

Readington Township Public Schools

Grade 4 Advanced Math

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Readington Township Public Schools

www.readington.k12.nj.us

I. OVERVIEW

The Mathematics 4 ACM course is designed to teach students grade level mathematics in addition to half of the fifth grade curriculum while promoting higher order thinking skills. The course is directly correlated to the Common Core Standards for both grades and covers such topics as number sense, geometry, measurement, number operations in base ten and fractions, and algebraic thinking. The course also promotes and instills the skills of problem solving, communication in mathematics, and making mathematical connections. Students will utilize various tools and technology in the process, including manipulatives, calculators, websites, and computers to better enhance a well-rounded understanding of course topics. A strong focus of the program is on promoting high levels of mathematical thought through experiences which extend beyond traditional computation. Students will use websites such as Reflex Math and IXL.

II. STUDENT OUTCOMES (Linked to New Jersey Core Curriculum Standards)

Operations & Algebraic Thinking

All students will generalize place value understanding for multi-digit whole numbers and use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations in Base Ten

All students will generalize place value understanding for multi-digit whole numbers and use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

All students will extend understanding of fraction equivalence and ordering, build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers, and understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

All students will solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, represent and interpret data, and understand concepts of angles and measuring angles.

Geometry

All students will draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Students will:

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS.MATH.CONTENT.4.OA.A.1)
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS.MATH.CONTENT.4.OA.A.2)
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the

reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS.MATH.CONTENT.4.OA.A.3)

Gain familiarity with factors and multiples.

1. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. (CCSS.MATH.CONTENT.4.OA.B.4)

Generate and analyze patterns.

1. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (CCSS.MATH.CONTENT.4.OA.C.5)

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division. (CCSS.MATH.CONTENT.4.NBT.A.1)
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS.MATH.CONTENT.4.NBT.A.2)
3. Use place value understanding to round multi-digit whole numbers to any place. (CCSS.MATH.CONTENT.4.NBT.A.3)

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS.MATH.CONTENT.4.NBT.B.4)
2. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS.MATH.CONTENT.4.NBT.B.5)
3. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS.MATH.CONTENT.4.NBT.B.6)

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS.MATH.CONTENT.4.NF.A.1)
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS.MATH.CONTENT.4.NF.A.2)

Build fractions from unit fractions.

1. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. (CCSS.MATH.CONTENT.4.NF.B.3)
 - a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS.MATH.CONTENT.4.NF.B.3.A)
 - b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$. (CCSS.MATH.CONTENT.4.NF.B.3.B)
 - c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS.MATH.CONTENT.4.NF.B.3.C)
 - d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS.MATH.CONTENT.4.NF.B.3.D)
2. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
 - a. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. (CCSS.MATH.CONTENT.4.NF.B.4.A) (CCSS.MATH.CONTENT.4.NF.B.4)
 - b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.) (CCSS.MATH.CONTENT.4.NF.B.4.B)
 - c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? (CCSS.MATH.CONTENT.4.NF.B.4.C)

Understand decimal notation for fractions, and compare decimal fractions.

1. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (CCSS.MATH.CONTENT.4.NF.C.5)
2. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (CCSS.MATH.CONTENT.4.NF.C.6)
3. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model. (CCSS.MATH.CONTENT.4.NF.C.7)

Solve problems involving measurement and conversion of measurements.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS.MATH.CONTENT.4.MD.A.1)

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS.MATH.CONTENT.4.MD.A.2)
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS.MATH.CONTENT.4.MD.A.3)

Represent and interpret data.

1. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. (CCSS.MATH.CONTENT.4.MD.B.4)

Geometric measurement: understand concepts of angle and measure angles.

1. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (CCSS.MATH.CONTENT.4.MD.C.5)
2. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS.MATH.CONTENT.4.MD.C.5.A)
3. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS.MATH.CONTENT.4.MD.C.5.B)
4. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS.MATH.CONTENT.4.MD.C.6)
5. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS.MATH.CONTENT.4.MD.C.7)

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS.MATH.CONTENT.4.G.A.1)
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS.MATH.CONTENT.4.G.A.2)
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS.MATH.CONTENT.4.G.A.3)

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. (CCSS.MATH.CONTENT.5.NBT.A.1)
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided

by a power of 10. Use whole-number exponents to denote powers of 10. (CCSS.MATH.CONTENT.5.NBT.A.2)

3. Read, write, and compare decimals to thousandths. (CCSS.MATH.CONTENT.5.NBT.A.3)
4. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. (CCSS.MATH.CONTENT.5.NBT.A.3.A)
5. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS.MATH.CONTENT.5.NBT.A.3.B)
6. Use place value understanding to round decimals to any place. (CCSS.MATH.CONTENT.5.NBT.A.4)

Perform operations with multi-digit whole numbers and with decimals to hundredths.

1. Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS.MATH.CONTENT.5.NBT.B.5)
2. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS.MATH.CONTENT.5.NBT.B.7)

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.) (CCSS.MATH.CONTENT.5.NF.A.1)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$. (CCSS.MATH.CONTENT.5.NF.A.2)

Apply and extend previous understandings of multiplication and division.

1. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS.MATH.CONTENT.5.NF.B.3)
2. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS.MATH.CONTENT.5.NF.B.4)
 - a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.) (CCSS.MATH.CONTENT.5.NF.B.4.A)
 - b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS.MATH.CONTENT.5.NF.B.4.B)
3. Interpret multiplication as scaling (resizing), by: (CCSS.MATH.CONTENT.5.NF.B.5)
 - a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS.MATH.CONTENT.5.NF.B.5.A)

- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1. (CCSS.MATH.CONTENT.5.NF.B.5.B)
- 4. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS.MATH.CONTENT.5.NF.B.6)
- 5. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1 (CCSS.MATH.CONTENT.5.NF.B.7)
 - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(\frac{1}{3}) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$. (CCSS.MATH.CONTENT.5.NF.B.7.A)
 - b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (\frac{1}{5})$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$ because $20 \times (\frac{1}{5}) = 4$. (CCSS.MATH.CONTENT.5.NF.B.7.B)
 - c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$ -cup servings are in 2 cups of raisins? (CCSS.MATH.CONTENT.5.NF.B.7.C)
 - d.

Convert like measurement units within a given measurement system.

- 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (CCSS.MATH.CONTENT.5.MD.A.1)

Represent and interpret data.

- 1. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS.MATH.CONTENT.5.MD.B.2)

IV. ESSENTIAL QUESTIONS AND CONTENT

Grade 4

Unit 1 Place Value and Multi-digit Addition and Subtraction

How can I use place value to understand very large numbers?

What strategies and understandings allow me to successfully add and subtract very large numbers?

Unit 2 Multiplication with Whole Numbers

How do arrays of ones, tens and hundreds help to understand with multiplication?

What is the process for multiplying one digit by two- and three-digit numbers?

Unit 3 Division with Whole Numbers

How are multiplication and division related?

What happens when a quotient isn't a whole number?

Are there different strategies for solving division problems?

Can factors or prime numbers help me understand division?

Unit 4 Equations and Word Problems

How are equations and expressions related to what I know about math and the real world?

How can I decide which operation to use when I'm faced with a word problem?

If I see a number pattern, can I write an equation or expression to represent it?

Unit 5 Measurement

What are the different ways to measure things?

How do I convert a smaller unit of measurement to a larger one?

Unit 6 Fraction Concepts and Operations

What are fractions, and how can I compare them?

How do I perform mathematical operations with fractions?

Unit 7 Fractions and Decimals

How do decimals relate to fractions?

Unit 8 Geometry

How can I create and measure angles?

What do angles and lines tell me about two-dimensional shapes?

Grade 5**Unit 1 Addition and Subtraction with Fractions**

What strategies can be used to compare fractions?

How can I add or subtract fractions and mixed numbers?

Unit 2 Addition and Subtraction with Decimals

What is the correct way to read and write decimals and fractions?

What is the procedure for adding and subtracting decimals?

How do I use what I know about fractions and decimals to solve problems?

Unit 3 Multiplication and Division with Fractions

Can I multiply and divide fractions just like whole numbers?

How can I use multiplication or division of fractions to solve real-world problems?

Unit 4 Multiplication with Whole Numbers and Decimals

How is multiplying with decimals related to multiplying whole numbers?

V. STRATEGIES

- Group discussions
- Teacher presentation
- Student projects
- Guided groups
- One to one instruction
- Interactive SmartBoard lessons
- Tutorials
- Online practice such as Reflex Math and IXL

VI. EVALUATION

Assessments may include but are not limited to:

Teacher Observations
Class Participation
Class Discussions
Class Assignments
Homework Assignments
Notebooks
Student Projects
Unit Tests and Quizzes
Anecdotal Records
End of year Assessment

VII. REQUIRED RESOURCES

Textbook for course:

Fuson, Karen (2013). *Math Expressions Common Core Grade 4*. Orlando, FL: Houghton Mifflin Harcourt.

Fuson, Karen (2013). *Math Expressions Common Core Grade 5*. Orlando, FL: Houghton Mifflin Harcourt.

VIII. SCOPE AND SEQUENCE

GRADE 4

Unit 1 Place Value and Multidigit Addition and Subtraction (15 days)

- Place Value to One Million
- Addition with Greater Numbers
- Subtraction with Greater Numbers

Unit 2 Multiplication with Whole Numbers (23 days)

- Multiplication with tens and Hundreds
- Multiply by One-Digit Numbers
- Multiplication with Two-Digit Numbers
- Multiplication with Thousands

Unit 3 Division with Whole Numbers (13 days)

- Dividing Whole Numbers
- Division Issues and Word Problems

Unit 4 Equations and Word Problems (16 days)

- Reasoning and Solving Problems
- Comparison Word Problems
- Problems with More than One Step
- Analyzing Patterns

Unit 5 Measurement (10 days)

- Converting Measurements
- Perimeter and Area

Unit 6 Fraction Concepts and Operations (13 days)

- Fractions with Like Denominators
- Mixed Numbers with like Denominators
- Multiply Fractions and Whole Numbers

Unit 7 Fractions and Decimals (16 days)

- Comparing Fractions
- Equivalent Fractions
- Understanding Decimals

Unit 8 Geometry (14 days)

- Measuring and Drawing Angles
- Triangles and Angle Measurements
- Analyzing Quadrilaterals
- Analyzing Polygons

GRADE 5

Unit 1 Addition and Subtraction with Fractions (15 days)

- Equivalent Fractions
- Addition and Subtraction with Fractions

Unit 2 Addition and Subtraction with Decimals (11 days)

- Read and Write Whole Numbers and Decimals
- Addition and subtraction of Whole and Decimal Numbers
- Round and Estimate with Decimals

Unit 3 Multiplication and Division with Fractions (20 days)

- Multiplication with Fractions
- Multiplication Links
- Division with Fractions

Unit 4 Multiplication with Whole Numbers and Decimals (14 days)

- Multiplication with Whole Numbers and Decimals
- Multiplication with Decimal Numbers

Additional time will be spent on reviewing concepts that may need to be revisited and looking ahead to next year's curriculum.