learning	**Teacher Edition** Learn **Technology** Digi **Student Book** Guided Learning Pages Hands-on Activity	Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking **Small Group w/Teacher circulating among groups** Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	**Independent Practice** *A formal formative assessment*	**Teacher Edition** Let's Practice **Student Book** Let's Practice **Differentiation Options** All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	**Let's Practice** determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives **CAN** be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
ADDITIONAL PRACTICE	**Extending the Lesson**	Math Journal Problem of the Lesson Interactivities Games	
	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	**End of Chapter Wrap Up and Post Test**	**Teacher Edition** Chapter Review/Test Put on Your Thinking Cap **Student Workbook** Put on Your Thinking Cap **Assessment Book** Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed • Individually (e.g. for homework) then reviewed in class • As a 'mock test' done in class and doesn't count • As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.

Unit 2 Math Background

During their elementary mathematics education, students were exposed to the following:

- Understand the meanings and uses of fractions, not including fraction of a set.
- Understand that the size of a fractional part is relative to the size of the whole.
- Compare fractions using models, and number lines.
- Recognize equivalent fractions through the use of models, multiplication, division, and number lines.
- Write whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
- Use the dollar sign and decimal point in money amounts.
- Add and subtract like fractions.

In this unit, the students extend their learning to the following:

- Recognize, write, name and illustrate mixed numbers and improper fractions in various forms.
- Find a fraction of a set.
- Generate equivalent fractions.
- Compare fractions by creating common denominators or numerators, or by comparing with benchmark fractions. Use <, >, = symbols.
- Convert mixed numbers and improper fractions.
- Model decimals using tenths and hundredths.
- Understand decimal notation through hundredths as an extension of the base-ten system.
- Read and write decimals that are greater than or less than 1.
- Compare and order decimals.
- Identify equivalent decimals.
- Identify equivalent fractions and decimals.
- Add and subtract unlike fractions.

Potential Student Misconceptions

Chapter 6

- When adding fractions students may try to add both the numerators and the denominators
- When solving a real-world problem, for which a difference is required, student may not know whether to express the difference as 1/2- 3/8 or 3/8 –1/2.
- Students may order the improper fractions based on the numerators, thus writing 24/12 as the last number on the number line.
- Students may multiply the numerator by the whole number and then add the denominator when renaming mixed numbers as improper fractions.
- Students may not model finding the fractional part of each number correctly.
- Students often choose the wrong operation when solving real-world problems.
- Students may forget to label number lines.

Chapter 7

- Students may have difficulty assigning value to unlabeled marks on the number lines.
- Students may have difficulty differentiating > from <.
- Students may choose the incorrect rounding place.

PARCC Assessment Evidence/Clarification Statements

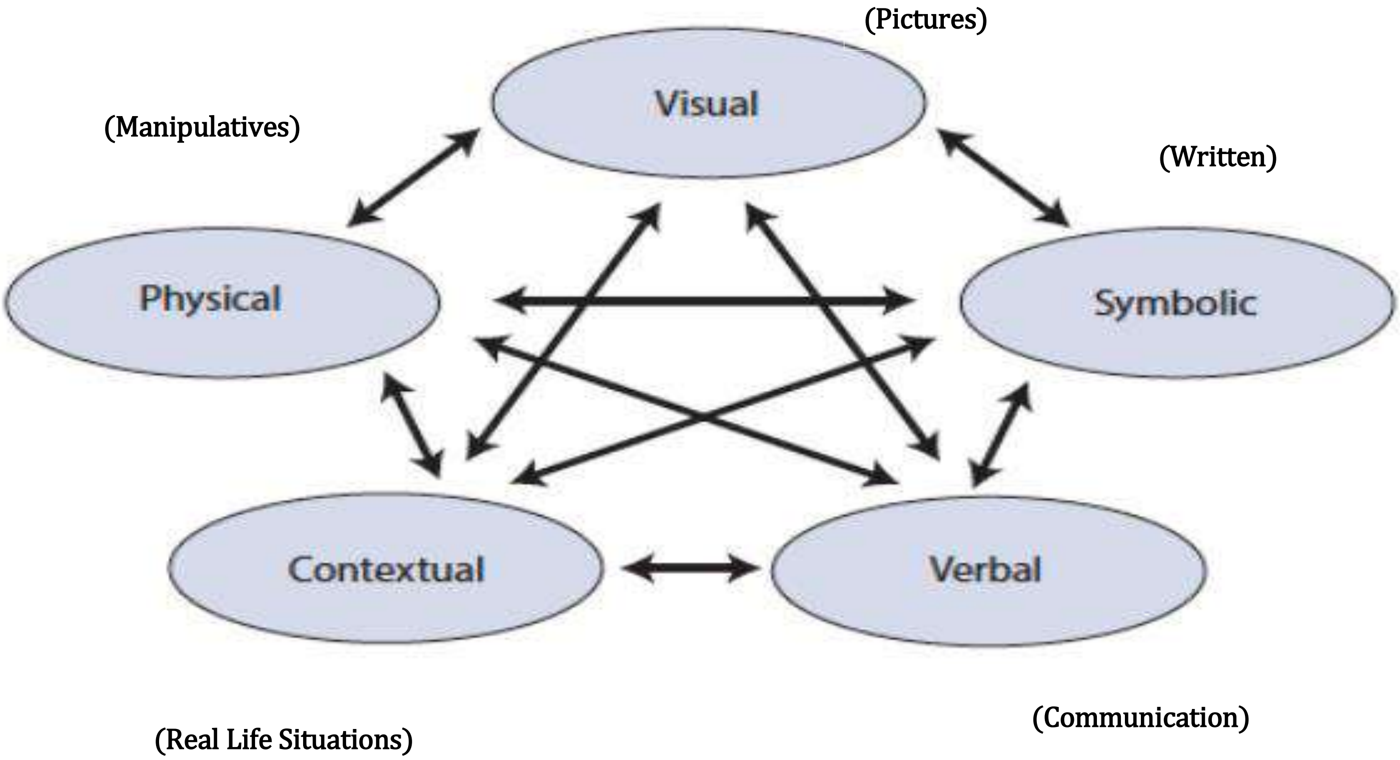
	Evidence Statement	Clarification	Math Practices
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	i) See the OA Progression document, especially p. 29 and Table 2, Common Multiplication and Division situations on page 89 of NJSLS. ii) Tasks sample equally the situations in the third row of Table 2 on page 89 of NJSLS.	MP.2, MP.4
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	i) Tasks do not require students to determine a rule; the rule is given. ii) 75% of patterns should be number patterns.	MP.8
4.NF.1a-b	1 Apply conceptual understanding of fraction equivalence and ordering to solve simple word problems requiring fraction comparison.	i) Tasks have "thin context." ii) Tasks do not require adding, subtracting, multiplying, or dividing fractions. iii) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. iv) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100	MP.1, MP.4, MP.5
4.NF.3.a	Understand a fraction a b/ with a > 1 as a sum of fractions 1/ b . a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	i) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP.2, MP.7, MP.8
4.NF.3.b	Understand a fraction a/b with a > 1 as a sum of fractions 1/ b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.	i) Only the answer is required (methods, representation, etc. are not assessed here). ii) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (NJSLS footnote, p. 30). iii) Tasks may include fractions that equal whole numbers.	MP.7, MP.8
4.NF.3.c	Understand a fraction a b/ with a >1 as a sum of fractions 1/ b. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by	i) Tasks do not have a context. ii) Denominators are limited to grade 3 possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower (NJSLS footnote, p. 24).	MP.8

	using properties of operations and the relationship between addition and subtraction.		
4.NF.4.a	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a /b as a multiple of 1/ b. For example, use a visual fraction model to represent 5/4 as the product 1 5 , 4 × recording the conclusion by the equation 5 1 5 . 4 4	i) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100	MP.5, MP.7
4.NF.4.b.1	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Understand a multiple of a b/ as a multiple of 1/ b.	i) Tasks do not have a context. ii) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. iii) Tasks involve expressing a multiple of a b/ as a fraction. iv) Results may equal fractions greater than 1 (including those equal to whole numbers). v) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100	MP.5, MP.7
4.NF.4.b.2	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Use the understanding that a multiple of a b/ is a multiple of 1/ b to multiply a fraction by a whole number.	i) Tasks do not have a context. ii) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. iii) Tasks involve expressing a multiple of a/b as a fraction. iv) Results may equal fractions greater than 1 (including fractions equal to whole numbers). v) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100	MP.5, MP.7
4.NF.4.c	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	i) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. ii) Situations are limited to those in which the product is unknown (situations do not include those with an unknown factor). iii) Situations involve a whole number of fractional quantities, not a fraction of a whole-number quantity. iv) Results may equal fractions greater than 1 (including fractions equal to whole numbers). v) Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100	MP.1, MP.4, MP.5
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique	i) Tasks do not have a context.	MP.7

	to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 30/100=		
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	i) Measuring to the nearest mm or cm is equivalent to measuring on the number line.	MP.7
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols > =, , or	i) Tasks have “thin context” or no context. ii) Justifying conclusions is not assessed here. iii) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP.5, MP.7
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12) , (2, 24) , and (3, 36) ,…	None	MP.5, MP.8
4.MD.2.1	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, in problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	i) Situations involve whole-number measurements and require expressing measurements given in a larger unit in terms of a smaller unit. ii) Tasks may present number line diagrams featuring a measurement scale. iii) Tasks may include measuring to the nearest cm or mm	MP.4, MP.5
4.MD.2.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, in problems involving simple fractions or decimals. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	i) Situations involve two measurements given in the same units, one a whole-number measurement and the other a non-whole number measurement (given as a fraction or a decimal). ii) Tasks may present number line diagrams featuring a measurement scale. iii) Tasks may include measuring distances to the nearest cm or mm.	MP.4, MP.5
4.MD.4.1	Make a line plot to display a data set of measurements in fractions of a unit (1/2,	None	MP.5

	1/4, 1/8).		
4.MD.4.2	Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	None	MP.4, MP.5

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

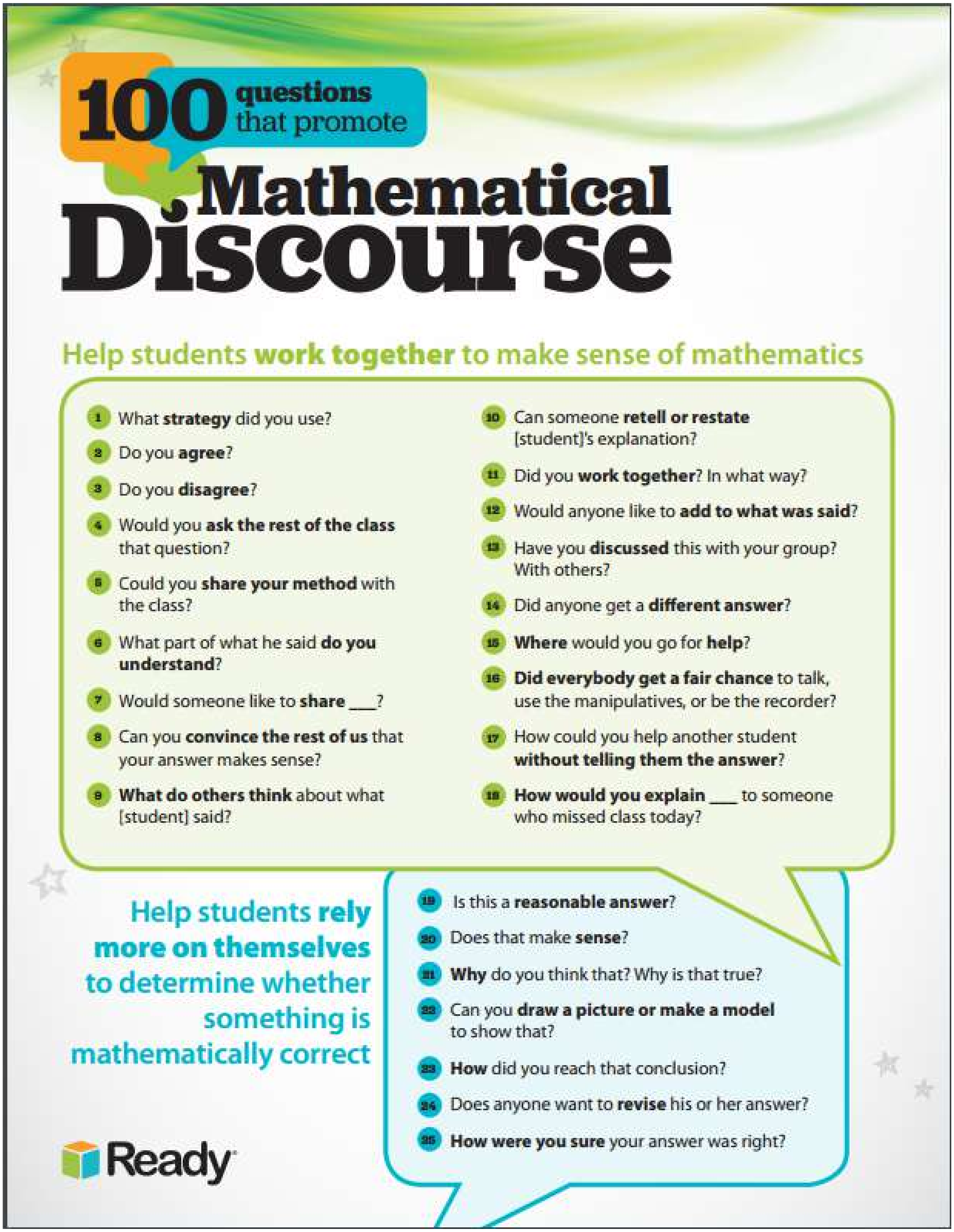
<u>Teacher Questioning:</u>

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

The most important thing is to NEVER stop questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



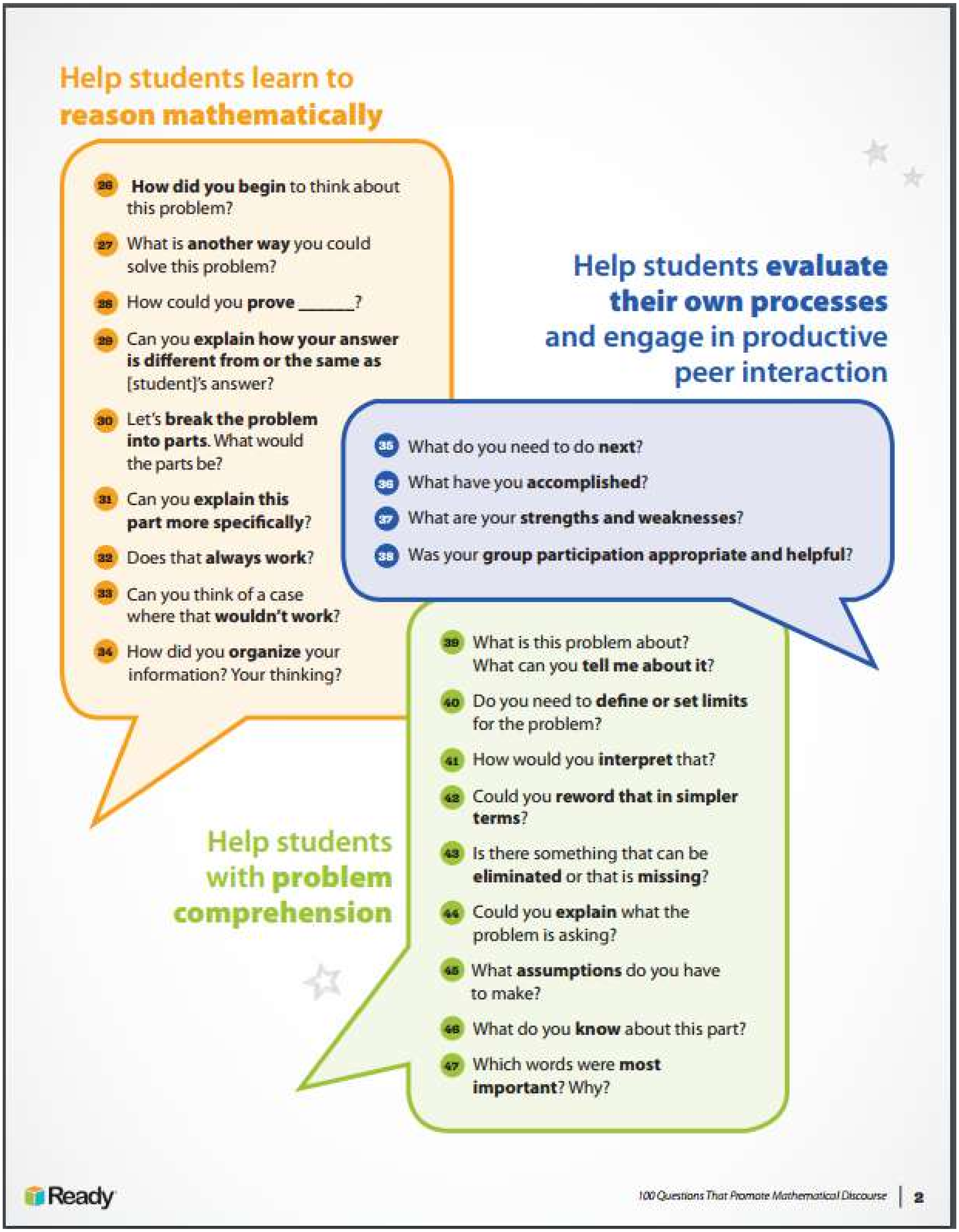
100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?



Help students learn to **reason mathematically**

㉖ **How did you begin** to think about this problem?

㉗ What is **another way** you could solve this problem?

㉘ How could you **prove** ______?

㉙ Can you **explain how your answer is different from or the same as** [student]'s answer?

㉚ Let's **break the problem into parts**. What would the parts be?

㉛ Can you **explain this part more specifically**?

㉜ Does that **always work**?

㉝ Can you think of a case where that **wouldn't work**?

㉞ How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

㉟ What do you need to do **next**?

㊱ What have you **accomplished**?

㊲ What are your **strengths and weaknesses**?

㊳ Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

㊴ What is this problem about? What can you **tell me about it**?

㊵ Do you need to **define or set limits** for the problem?

㊶ How would you **interpret** that?

㊷ Could you **reword that in simpler terms**?

㊸ Is there something that can be **eliminated** or that is **missing**?

㊹ Could you **explain** what the problem is asking?

㊺ What **assumptions** do you have to make?

㊻ What do you **know** about this part?

㊼ Which words were **most important**? Why?

Ready

100 Questions That Promote Mathematical Discourse | 2

Help students learn to **conjecture, invent, and solve** problems

㊽ **What would happen if ___?**

㊾ Do you see a **pattern**?

㊿ What are some **possibilities** here?

(51) Where could you find the **information** you need?

(52) How would you **check your steps** or your answer?

(53) What **did not work**?

(54) How is your solution method the **same as or different from** [student]'s method?

(55) Other than retracing your steps, **how can you determine** if your answers are appropriate?

(56) How did you **organize** the information? Do you have a **record**?

(57) How could you solve this using **tables, lists, pictures, diagrams**, etc.?

(58) What have you tried? What **steps** did you take?

(59) How would it look if you used this **model** or these **materials**?

(60) How would you draw a **diagram or make a sketch** to solve the problem?

(61) Is there **another possible answer**? If so, explain.

(62) Is there **another way to solve** the problem?

(63) Is there **another model** you could use to solve the problem?

(64) Is there anything you've **overlooked**?

(65) **How did you think** about the problem?

(66) What was your **estimate or prediction**?

(67) How **confident** are you in your answer?

(68) **What else** would you like to know?

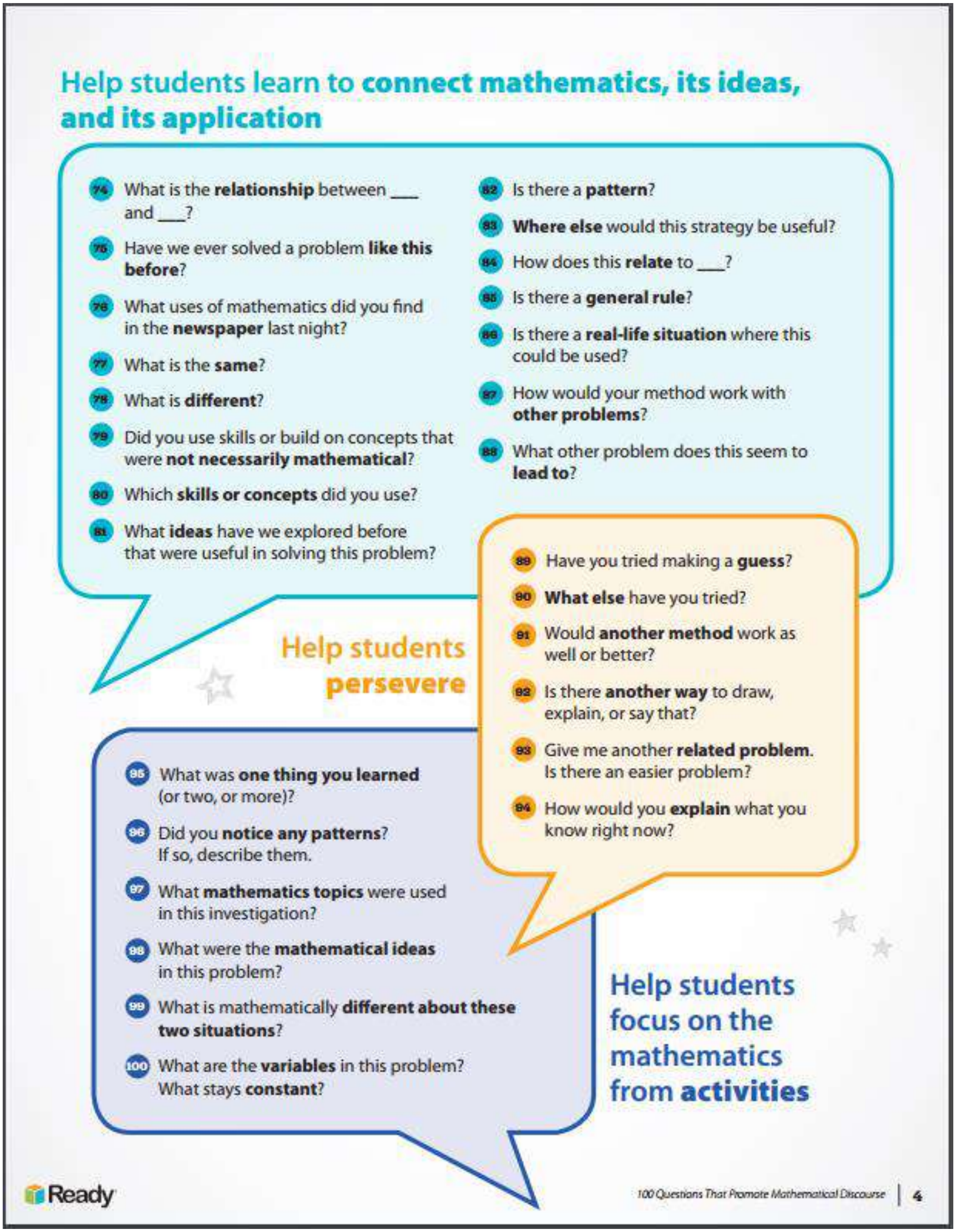
(69) What do you think comes **next**?

(70) Is the solution **reasonable**, considering the context?

(71) Did you have a **system**? Explain it.

(72) Did you have a **strategy**? Explain it.

(73) Did you have a **design**? Explain it.



Help students learn to **connect mathematics, its ideas, and its application**

74. What is the **relationship** between ___ and ___?
75. Have we ever solved a problem **like this before**?
76. What uses of mathematics did you find in the **newspaper** last night?
77. What is the **same**?
78. What is **different**?
79. Did you use skills or build on concepts that were **not necessarily mathematical**?
80. Which **skills or concepts** did you use?
81. What **ideas** have we explored before that were useful in solving this problem?
82. Is there a **pattern**?
83. **Where else** would this strategy be useful?
84. How does this **relate** to ___?
85. Is there a **general rule**?
86. Is there a **real-life situation** where this could be used?
87. How would your method work with **other problems**?
88. What other problem does this seem to **lead to**?

Help students **persevere**

89. Have you tried making a **guess**?
90. **What else** have you tried?
91. Would **another method** work as well or better?
92. Is there **another way** to draw, explain, or say that?
93. Give me another **related problem**. Is there an easier problem?
94. How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95. What was **one thing you learned** (or two, or more)?
96. Did you **notice any patterns**? If so, describe them.
97. What **mathematics topics** were used in this investigation?
98. What were the **mathematical ideas** in this problem?
99. What is mathematically **different about these two situations**?
100. What are the **variables** in this problem? What stays **constant**?

Ready

100 Questions That Promote Mathematical Discourse | 4

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)

3.NBT.A.2: Add/subtract within 1000

4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm

5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

> *Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*
>
> (Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

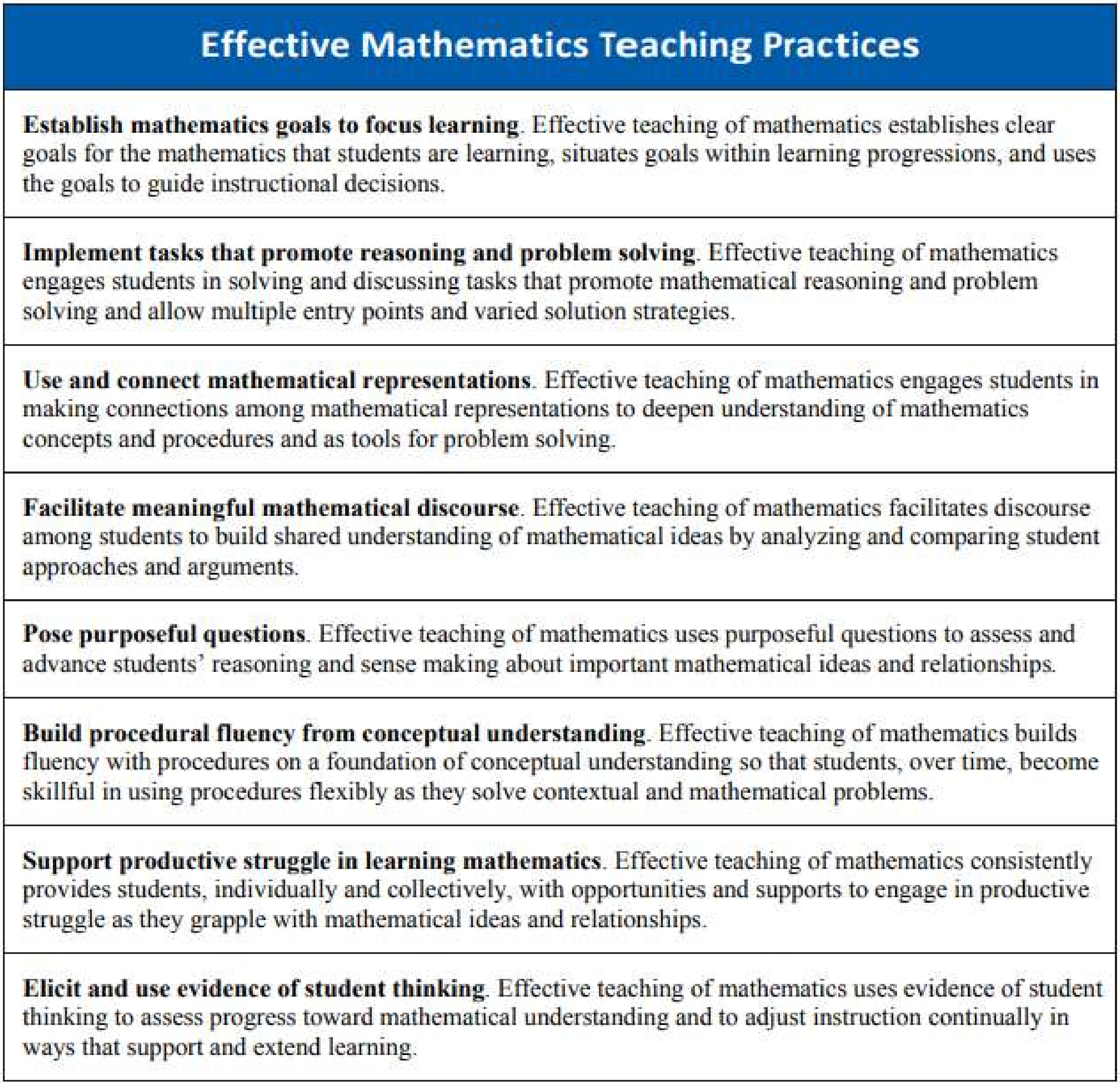
Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them**
	Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively**
	Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others**
	In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

4	**Model with mathematics**
	Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically**
	Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision**
	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure**
	In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning**
	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.



Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discourse

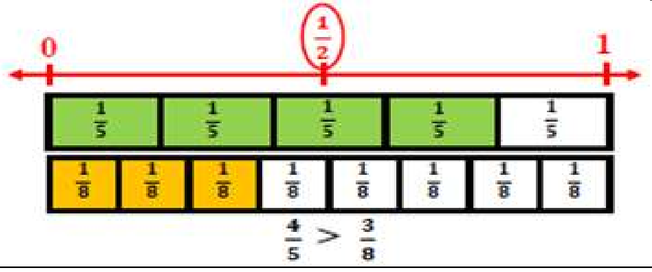
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

Visual Vocabulary

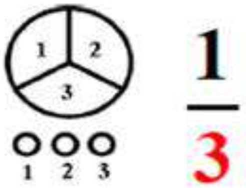
Visual Definition

The terms below are for teacher reference only and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

benchmark fractions



denominator

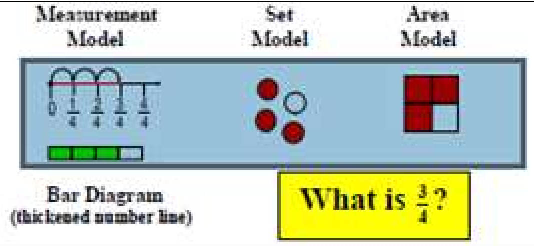


- Equal parts described in fraction
- Equal parts in the whole

equivalent fractions

Fractions that have the same value.

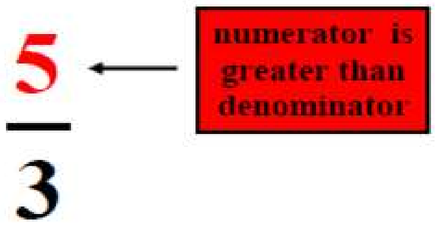
fraction



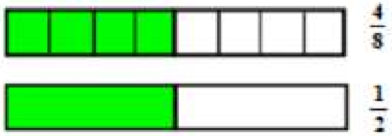
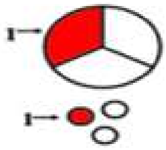
fraction bar

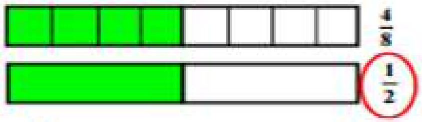
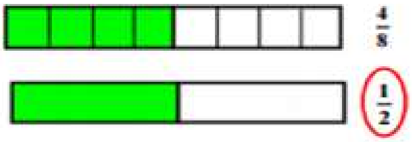
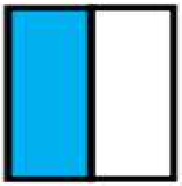
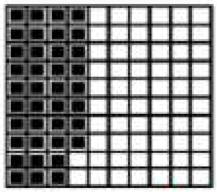
A horizontal bar that separates the numerator and the denominator.

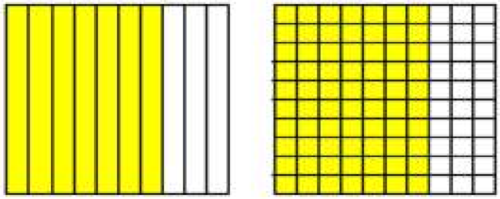
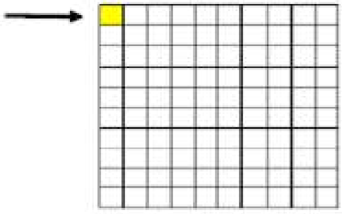
fraction greater than one

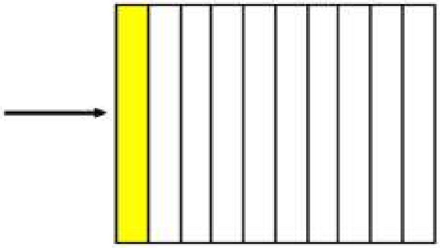
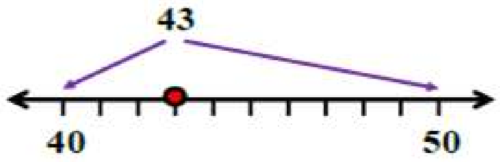


numerator is greater than denominator

fraction less than one	$\frac{3}{5}$ ← numerator is less than denominator
like denominators	$\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$
like numerators	$\frac{3}{4}$ $\frac{3}{5}$ $\frac{3}{8}$
lowest terms	 $\frac{4}{8}$ in lowest terms is $\frac{1}{2}$.
mixed number	$3\frac{3}{7}$ 
numerator	 $\frac{1}{3}$ • Equal parts described in fraction • Equal parts in the whole
order	$\frac{2}{8}$ $\frac{2}{6}$ $\frac{2}{4}$ In order from least to greatest.

simplest form	 $\frac{4}{8}$ in simplest form is $\frac{1}{2}$.
simplify	
unit fraction	$\frac{1}{2}$ 
unlike denominators	$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$
unlike numerators	$\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$
decimal	$29.45 53.0 0.02
decimal fraction	 $0.38 = \frac{38}{100}$

decimal point	$1.55 3.2 ↑ ↑ decimal point
equivalent decimals	 0.7 = 0.70
greater than	 5 > 3
hundredth	 One of the equal parts when a whole is divided into 100 equal parts.
hundredths	4.38 In the decimal numeration system, hundredths is the name of the next place to the right of tenths.
order	$\frac{2}{8}$ $\frac{2}{6}$ $\frac{2}{4}$ In order from least to greatest.

tenth		One of the equal parts when a whole is divided into 10 equal parts.
tenths		In the decimal numeration, tenths is the name of the place to the right of the decimal point.
round a whole number		

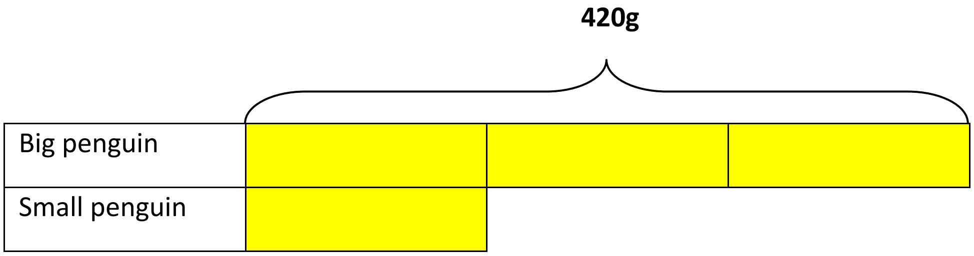
Teaching Multiple Representations

Multiple Representations Framework

<table>
<tr>
<td>Multiplicative Comparison
What factor would multiply
Multiplicative Comparison
<i>What factor would multiply one quantity in order to result in the other?</i></td>
<td>

A tape diagram used to solve a Compare problem

A big penguin will eat 3 times as much fish as a small penguin. The big penguin will eat 420 grams of fish. All together, how much will the two penguins eat?



B=number of grams the big penguin eats

S=number of grams the small penguin eats

3 x S=B

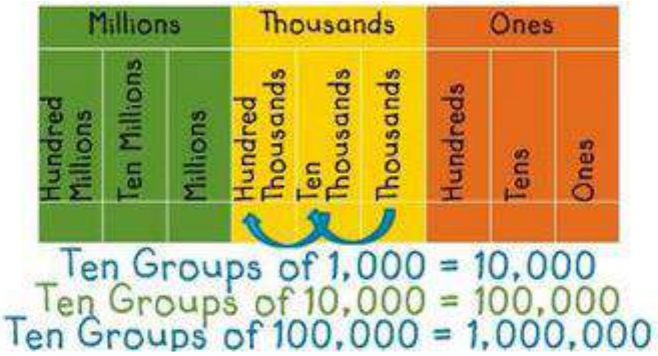
3 x S=420

S=140

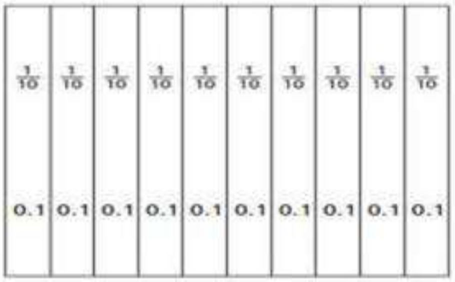
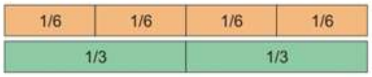
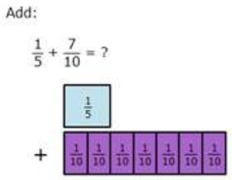
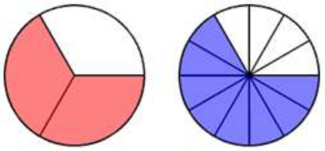
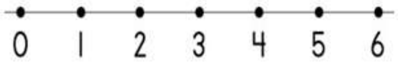
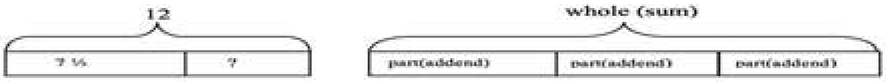
S + B=140 + 420

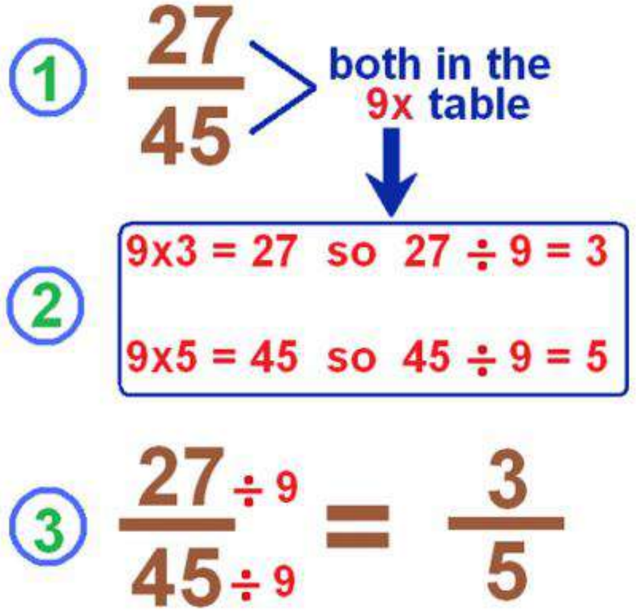
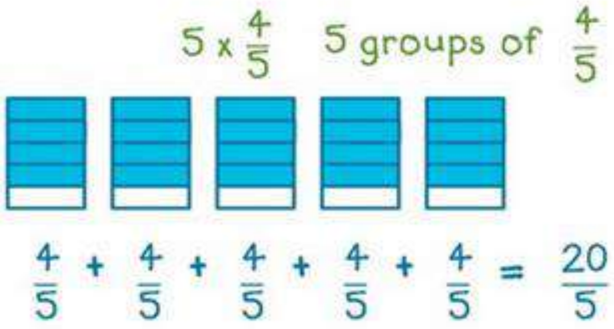
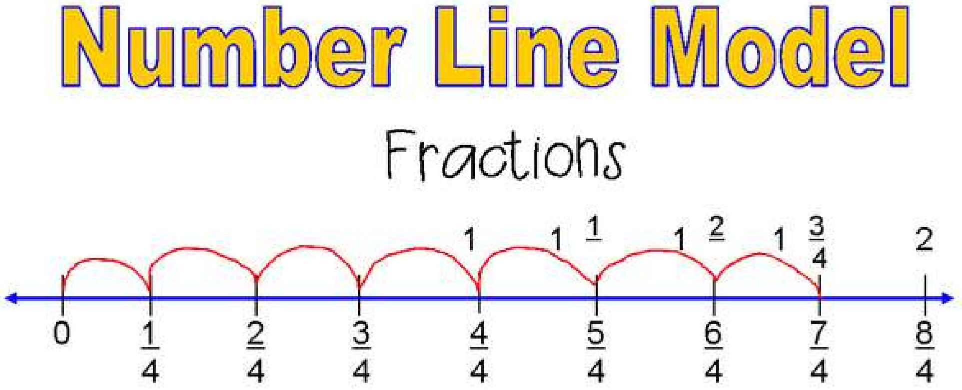
=560

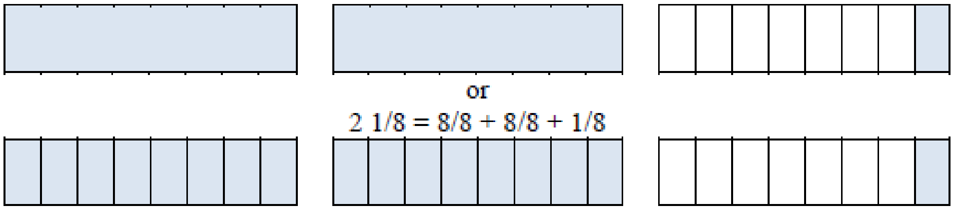
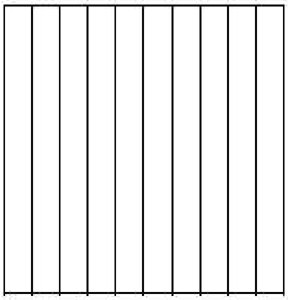
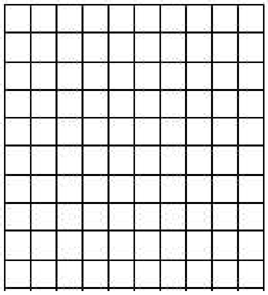
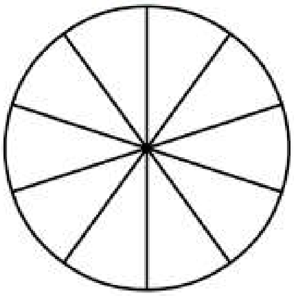
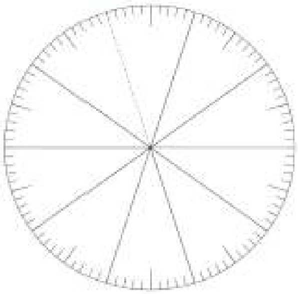
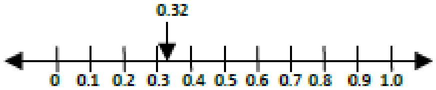
</td>
</tr>
<tr>
<td>Understanding a digit in one place represents ten times what it represents in the place to its right.</td>
<td>



</td>
</tr>
</table>

Fractional Strips	
Equal Partitioning and Unitizing *Using Visual Fraction Models* • Fraction Strips • Fraction Circles • Number line	   
Bar Model 	Leticia read 7 ½ books for the read-a-thon. She wants to read 12 books in all. How many more books does she have to read?  12-7 ½=? Or 7 ½ + ?=12 so Leticia needs to read 4 ½ more books.
Equivalent Fractions	

Simplifying Fractions	
Benchmark Fractions	1/2, 1/3, 1/4, 1/5, 1/6. 1/8, 1/10
Multiply Fraction by a Whole Number	
Multiply Fraction by a Whole Number	 What is 1/4 x 7? 1/4 x 7 = 7/4 or 1 3/4

<table>
<tr><td>Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an Equation.</td><td>
2 1/8 = 1 + 1 + 1/8
or
2 1/8 = 8/8 + 8/8 + 1/8</td></tr>
<tr><td>Represent a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions
with respective denominators 10 and
100.</td><td>Example:<table><tr><td>Ones</td><td>.</td><td>Tenths</td><td>Hundredths</td></tr></table>Tenths Grid

.3 = 3 tenths = 3/10
Hundredths Grid

.30 = 30 hundredths = 30/100
Example:
Represent 3 tenths and 30 hundredths on the models below.
10th's circle

100th's circle
</td></tr>
<tr><td>Decimal notation for fractions with denominators 10 or 100.
Place Value Chart
Number line</td><td><table><tr><td>Hundreds</td><td>Tens</td><td>Ones</td><td>•</td><td>Tenths</td><td>Hundredths</td></tr><tr><td></td><td></td><td></td><td>•</td><td>3</td><td>2</td></tr></table> </td></tr>
</table>

Assessment Framework

Unit 1 Assessment/Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Chapter 6			
Optional Chapter Test 6/Performance Assessment	4.NF.1, 4.NF.3a-d, 4.NF.4a-c, 4.MD.1, 4.MD.4. 4.OA.2	1/2 Block	Individual
Portfolio/Authentic Assessment Chocolate Bar Fractions	4.NF.2, 4.NF.4	1/2 Block	Individual
Authentic Assessment Cynthia's Perfect Punch	4.NF.b.3.c	30 minutes	Individual
Eureka Math Module 5: Fractions Equivalence, Ordering, and Operations (TOPICS B, C, D)			
Portfolio/Authentic Assessment Raising Money	4.NF.1, 4.NF.2	1/2 Block	Individual
Optional Mid-Module Assessment	4.NF.1, 4.NF.3b, 4.NF.2, 4.NF.3ad, 4.MD.2	1 Block	Individual
Chapter 7			
Optional Chapter Test 7/Performance Task	4.NBT.1-2, 4.NBT.4, 4.NF.5, 4.MD1-2	1/2 Block	Individual
Grade 4 Interim Assessment 2 (i-Ready)	4.OA.2, 4.OA.5, 4.NF.1, 4.NF.3a-d, 4.NF.4a-c, 4.NF.5, 4.NF.6, 4.NF.7, 4.MD.1, 4.MD.2, 4.MD.4	1-2 Blocks	Individual

	PLD	**Genesis Conversion**
Rubric Scoring	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

Name:____

A. John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy? Use words, a model, or an equation to justify your answer.

B. William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? Use a model and words to justify your answer.

Chocolate Bar Fractions

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify conclusions, e.g. by using a visual fraction model

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

Mathematical Practice: 1,3,4,6,and 7

SOLUTION:

A. **9 candy bars**
B. **William is correct because 4/4 is equivalent to 2/2**

Level 5: Distinguished Command
Student correctly multiplies 13 by 2/3 to arrive at a correct product of 26/3. Student correctly interprets 26/3 as 8 and 2/3 and recognizes that John requires 9 candy bars to share among his friends. Student uses an appropriate strategy such as a number line, visual fraction model, or algorithm to multiply the fraction by the whole number and explains that John needs 9 candy bars because he cannot buy 8 and 2/3 candy bars. Student correctly identifies that 4/4 and 2/2 are equivalent and states that William is correct. Student includes a model and justifies answer using reasoning such as: • 4 pieces that are each ¼ make one whole and 2 pieces that are each ½ make one whole because in both cases we have all of the pieces or one whole, so 4/4=2/2 • 2/2 x 2/2 equals 4/4 or 4/4 x (½)/(½) equals 2/2 • If you have 2 pieces that are each a half and you cut the two halves into two equal pieces you get fourths. Since both halves belonged to you because you had 2/2, now you have 4/4, or the same amount. Student makes sense of the problem and applies knowledge of fractions to provide an accurate solution. Student uses clear language to communicate written responses. In written explanations, student refers to labels, quantities, and units precisely such as referring correctly to units as either chocolate bars in part 1 or pounds of chocolate in part 2. Models including number lines or area models are appropriate, clearly reflecting the problem situation. The student supports his/her responses with logical and appropriate reasoning.

Level 4: Strong Command
Student correctly multiplies 13 by 2/3, with a correct product of 26/3. Student may not interpret 26/3 as 8 and 2/3 or may not recognize that John requires 9 candy bars to share among his friends. Student uses an appropriate strategy such as a number line, visual fraction model, or algorithm to multiply the fraction by the whole number. Student correctly identifies that 4/4 and 2/2 are equivalent. Student explains answer in words or uses a diagram such as a number line, area model, or an equation. Reasoning is generally correct, though explanation may be limited. Student makes sense of the problem and applies knowledge of fractions and operations to provide an accurate solution. Student uses clear language to communicate written responses. In written explanations, student refers to labels, quantities, and units. Models are appropriate, reflecting the problem situation. The student supports his/her responses with reasoning.

Level 3: Moderate Command
Student attempts to multiply 13 by 2/3, with an incorrect product or a number line, visual fraction model, or algorithm that indicates a conceptual error. Student may add 2/3 repeatedly or try to partition 13 into 3 equal groups, with limited success. Student is unable to identify either the number of candy bars that John intends to distribute (8 and 2/3) or the number he needs to buy (9). Student attempts to explain why fractions are/are not equivalent using an appropriate strategy, but may incorrectly multiply ½ by 2 or ¼ by 4. Student communicates an incomplete argument with unclear reference to quantities, units, and labels. The student may generally describe fractional equivalence. Student may apply an algorithm inappropriately or with limited evidence of understanding.

Level 2: Partial Command
Student attempts to solve the problem, but work demonstrates major conceptual flaws. Student provides very limited evidence of understanding the operations required to solve the problem such as being unable to generate the correct weight of the chocolate or demonstrate fractional equivalence. Work may include an answer such as "William" or "Mary" with no work or justification or an incorrect justification that indicates a major conceptual error.

Level 1: No Command
The student shows no work or justification.

Name:______________________________________

Cynthia is making her famous "Perfect Punch" for a party. After looking through the recipe, Cynthia knows that she needs to mix $4\frac{5}{8}$ gallons of fruit juice concentrate with $3\frac{7}{8}$ gallons of sparkling water.

a. Just as she is about to get started she realizes that she only has one 10-gallon container to use for mixing. Will this container be big enough to hold all the ingredients?

b. How much punch will this recipe make?

Cynthia’s Perfect Punch

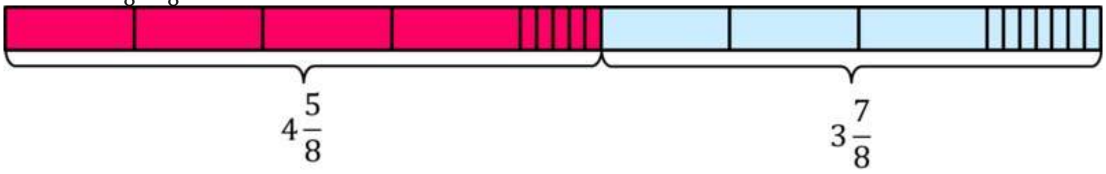
4.NF.B.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Mathematical Practice: 1 and 6

SOLUTION:

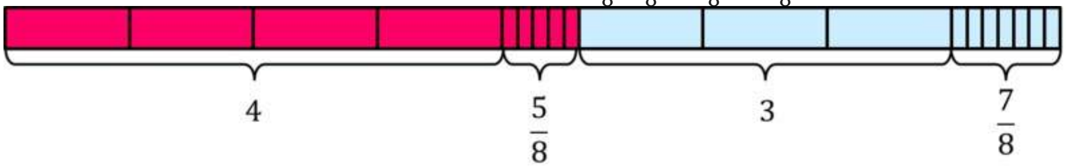
The container is large enough to hold all of the ingredients. Perhaps the easiest way to see this is by observing that $4\frac{5}{8}$is less than 5 and $3\frac{7}{8}$ is less than 4, so $4\frac{5}{8}+3\frac{7}{8}$ is less than 9. Since there are less than 9 gallons of ingredients altogether they will certainly all fit in a 10-gallon container.

To see how much total punch is made we need to add the amount of lemon lime soda to the amount of fruit juice. The picture below represents $4\frac{5}{8}+3\frac{7}{8}$



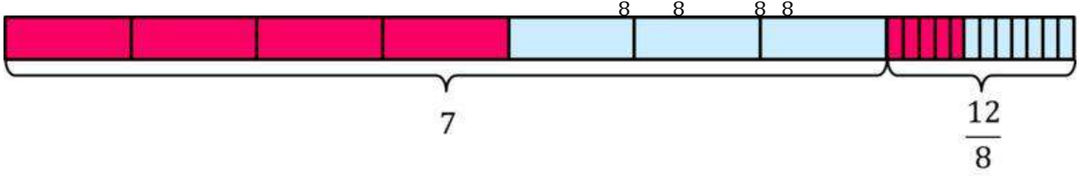
We can write the mixed numbers as a sum of a whole number and a fraction.

$$4\frac{5}{8}+3\frac{7}{8}=(4+\frac{5}{8})+(3+\frac{7}{8})$$



Since addition is commutative and associative, we can add the numbers in any order we wish. Let's add the whole numbers together and the fractions together.

$$4+\frac{5}{8}+3+\frac{7}{8}=4+3+\frac{5}{8}+\frac{7}{8}$$

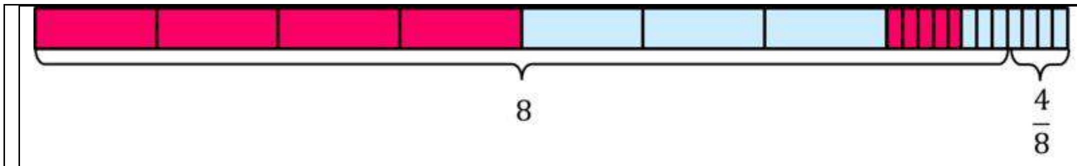


$$4+3+\frac{5}{8}+\frac{7}{8}=7+5+\frac{7}{8}=7+\frac{12}{8}$$

Next we can re-write $\frac{12}{8}$ as a mixed number...

$$7+\frac{12}{8}=7+\frac{8+4}{8}=7+\frac{8}{8}+\frac{4}{8}=7+1+\frac{4}{8}$$

and add the whole numbers once again.



$$7+1+\frac{4}{8}=8+\frac{4}{8}$$

Since $\frac{4}{8}=\frac{1}{2}$, we can write the sum as $8\frac{1}{2}$. So we see that this recipe makes $8\frac{1}{2}$ gallons of punch.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Student correctly answers both questions and clearly constructs and communicates a complete response based on explanations/reasoning using : • Properties of Operations • Relationship between addition and subtraction • Equivalent Fractions using mixed numbers Response includes an **efficient** and logical progression of steps.	Student correctly answers both questions and clearly constructs and communicates a complete response based on explanations/reasoning using : • Properties of Operations • Relationship between addition and subtraction • Equivalent Fractions using mixed numbers Response includes a **logical** progression of steps.	Student correctly answers one question and clearly constructs and communicates a complete response based on explanations/reasoning using : • Properties of Operations • Relationship between addition and subtraction • Equivalent Fractions using mixed numbers Response includes a **logical but incomplete** progression of steps.	Student correctly answers one question and clearly constructs and communicates a complete response based on explanations/reasoning using : • Properties of Operations • Relationship between addition and subtraction • Equivalent Fractions using mixed numbers Response includes a il**logical or incomplete** progression of steps.	The student shows no work or justification

Name:____

Raising Money

The bicycle, track, and band clubs are all trying to raise money for new uniforms. The principal wants to make sure all the clubs get an equal amount of money from the school. The bicycle club will get a total of $\frac{2}{5}$ of the money. The track club will get $\frac{4}{10}$ and the band club will get $\frac{30}{100}$. Did the principal share the money equally among all three clubs? Explain or show your thinking.

4.NF.1: Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Mathematical Practice: 1, 3, 6

Type: Individual or Individual w/Interview

SOLUTION: **See below**				
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Students used representation to find equivalents fractional benchmarks. Students were able to use benchmarks to help estimate the size of the number and compare fractions to see if they were equal. Students were able to develop and use benchmarks that relates to different forms of representation of rational numbers (for example, 25 out of 100 is the same as ¼). By doing so, students were able to determine that two out of the three fractions were equal and 30/100 would give the band club less money. Students showed their work and gave a clear explanation of the answer to their problem.		Students did not use benchmarks to solve the problem, however, they were able to determine that two out of the three fractions were equal and 30/100 would give the band club less money. Students showed their work and gave a clear explanation of the answer to their problem.	Students attempted to compare the fractions using representation; however, their answer did not come up with the correct solution. An understanding of using benchmark fractions was not evident in their work.	Does not address task, unresponsive, unrelated or inappropriate.

Response includes an **efficient** and logical progression of steps. Compares fractions, with like or unlike numerators and denominators, by creating equivalent fractions with common denominators, comparing to a benchmark fraction and generating equivalent fractions Demonstrates the use of conceptual understanding of fractional equivalence and ordering when solving simple word problems requiring fraction comparison.	Response includes a **logical** progression of steps Compares fractions, with like or unlike numerators and denominators, by creating equivalent fractions with common denominators, comparing to a benchmark fraction and generating equivalent fractions Demonstrates the use of conceptual understanding of fractional equivalence and ordering when solving simple word problems requiring fraction comparison.	Response includes a **logical but incomplete** progression of steps. Minor calculation errors. Given a visual model and/or manipulatives, compares fractions, with like or unlike numerators and denominators, by **creating equivalent fractions with common denominators** and comparing to a benchmark fraction.	Response includes an **incomplete or Illogical** progression of steps. Given a visual model and/or manipulatives, compares fractions, with like or unlike numerators and denominators, by **creating equivalent fractions with common denominators** and comparing to a benchmark fraction.	The student shows no work or justification

Additional Assessment Resources

Literature

Literature Fractions and Decimals Made Easy, by Rebecca Wingard-Nelson Fun Food Word Problems Starring Fractions, by Rebecca Wingard-Nelson
The Hershey's Milk Chocolate Fractions Book, by Jerry Pallotta
Jump, Kangaroo, Jump!, by Stuart J. Murphy Polar Bear
Math: Learning About Fractions from Klondike and Snow, by Ann Whitehead Nagda
The Wishing Club: A Story About Fractions, by Donna Jo Napoli
Working With Fractions, by David A. Adler

Project Ideas:
Doubling A Recipe
Mozaic Art

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .