# Readington Township Public Schools

# Algebra 2

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## I. OVERVIEW

This full-year Algebra 2 course is designed to provide advanced 8th grade students with the opportunity to strengthen their skills in the areas of linear, quadratic, and exponential functions, while extending their content base and knowledge to include higher-degree polynomial functions (including now solving these over the set of complex numbers), rational exponents and radical functions, solving exponential and logarithmic equations, rational functions, arithmetic and geometric sequences and series, trigonometric ratios and functions, probability, data analysis and statistics.

In addition to these topics, and in keeping with the Common Core Mathematical Practice Standards, students will experience the course content as an integrated, useful, and coherent whole, continually refining their abilities to model with mathematics, reason abstractly and quantitatively while attending to precision both in calculations and vocabulary, and to make sense of problem situations as an essential part of the solution process.

## II. STUDENT OUTCOMES (Linked to Common Core State Standards/Common Core Mathematics)

*N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

*N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).

N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context. (Linear, exponential, quadratic for Alg.1; Polynomial and rational for Alg. 2)

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

*A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15% .

A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.4 Prove Polynomial identities and use them to describe numerical relationships and solve problems. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

*A.CED.1, 2, 3, 4 Create equations that describe numbers or relationships. (This is an Alg. 1 cluster that limits discussion to linear, quadratic, and exponential functions with integer inputs only. Algebra 2 extends this to equations using all types of expressions, including simple root functions).

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

*A.REI.4 Solve quadratic equations in one variable.

*a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

*b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

A-REI.6 Solve algebraically a system of *three* linear equations in three unknowns.

*A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y=-3x and the circle $x^2 + y^2 = 3$.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF.1 Write a function that describes a relationship between two quantities.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

*F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.4 Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2 x3 or f(x) = (x+1)/(x-1) for $x \neq 1$.

*F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

*F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

*S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

*S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

*S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

*S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

*S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

S-ID.6a-1 Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID.6a, excluding normal distributions and limiting function fitting to exponential functions

S-ID.6a-2 Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course level knowledge and skills articulated in S-ID.6a limiting function fitting to trigonometric functions.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. E.g., a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.6 Evaluate reports based on data.

**S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

**S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

**S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

**S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

**S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

**S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

**S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

**S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

**S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

**S-CP.Int.1 Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-CP.

S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## III ESSENTIAL SKILLS AND CONTENT

**Polynomial, Rational, and Radical Relationships**

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.
- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.
- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.
- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.
- Analyze functions using different representations.

**Trigonometric Functions**

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric function.
- Prove and apply trigonometric identities.

**Modeling with Functions**

- Create equations that describe numbers or relationships.
- Interpret functions that arise in applications in terms of a context.
- Analyze functions using different representations.
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.
- Construct and compare linear, quadratic, and exponential models and solve problems.

**Inferences and Conclusions from Data**

- Summarize, represent, and interpret data on single count or measurement variable.
- Understand and evaluate random processes underlying statistical experiments.

- Make inferences and justify conclusions from sample surveys, experiments and observational studies.
- Use probability to evaluate outcomes of decisions.

## IV. STRATEGIES

Intellectual engagement and active involvement of students in daily lessons will be achieved by a variety of teaching strategies, including:

1. Comprehensive direct instruction utilizing Interactive SmartBoard technology
2. Guided practice using worked-out study examples, followed immediately by
3. Independent Practice using 'Now You Try' examples
4. Student presentation of work and solutions, with explanation and justification of solutions
5. High-level questioning and encouragement of student participation
6. Student group discussion and mutual help as part of point 3 above
7. Regular written 'check for understanding' assessments.
8. End of unit formative assessments.

## V. EVALUATION

Chapter Tests and Quizzes
Mid-Term Exam
Final Exam
Class Participation
Homework Assignments
Individual-lesson and unit-based formative assessments

## VI. REQUIRED RESOURCES

**Textbook for course:**
*McDougal Littell Algebra 2.* Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2004 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

**Supplemental resources for course:**
Chapter Resource Books. Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2001 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

Standardized Test Practice Workbook. Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2001 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

*Big Ideas MATH Algebra 2*, Larson, R., Boswell, L. Copyright 2015 by Big Ideas Learning, LLC.

**Suggested supplemental companion Web sites:**
www.classzone.com [eEdition Plus Online (online textbook)]
www.shmoop.com/common-core-standards/
www.illustrativemathematics.org

## VII. SCOPE AND SEQUENCE

A. Polynomials (Quadratics) (34 days)
   1. Review rewriting equations and formulas
   2. Functions and their graphs
   3. Review quick-sketch graphing techniques of linear equations

4. Piecewise functions
5. Absolute value functions
6. Review solving systems of two equations graphically, and algebraically using elimination and substitution methods
7. Solving systems of linear equations in three variables
8. Graphing quadratic functions
9. Solving quadratic functions by factoring
10. Solving quadratic functions by finding square roots
11. Complex numbers
12. Completing the square
13. The quadratic formula and the discriminant
14. Modeling with quadratic functions
15. Solving quadratic systems (i.e. a linear – quadratic system)
16. Parabolas as conic sections (derive equation of a parabola given a focus and directrix)

**Formative Assessments for PARCC readiness:**
"Average Cost" Domain: F-IF, Resource: www.illustrativemathematics.org/illustrations/387
"Throwing Baseballs" Domain: F-IF, Resource: www.illustrativemathematics.org/illustrations/1279

B. Polynomials (Higher Order Polynomials) (20 days)
1. Evaluating and graphing polynomial functions
2. Using properties of exponents
3. Adding, subtracting, and multiplying polynomials
4. Factoring and solving polynomial equations
5. The Remainder and Factor Theorems and dividing polynomials
6. The Rational Zero Theorem
7. The Fundamental Theorem of Algebra
8. Analyzing graphs of polynomial functions

**Formative Assessments for PARCC readiness:**
"Missing Coefficient" Domain:A-APR, Resource: www.illustrativemathematics.org/illustrations/592
"Seeing Dots" Domain: A-SSE, Resource: www.illustrativemathematics.org/illustrations/21

C. Radical and Rational Functions (19 days)
1. Nth roots and rational exponents
2. Properties of rational exponents
3. Power functions and function operations ( limited to arithmetic operations)
4. Inverse functions
5. Graphing square root and cube root functions
6. Solving radical equations
7. Inverse and joint variation
8. Graphing simple rational functions (Suppl. Required to solve rational eqn's. for the intersection of two fns. using technology – www.shmoop.com/common-core-standards/ccss-hs-a-rei-11.html)
9. Graphing general rational functions
10. Multiplying and dividing rational expressions
11. Adding and subtracting rational expressions and simplifying complex fractions
12. Solving rational equations

**Formative Assessments for PARCC readiness:**
"Basketball" Domain: A-CED, Resource: www.illustrativemathematics.org/illustrations/702
"Radical Equations" Domain: A-REI, Resource: www.illustrativemathematics.org/illustrations/391

D. Exponential and Logarithmic Functions (20 days)
   1. Exponential growth
   2. Exponential decay
   3. The number ***e***
   4. Logarithmic functions
   5. Properties of Logarithms (cover only those needed to solve exp. and log. Equations, i.e. Power Property and Change-of-Base Formula. Cover others if time.)
   6. Solving exponential and logarithmic equations (Suppl. same as C.8 above for solving *systems* of exponential and logarithmic equations)
   7. Modeling with exponential functions (and power functions – time allowing)

**Formative Assessments for PARCC readiness:**
"Ice Cream" Domain: A-SSE, Resource: www.illustrativemathematics.org/illustrations/551
"Basketball Rebounds" Domain: F-LE, Resource: www.illustrativemathematics.org/illustrations/347

E. Arithmetic and Geometric Sequences and Series (12 days)
   1. Introduction to sequences and series
   2. Arithmetic sequences and series
   3. Geometric sequences and series
   4. Infinite geometric series (Time-allowing. Not in PARCC PBA or EOY)
   5. Recursive rules for sequences (11.5)
   6. Mathematical induction (Chpt. 11 Extension)

**Formative Assessments for PARCC readiness:**
"Course of Antibiotics" Domain: A-SSE, Resource: www.illustrativemathematics.org/illustrations/805
"Compounding 5% Interest" Domain: F-LE, Resource: www.illustrativemathematics.org/illustrations/572

F. Probability (13 days)
   1. Introduction to probability (Suppl. vocab.: outcomes, event, sample space, with Big Ideas Algebra 2, BI-10.1)
   2. Probability of compound events
   3. Probability of independent and dependent events (Suppl. with Big Ideas Algebra 2 for 'Two-Way Tables and Probability')
   4. The Fundamental Counting Principle and permutations (Time-allowing)
   5. Combinations and the Binomial Theorem (Time-allowing)
   6. Binomial distributions (Time-allowing)

G. Data Analysis and Statistics (24 days)
   1. Statistics and statistical graphs
   2. Using Normal Distributions (Optional - Suppl. with shmoop.com/precalculus-statistics-probability/z-scores.html)
   3. Populations, Samples, and Hypotheses
   4. Collecting data
   5. Experimental design
   6. Making inferences from sample surveys
   7. Making inferences from experiments

**Formative Assessments for PARCC readiness:**
"Strict Parents" Domain: S-IC, Resource: www.illustrativemathematics.org/illustrations/122

"SAT Scores" Domain: S-ID, Resource: www.illustrativemathematics.org/illustrations/216

H. Trigonometry (18 days)
1. Right triangle trigonometry
2. The unit circle (BI-Chpt.9 Mathematical Practices, p.460)
3. General angles and radian measure
4. Trigonometric functions of any angle
5. Graphing sine, cosine, and tangent functions
6. Translations and reflections of trigonometric graphs (Suppl. with BI-9.4 for 'midline' as vocab., and other points as needed)
7. Verifying trigonometric identities (Suppl. with BI-9.7 unit circle discussion of why $\sin^2 + \cos^2 = 1$, p.514)
8. Solving trigonometric equations
9. Modeling with trigonometric functions
10. Using Sum and Difference formulas (time-allowing)

**Formative Assessments for PARCC readiness:**
"Trigonometric Identities and Rigid Motion" Domain: F-TF, G-CO,
Resource: www.illustrativemathematics.org/illustrations/1698

"Properties of Trigonometric Functions" Domain: F-TF,
Resource: www.illustrativemathematics.org/illustrations/1704

I. **Introduction to Geometry, Logical Reasoning and Proofs (20 days)**
1. Patterns and inductive reasoning (MLG-1.1) 1 (McDougal Littell Geometry 2004)
2. Points, lines, and planes (MLG-1.2) 1
3. Segments and their measure (MLG-1.3) 1
4. Angles and their measure (MLG-1.4) 1
5. Segment and angle bisectors (MLG-1.5) 1
6. Angle pair relationships (MLG-1.6) 1
7. Introduction to perimeter, circumference, and area (MLG-1.7) 1
8. Conditional statements (MLG-2.1) 2
9. Definitions and biconditional statements (MLG-2.2) 2
10. Deductive reasoning (MLG-2.3) 2
11. Reasoning with properties from algebra (MLG-2.4) 2
12. Proving statements about segments (MLG-2.5) 1
13. Proving statements about angles (MLG-2.6) 1
14. Indirect proofs (MLG-5.6) 2
15. Logical reasoning (McDougal Littell Alg. 2 'Skills Review Handbook' pp. 924-929) 2